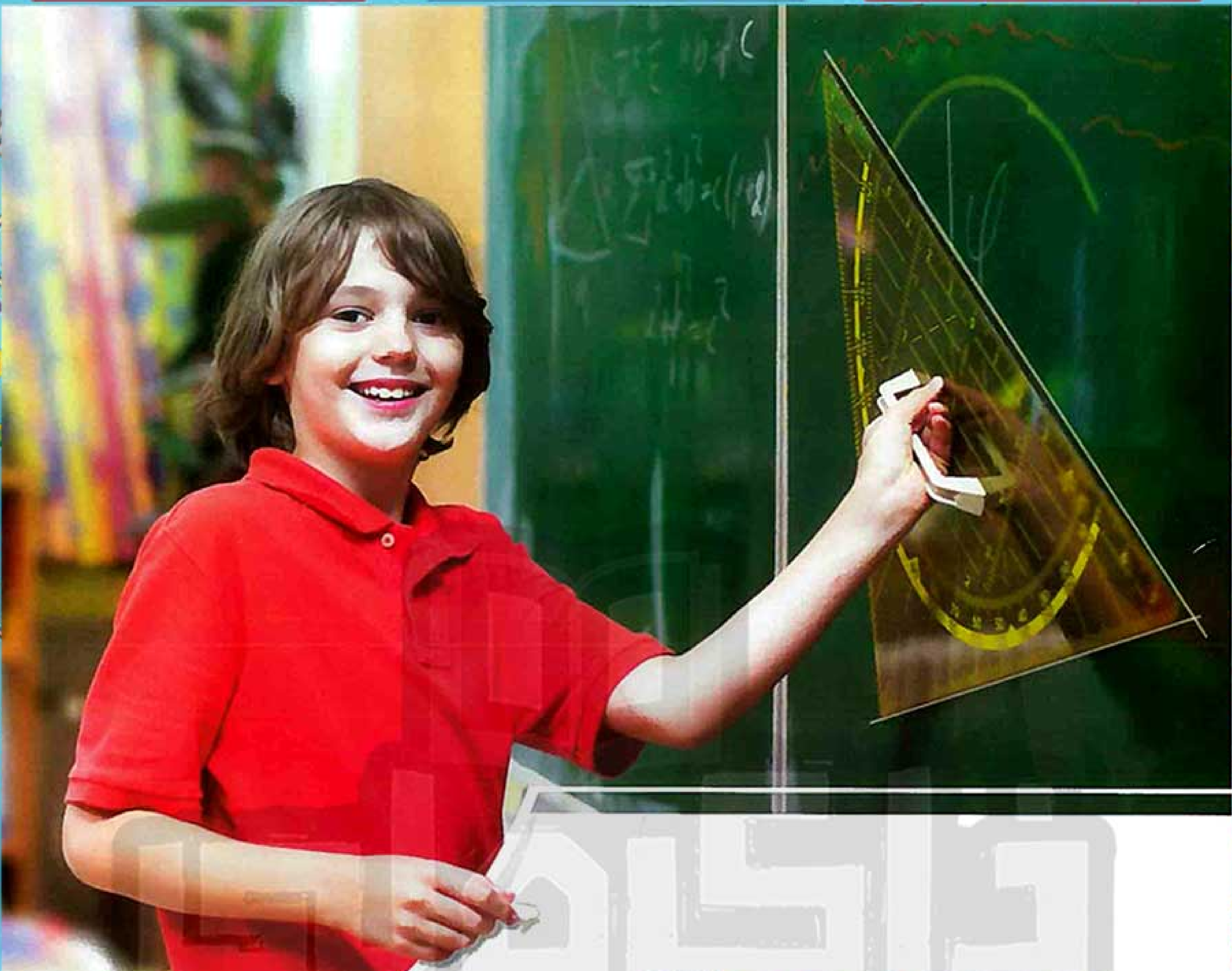

EL-MOASSER

In Mathematics

The Main Book

For 1st Prep

First Term

By
A group of supervisors

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CONTENTS

First Algebra and Statistics

UNIT

1

Rational Numbers

Lessons of the unit :

1. Set of rational numbers.
2. Comparing and ordering rational numbers.
3. Adding and subtracting rational numbers.
4. Multiplying and dividing rational numbers.
5. Applications on rational numbers.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the rational number in its different forms.
- put the rational number in the simplest form.
- represent the rational numbers on the number line.
- compare two rational numbers.
- arrange a set of rational numbers.
- add rational numbers.
- recognize the properties of addition of rational numbers.
- subtract two rational numbers.
- multiply rational numbers.
- recognize the properties of multiplication of rational numbers.
- divide two rational numbers.
- solve different problems on the operations of the rational numbers.
- find a rational number lying between two rational numbers.

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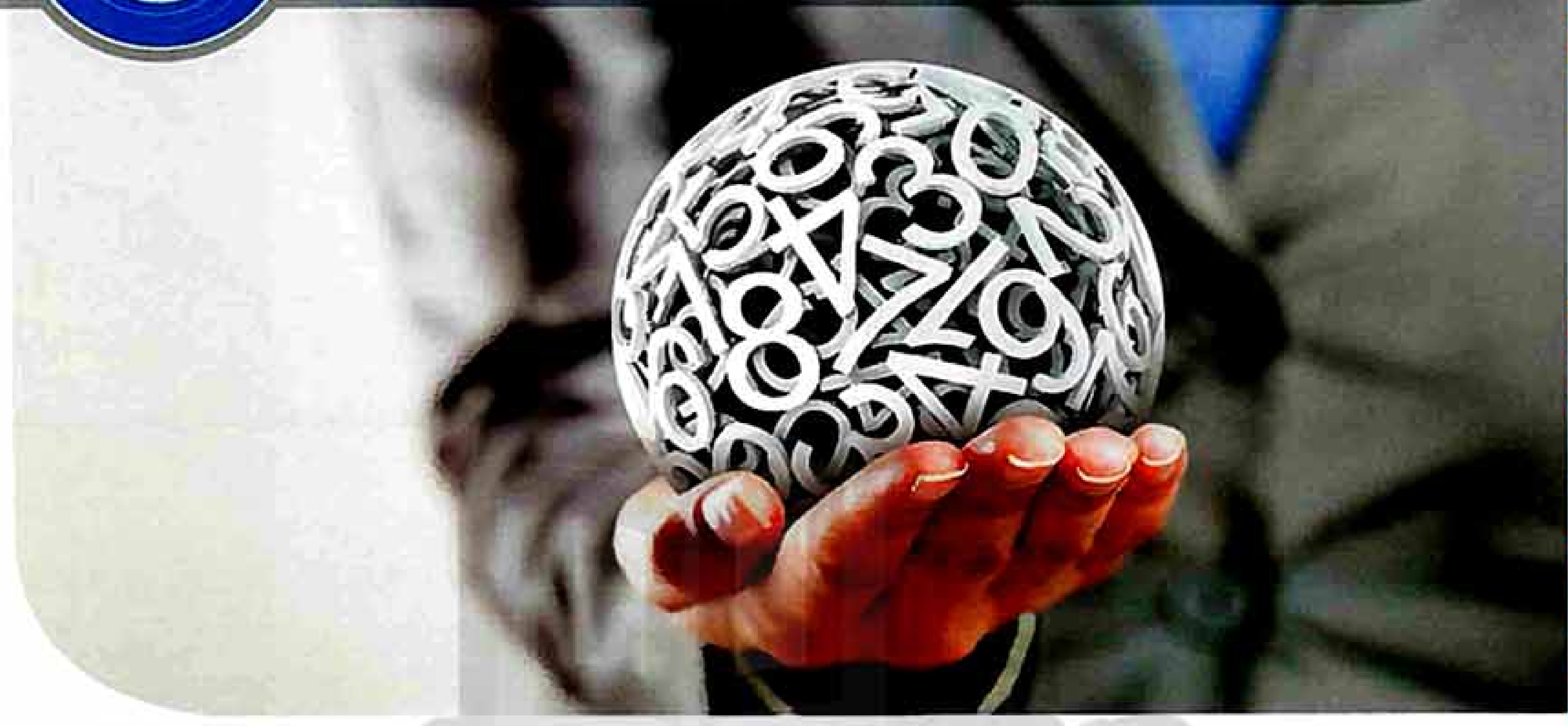
Al Bairony

Mohamed Ibn Ahmed Abo Al Rihany Al Bairony (Born in 363 H-973 A.D.)

He is one of the Arab mathematicians, he stated that : letters and digits vary in India by local variation, he stated that the indian numbers are :
١ , ٢ , ٣ , ٤ , ٥ , ٦ , ٧ , ٨ , ٩ , ٠
and are used in Arab east and Andalusian numbers are :
0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 and are used in Al Maghreb and Andalusian.

LESSON 1

Set of Rational Numbers



Prelude

- **You studied in the primary stage some sets of numbers as :**
  - * Set of counting numbers = $\{1, 2, 3, 4, ...\}$
  - * Set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4, ...\}$
  - * Set of integers $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- In this unit , you will recognize another set of numbers called "The set of rational numbers" and it is denoted by the symbol $\mathbb{Q}$

Rational numbers

The numbers : $\frac{1}{2}$, $-\frac{5}{8}$, 3 , 0 , $3\frac{1}{2}$, 0.7 , 2.5 and 15% are rational numbers.

Definition of the rational number

A rational number is a number that can be expressed in the form of a quotient of an integer divided by an integer other than 0

i.e. The rational numbers are all numbers can be expressed as $\frac{a}{b}$

where a and b are integers , $b \neq 0$

, where a and b are called the two terms of the rational number $\frac{a}{b}$

So , we can express the set of rational numbers as the following :

The set of rational numbers $\mathbb{Q} = \{x : x = \frac{a}{b}, a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\}$

Based on the previous definition , we can say that :

1 All the decimal numbers are rational numbers.

because any decimal number or decimal fraction can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

For example:

- 2.5 can be expressed in the form $\frac{25}{10}$ or $\frac{250}{100}$ or ...
- 0.7 can be expressed in the form $\frac{7}{10}$ or $\frac{70}{100}$ or ...

2 All percents are rational numbers.

because any percentage can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

For example:

15 % can be expressed in the form $\frac{15}{100}$ or $\frac{150}{1000}$ or ...

3 All integers are rational numbers.

because any integer can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

For example:

- 3 can be expressed in the form $\frac{3}{1}$ or $\frac{6}{2}$ or $\frac{9}{3}$ or ...
- 0 can be expressed in the form $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ or ...
- -16 can be expressed in the form $-\frac{16}{1}$ or $-\frac{32}{2}$ or $-\frac{48}{3}$ or ...

Therefore :

The set of integers is a subset of the set of rational numbers.

i.e. $\mathbb{Z} \subset \mathbb{Q}$

and since $\mathbb{N} \subset \mathbb{Z}$ then $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

and the opposite diagram shows that.

Remark

Each integer is a rational number , but not each rational number is an integer.

For example:

- $\frac{12}{6}$ expresses an integer because : 12 is divisible by 6 and the result is 2
- $\frac{25}{4}$ does not express an integer because : 25 is not divisible by 4

UNIT 1

Example 1 **Show why each of the following is a rational number :**

1. $3\frac{2}{5}$
2. -0.17
3. 0.006
4. $27\ \%$

Solution Each of the previous numbers is a rational number because each of them can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$ as follows :

1. $3\frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$
2. $-0.17 = -\frac{17}{100}$
3. $0.006 = \frac{6}{1000}$
4. $27\ \% = \frac{27}{100}$

TRY 1 by yourself

Show why each of the following is a rational number :

$1\frac{2}{3}$, 3.07 , -51 , $30\ \%$, $102\ \%$

Final answers of try by yourself questions are at the end of each lesson to check your answer.

Remark

If $\frac{a}{b}$ is a rational number , then $b \neq 0$

Example 2 **If x is an integer , write the required condition to make each of the following a rational number :**

1. $\frac{3}{2x}$
2. $\frac{7}{x-3}$

Solution

1. $\frac{3}{2x}$ will be a rational number if : $2x \neq 0$

therefore the required condition is : $x \neq 0$

2. $\frac{7}{x-3}$ will be a rational number if : $x - 3 \neq 0$

therefore the required condition is : $x \neq 3$

TRY 2 by yourself

Complete the following table :

The number	$\frac{5}{x-3}$	$\frac{3}{4-x}$	$\frac{7}{8x}$	$\frac{6x}{x}$
Expresses a rational number if $x \neq$	....	....	....	....

Remark

If the rational number $\frac{a}{b} = 0$, then $a = 0$

Example 3 If the rational number $\frac{x-3}{x+3}$ equals 0 , find the value of x

Solution

Since $\frac{x-3}{x+3} = 0$

therefore $x - 3 = 0$ i.e. $x = 3$

TRY 3 by yourself

Complete the following table :

The rational number	$\frac{x-2}{x-1}$	$\frac{6-x}{x-4}$	$\frac{2x}{x+5}$	$\frac{2x-4}{x+3}$
Equals zero if $x =$	....	....	....	....

Different forms of a rational number

The rational number $\frac{a}{b}$ can be written in the form of another rational number $\frac{c}{d}$ equal to it by applying the following property :

Property

The value of the rational number $\frac{a}{b}$ does not change if its two terms are multiplied or divided by an integer ≠ zero.

For example:

- $\frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{6}{14}$, $\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$

  i.e. $\frac{3}{7}$, $\frac{6}{14}$, $\frac{9}{21}$ are different forms which represent the same number.

- $\frac{24}{36} = \frac{24 \div 2}{36 \div 2} = \frac{12}{18}$, $\frac{24}{36} = \frac{24 \div 4}{36 \div 4} = \frac{6}{9}$

  i.e. $\frac{24}{36}$, $\frac{12}{18}$, $\frac{6}{9}$ are different forms which represent the same number.

TRY 4 by yourself

Write in three other forms each of the following rational numbers :

1. $\frac{2}{3}$
2. $\frac{16}{64}$

UNIT 1

Writing a rational number $\frac{a}{b}$ in its simplest form

- For any rational number expressed as $\frac{a}{b}$, we say that this rational number is in its simplest form if each of its terms has the smallest possible value.

For example:

- The simplest form of the rational number $\frac{16}{32}$ is $\frac{1}{2}$, and note that : $\frac{16}{32}$ and $\frac{1}{2}$ represent the same rational number.
- The rational number $\frac{3}{14}$ is in its simplest form and can not be simplified to more simple form.
- To put a rational number $\frac{a}{b}$ in its simplest form ,
  divide each of its terms by the highest common factor (H.C.F.) between them.

Example 4 **Put each of the following numbers in its simplest form :**

1. $\frac{8}{12}$
2. $-\frac{12}{36}$

Solution

1. The (H.C.F.) of 8 and 12 is 4
   Dividing the two terms of $\frac{8}{12}$ by 4, we get : $\frac{8}{12} = \frac{2}{3}$
2. The (H.C.F.) of 12 and 36 is 12
   Dividing the two terms of $-\frac{12}{36}$ by 12, we get : $-\frac{12}{36} = -\frac{1}{3}$

TRY 5 *by yourself*

Complete the following table :

The number	$\frac{5}{25}$	$-\frac{6}{9}$	$\frac{27}{45}$	$-\frac{12}{30}$
Its simplest form	....	....	....	....

Writing the rational number in the form of percentage

To write the rational number in the form of percentage we express it as $\frac{a}{100}$ which equals a %

Example 5 **Write each of the following numbers in the form of percentage :**

1. $\frac{9}{20}$
2. $\frac{5}{16}$
3. $\frac{17}{1000}$
4. $5\frac{12}{125}$
5. 3.2

Solution

1. $\frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 45\ \%$

   Another solution : $\frac{9}{20} = \frac{\frac{9}{20} \times 100}{100} = \frac{45}{100} = 45\ \%$

2. $\frac{5}{16} = \frac{\frac{5}{16} \times 100}{100} = \frac{31.25}{100} = 31.25\ \%$

3. $\frac{17}{1000} = \frac{\frac{17}{1000} \times 100}{100} = \frac{1.7}{100} = 1.7\ \%$

4. $5\frac{12}{125} = \frac{637}{125} = \frac{\frac{637}{125} \times 100}{100} = \frac{509.6}{100} = 509.6\ \%$

5. $3.2 = \frac{32}{10} = \frac{32 \times 10}{10 \times 10} = \frac{320}{100} = 320\ \%$

TRY 6 *by yourself*

Write each of the following numbers in the form of percentage :

1. $\frac{4}{5}$ 2. $\frac{3}{1000}$ 3. 2.5

Changing a rational number from the form $\frac{a}{b}$ to a decimal form

Some rational numbers could be changed from the form $\frac{a}{b}$ into a terminated decimal.

For example: • The rational number $\frac{3}{5}$ can be changed into 0.6

• The rational number $\frac{3}{2}$ can be changed into 1.5

To write a rational number in the form of a terminated decimal , we make its denominator equal to 10 , 100 , 1000 or ...

Example 6 **Write each of the following numbers in the form of a terminating decimal :**

1. $\frac{2}{5}$ 2. $|-\frac{3}{8}|$ 3. $-2\frac{7}{25}$

Solution

1. $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$

2. $|-\frac{3}{8}| = \frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$

3. $-2\frac{7}{25} = -2\,\frac{7 \times 4}{25 \times 4} = -2\,\frac{28}{100} = -2.28$

Check your answer using calculator

UNIT 1

TRY 7 by yourself

Write each of the following rational numbers in the form of a terminating decimal :

1 $\frac{3}{4}$ 2 $\frac{11}{20}$

Remark

Some rational numbers could not be changed into terminated decimal as the rational number $\frac{1}{3}$, then using calculator , we find that : $\frac{1}{3} = 0.333333 ...$

We express that as ($0.\dot{3}$) and read it as the infinite repeating decimal 0.3 (the recurring decimal 0.3) where the dot above the digit 3 means the digit is repeating (recurring).

Example 7 **Using a calculator , write each of the following rational numbers in the form of a recurring decimal :**

1 $\frac{2}{3}$ 2 $\frac{2}{11}$ 3 $5\frac{71}{333}$

Solution

1 Using the calculator , we get that :

$\frac{2}{3} = 0.6666666667$ i.e. $\frac{2}{3} = 0.\dot{6}$

2 Using the calculator , we get that :

$\frac{2}{11} = 0.1818181818$ i.e. $\frac{2}{11} = 0.\dot{1}\dot{8}$

3 Using the calculator , we get that :

$\frac{71}{333} = 0.2132132132$ i.e. $5\frac{71}{333} = 5.\dot{2}1\dot{3}$

Notice that :

Putting dots above the first and last digits means repeating all digits (first , last and between them)

TRY 8 by yourself

Write each of the following rational numbers in the form of a recurring decimal :

1 $\frac{3}{11}$ 2 $\frac{41}{333}$

Remark

It is possible to write the recurring decimal in the form of $\frac{a}{b}$ by using scientific calculators of type CASIO fX-95ES plus or a different type.

Notice that some scientific calculators can not be able to solve this problem.

For example:

- To write the number $0.\dot{2}\dot{1}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.21212121212121 ,
  then press = you will get the rational number $\frac{7}{33}$
- To write the number $0.1\dot{3}\dot{6}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.1363636363636 ,
  then press = you will get the rational number $\frac{3}{22}$

TRY 9 *by yourself*

Use the calculator to write each of the following in the form $\frac{a}{b}$:

1. $0.\dot{1}\dot{8}$
2. $0.1\dot{4}\dot{5}$

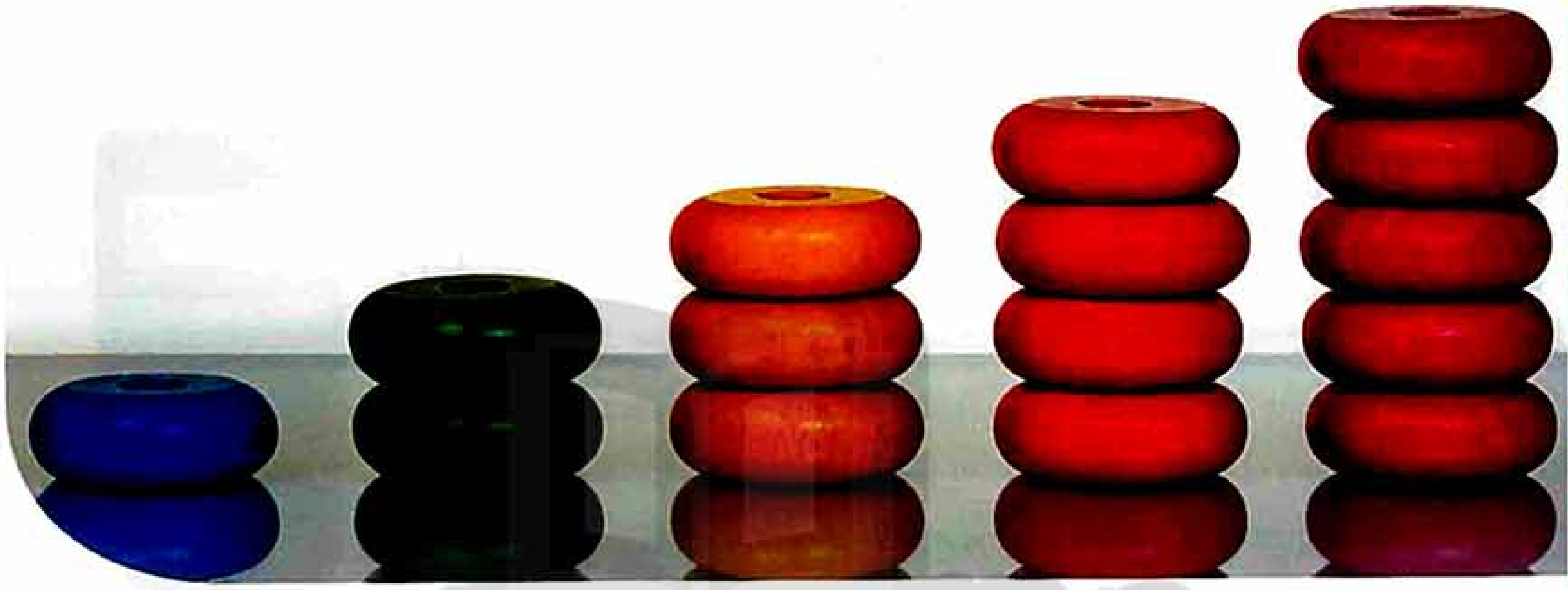
Answers *of try by yourself*

1. Because each of these numbers can be written in the form $\frac{a}{b}$ where a , b are integers , $b \neq 0$
   $\left(\frac{5}{3} , \frac{307}{100} , \frac{-51}{1} , \frac{30}{100} , \frac{102}{100} \right)$
2. 3 , 4 , 0 , 0
3. 2 , 6 , 0 , 2
4. (1) $\frac{4}{6} , \frac{12}{18} , \frac{10}{15}$ "There are other solutions"
   (2) $\frac{8}{32} , \frac{4}{16} , \frac{1}{4}$ "There are other solutions"
5. $\frac{1}{5} , -\frac{2}{3} , \frac{3}{5} , -\frac{2}{5}$
6. (1) 80% (2) 0.3% (3) 250%
7. (1) 0.75 (2) 0.55
8. (1) $0.\dot{2}\dot{7}$ (2) $0.\dot{1}2\dot{3}$
9. (1) $\frac{2}{11}$ (2) $\frac{8}{55}$

At the end of each lesson , you will find the final answers of try by yourself questions in the same form.

LESSON 2

Comparing and Ordering Rational Numbers



We will study how to represent a rational number on the number line before studying of comparing and ordering the rational numbers.

Representing the rational numbers on the number line

- Each rational number can be represented by a unique point on the number line.
- The positive rational numbers are represented on the number line by points lying on the right side of the point which represents the number zero and the negative rational numbers are represented by points lying on the left side of the point which represents the number zero.

The following examples show how to represent the rational numbers on the number line :

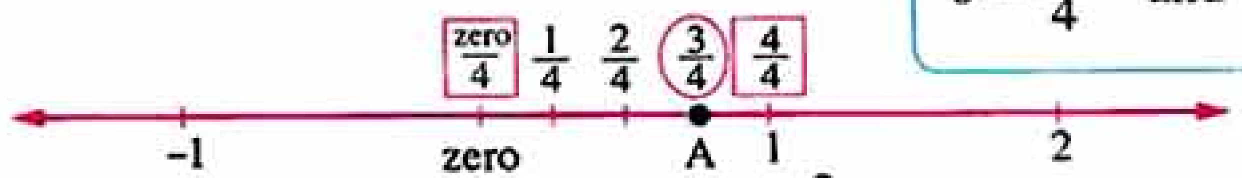
Example 1 **Represent the rational number $\frac{3}{4}$ on the number line.**

Solution

- Since the rational number $\frac{3}{4}$ lies between the two integers zero and 1 , then the point which represents the number $\frac{3}{4}$ lies between the two points which represent the two numbers zero and 1
- Divide the distance between the point representing the number zero and the point representing the number 1 to 4 equal parts as follows :

Notice that :

$0 = \frac{0}{4}$ and $1 = \frac{4}{4}$



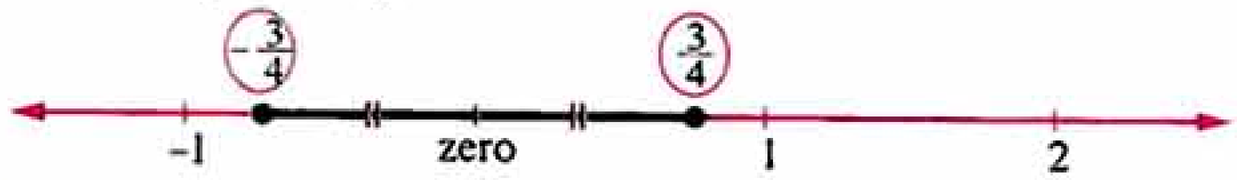
The point A represents the rational number $\frac{3}{4}$

Remark

The two rational numbers A and – A are represented on the number line by two points equidistant from the point representing the number zero and on two different sides of it.

For example:

The two rational numbers $\frac{3}{4}$ and $-\frac{3}{4}$ are represented on the number line as the following figure :



Example 2 **Represent on the number line each of the following rational numbers :**

1 $\frac{7}{5}$ **2** $-\frac{24}{9}$

Solution

1 Since $\frac{7}{5} = 1\frac{2}{5}$

, then $\frac{7}{5}$ lies between the two integers 1 and 2 therefore , we divide the distance between the point representing the number 1 and the point representing the number 2 into 5 equal parts as follows :

Notice that :
$1 = \frac{5}{5}$ and $2 = \frac{10}{5}$

The point A represents the rational number $\frac{7}{5}$

2 Before representing the rational number on the number line it is better to put the number in its simplest form.

$-\frac{24}{9} = -\frac{24 \div 3}{9 \div 3} = -\frac{8}{3}$ since $-\frac{8}{3} = -2\frac{2}{3}$, then $-\frac{24}{9} = -2\frac{2}{3}$

i.e. It lies between the two integers – 2 , – 3

Therefore , we divide the distance between the point representing the number – 2 and the point representing the number – 3 into 3 equal parts as follows :

Notice that :
$-2 = -\frac{6}{3}$ and $-3 = -\frac{9}{3}$

The point B represents the rational number $-\frac{24}{9}$

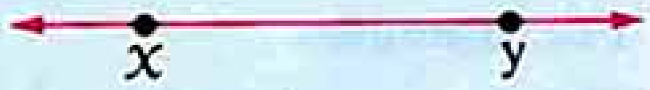
TRY 1 by yourself

Represent on the number line the rational number $\frac{18}{8}$

UNIT 1

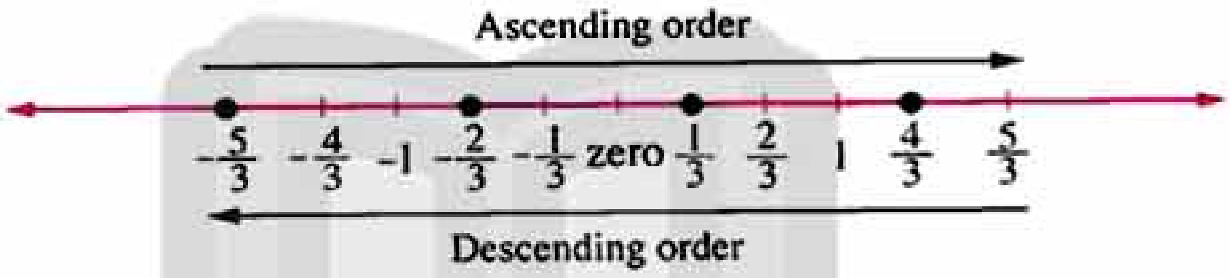
Comparing and ordering the rational numbers

If the point which represents the number x lies on the left of the point which represents the number y on the number line as shown in the opposite figure , then : $x < y$ or $y > x$



For example:

In the following figure we find that :



- $\frac{1}{3} < \frac{4}{3}$ or $\frac{4}{3} > \frac{1}{3}$

  Because :

  the point which represents the number $\frac{1}{3}$ lies on the left of the point which represents the number $\frac{4}{3}$

- $-\frac{5}{3} < -\frac{2}{3}$ or $-\frac{2}{3} > -\frac{5}{3}$

  Because :

  the point which represents the number $-\frac{5}{3}$ lies on the left of the point which represents the number $-\frac{2}{3}$

Example 3 **Represent the following rational numbers on the number line , then arrange them ascendingly : $\frac{7}{5}$, zero , $\frac{9}{5}$, 2 , – 1**

Solution

According to the positions of the numbers on the number line shown above we find that the ascending order is : – 1 , zero , $\frac{7}{5}$, $\frac{9}{5}$, 2

TRY 2 *by yourself*

Represent the following rational numbers on the number line , then arrange them descendingly : 2 , $-\frac{5}{2}$, $\frac{7}{2}$, zero , – 1

Comparing two rational numbers

- If the two numbers have different signs , then the positive number is greater than the negative number.

  For example: $0.05 > -\frac{15}{2}$

- If one of the two numbers is greater than a certain number x and the other number is less than this number x , then the first number is greater than the second.

  For example: $\frac{65}{63} > \frac{57}{59}$ (Because : $\frac{65}{63} > 1$, $\frac{57}{59} < 1$)

- If the two numbers are in the form $\frac{a}{b}$ and have the same positive denominator , then the number having the greater numerator will be the greater.

  For example: $\frac{7}{13} > \frac{5}{13}$ (Because : $7 > 5$)

- If the two numbers are in the form $\frac{a}{b}$ and have the same positive numerator , then the number having the greater denominator will be the smaller.

  For example: $\frac{2}{5} > \frac{2}{9}$ (Because : $9 > 5$)

- If the two numbers are in the form $\frac{a}{b}$ and different in numerator and denominator , convert the two numbers in order to have a common denominator taking care of the denominator should be positive , and then compare their numerators.

  For example: $\frac{2}{3} > \frac{8}{15}$ (Because : $\frac{2}{3} = \frac{10}{15}$, $\frac{10}{15} > \frac{8}{15}$)

Example 4 Compare the two numbers in each of the following :

1. $\frac{5}{12}$, $\frac{7}{12}$
2. $\frac{1}{4}$, $-\frac{5}{6}$
3. $\frac{11}{12}$, $\frac{11}{15}$
4. $\frac{6}{12}$, $\frac{2}{3}$
5. 3.2 , $\frac{11}{2}$
6. $23\ \%$, $\frac{3}{8}$

Solution

1. $\frac{5}{12} < \frac{7}{12}$ (Because the two numbers have the same denominator , $5 < 7$)
2. $\frac{1}{4} > -\frac{5}{6}$ (Because $\frac{1}{4}$ is positive , $-\frac{5}{6}$ is negative)
3. $\frac{11}{12} > \frac{11}{15}$ (Because the two numbers have the same numerator , $12 < 15$)

UNIT 1

4 $\frac{6}{12}$, $\frac{2}{3}$ are different in the numerator and the denominator ,
we convert the two numbers in order to have a common denominator.
Since the L.C.M. of the denominators = 12
Therefore $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and since $8 > 6$
Therefore $\frac{6}{12} < \frac{8}{12}$ **i.e.** $\frac{6}{12} < \frac{2}{3}$

5 $3.2 = 3\frac{2}{10}$, $\frac{11}{2} = 5\frac{1}{2}$
Therefore $3.2 < 5\frac{1}{2}$ (Because : $3 < 5$) **i.e.** $3.2 < \frac{11}{2}$
Another solution : $3.2 < \frac{11}{2}$ (Because : $\frac{11}{2} = 5.5$, $3.2 < 5.5$)

6 $23\ \% = \frac{23}{100}$ **i.e.** The two numbers are $\frac{23}{100}$ and $\frac{3}{8}$
Since the L.C.M. of the denominators = 200
Therefore $\frac{23}{100} = \frac{46}{200}$, $\frac{3}{8} = \frac{75}{200}$ and since $46 < 75$
Therefore $\frac{46}{200} < \frac{75}{200}$ **i.e.** $23\ \% < \frac{3}{8}$
Another solution : $23\ \% < \frac{1}{4}$ (Because : $\frac{1}{4} = 25\ \%$)
, $\frac{3}{8} > \frac{1}{4}$ (Because : $\frac{1}{4} = \frac{2}{8}$) **i.e.** $23\ \% < \frac{3}{8}$

Example 5 **Arrange the following rational numbers ascendingly :**

$-\frac{2}{3}$, $\frac{3}{4}$, $-\frac{7}{12}$, $\frac{5}{6}$, -1

Solution

Since L.C.M. of the denominators = 12
, then $-\frac{2}{3} = -\frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{5}{6} = \frac{10}{12}$, $-1 = -\frac{12}{12}$
, then the numbers after converting their denominators are :
$-\frac{8}{12}$, $\frac{9}{12}$, $-\frac{7}{12}$, $\frac{10}{12}$, $-\frac{12}{12}$
Since $-12 < -8 < -7 < 9 < 10$, then $-\frac{12}{12} < -\frac{8}{12} < -\frac{7}{12} < \frac{9}{12} < \frac{10}{12}$
i.e. $-1 < -\frac{2}{3} < -\frac{7}{12} < \frac{3}{4} < \frac{5}{6}$

TRY 3 by yourself

***Complete each of the following using the suitable sign* (< , > or =) :**

1 $\frac{7}{5}$ .... $\frac{4}{5}$
2 $-\frac{3}{4}$ .... $-\frac{2}{4}$
3 $\frac{1}{5}$ .... $\frac{1}{6}$
4 $\frac{3}{6}$ .... $\frac{2}{3}$
5 $\frac{4}{10}$ .... $\frac{14}{35}$
6 $\left|-\frac{10}{15}\right|$ .... $\frac{2}{3}$

The density of the rational numbers

Between every two different rational numbers there are an infinite number of rational numbers.

To illustrate this :

If we have two rational numbers as $\frac{1}{3}$ and $\frac{2}{3}$, we can deduce that there are other rational numbers between them as follows :

1 If we multiply both terms of the two numbers $\frac{1}{3}$ and $\frac{2}{3}$ by 2 ,
we get the two rational numbers : $\frac{2}{6}$ and $\frac{4}{6}$ which are equal to them.
It is clear that $\frac{3}{6}$ lies between $\frac{2}{6}$ and $\frac{4}{6}$

i.e. **The rational number $\frac{3}{6}$ $(= \frac{1}{2})$ is lying between the two numbers $\frac{1}{3}$ and $\frac{2}{3}$**

2 If we multiply both terms of the two numbers : $\frac{1}{3}$ and $\frac{2}{3}$ by 3 ,
then we get the two rational numbers $\frac{3}{9}$, $\frac{6}{9}$ which are equal to them.
It is clear that $\frac{4}{9}$ and $\frac{5}{9}$ are lying between $\frac{3}{9}$ and $\frac{6}{9}$

i.e. **The two rational numbers $\frac{4}{9}$ and $\frac{5}{9}$ are lying between the two numbers $\frac{1}{3}$ and $\frac{2}{3}$**

So, we can deduce that : between the two rational numbers $\frac{1}{3}$ and $\frac{2}{3}$ there are an infinite number of rational numbers.

Remarks

- There is no integer between any two consecutive integers , it means that the set of integers which is an infinite set , but doesn't have the property of density.
- For any integer , we can determine the integer which lies just after or just before it.
- For any rational number we cannot determine the rational number which lies just after or just before it.

Example 6 **Find four rational numbers lying between : $\frac{1}{2}$ and $\frac{5}{7}$**

Solution

Since L.C.M. of the denominators is 14

, then $\frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14}$

, $\frac{5}{7} = \frac{5 \times 2}{7 \times 2} = \frac{10}{14}$

Since $\frac{7}{14} < \frac{8}{14} < \frac{9}{14} < \frac{10}{14}$

, then $\frac{8}{14}$, $\frac{9}{14}$ are two rational numbers lying between : $\frac{1}{2}$, $\frac{5}{7}$

but the required is 4 numbers not two only.

Notice that :
We should convert their denominators to have common denominator at first.

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Therefore, we multiply the two terms of each of $\frac{7}{14}$ and $\frac{10}{14}$ by 2

, then $\frac{7}{14} = \frac{7 \times 2}{14 \times 2} = \frac{14}{28}$

, $\frac{10}{14} = \frac{10 \times 2}{14 \times 2} = \frac{20}{28}$

Since $\frac{14}{28} < \frac{15}{28} < \frac{16}{28} < \frac{17}{28} < \frac{18}{28} < \frac{19}{28} < \frac{20}{28}$

i.e. $\frac{1}{2} < \frac{15}{28} < \frac{4}{7} < \frac{17}{28} < \frac{9}{14} < \frac{19}{28} < \frac{5}{7}$

, then $\frac{15}{28}$, $\frac{4}{7}$, $\frac{17}{28}$, $\frac{9}{14}$, $\frac{19}{28}$ are five rational numbers between $\frac{1}{2}$ and $\frac{5}{7}$

From the five numbers choose any 4 numbers only as required.

Notice that :

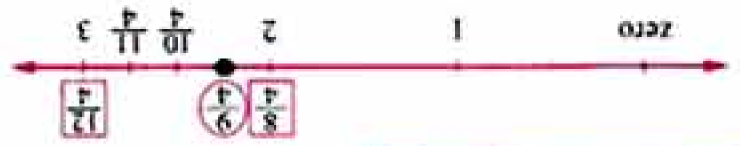
For facilitating the solution, we multiply each of the numerator and the denominator of the two numbers by 10

TRY 4 *by yourself*

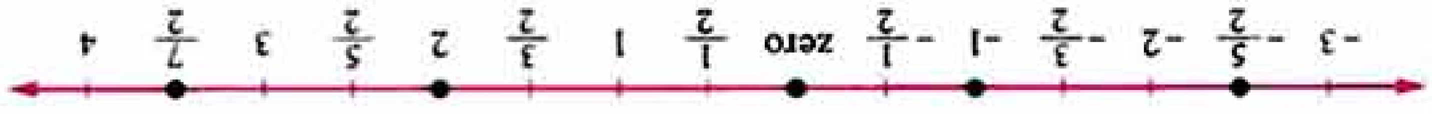
Find three rational numbers lying between : $\frac{1}{3}$ and $\frac{1}{2}$

Answers *of try by yourself*

1 $\frac{18}{8} = \frac{9}{4}$



2



The order is : $\frac{7}{2}$, 2 , zero , -1 and $-\frac{5}{2}$

3 1) > 2) < 3) >
4) < 5) = 6) =

4 $\frac{3}{8}$, $\frac{5}{12}$, $\frac{11}{24}$ "There are other solutions".

LESSON **3**

Adding and Subtracting Rational Numbers

First : Addition operation

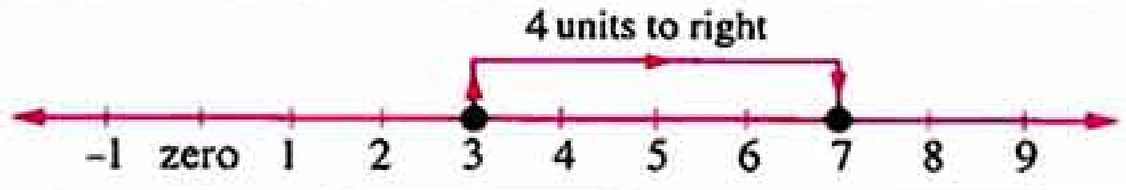
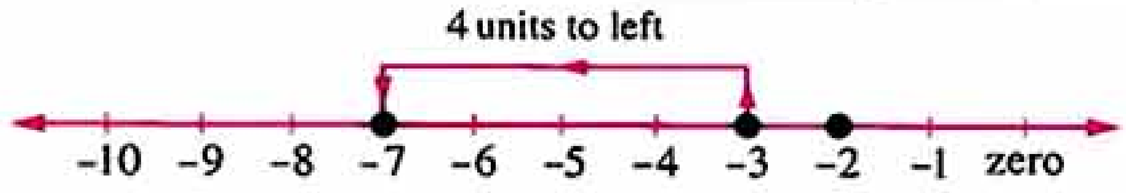
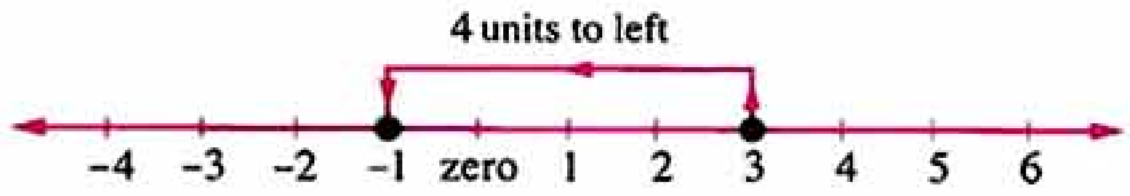
Prelude

We will use the number line to explain the concept of addition in $\mathbb{Q}$ as follows :

To find the sum of a + b on the number line :

1. Determine the point that represents the number a on the number line.
2. Move to right or left according to the sign of b and with its units till you reach the point that represents (a + b).

For example:

$3 + 4$		$3 + 4 = 7$
$(-3) + (-4)$		$(-3) + (-4) = -7$
$3 + (-4)$		$3 + (-4) = -1$
$(-3) + 4$		$(-3) + 4 = 1$

UNIT 1

From the previous, notice that

- The sum of two positive numbers is positive.

For example: $2 + 3 = 5$

- The sum of two negative numbers is negative.

For example: $(-4) + (-5) = -9$

- The sum of two numbers different in sign can be positive or negative or zero.

For example: $* \ 5 + (-3) = 2$ $\quad * \ 4 + (-7) = -3$ $\quad * \ 2 + (-2) = 0$

Adding two rational numbers expressed as $\frac{a}{b}$

1 Adding two rational numbers having the same denominator :

If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational numbers , then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

For example:

- $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$
- $\frac{3}{5} + \left(-\frac{1}{5}\right) = \frac{3+(-1)}{5} = \frac{2}{5}$

2 Adding two rational numbers with different denominators :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

For example:

$\frac{2}{5} + \frac{1}{7} = \frac{2 \times 7 + 1 \times 5}{5 \times 7} = \frac{14+5}{35} = \frac{19}{35}$

Example 1 Add :

1. $\frac{3}{8} + \frac{1}{4}$
2. $\frac{4}{12} + \left(-\frac{10}{15}\right)$
3. $\frac{2}{5} + 3$
4. $3\frac{1}{4} + \left(-2\frac{1}{5}\right)$

Solution

1. $\frac{3}{8} + \frac{1}{4} = \frac{3 \times 4 + 1 \times 8}{8 \times 4} = \frac{12+8}{32} = \frac{20}{32} = \frac{5}{8}$

Notice that :
After carrying out the operation of addition , the result should be put in its simplest form.

Another solution «by finding the common denominator» :

Since L.C.M. of the denominators 8 and 4 is 8

Therefore $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$ **i.e.** $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$

2 Since $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$,

$-\frac{10}{15} = -\frac{10 \div 5}{15 \div 5} = -\frac{2}{3}$

Therefore $\frac{4}{12} + (-\frac{10}{15}) = \frac{1}{3} + (-\frac{2}{3})$

$= \frac{1 + (-2)}{3} = -\frac{1}{3}$

> **Notice that :**
> Before adding the two rational numbers it is better to write them in the simplest form as shown in the opposite solution.

3 Since $3 = \frac{15}{5}$

Therefore $\frac{2}{5} + 3 = \frac{2}{5} + \frac{15}{5} = \frac{17}{5}$

Another solution :

$\frac{2}{5} + 3 = 3\frac{2}{5}$, then convert the mixed number into an improper fraction ,

we find $3\frac{2}{5} = \frac{17}{5}$

4 Since $3\frac{1}{4} = \frac{13}{4}$, $-2\frac{1}{5} = -\frac{11}{5}$ therefore $3\frac{1}{4} + (-2\frac{1}{5}) = \frac{13}{4} + (-\frac{11}{5})$

Since L.C.M. of the denominators 4 and 5 is 20

Therefore $\frac{13}{4} + (-\frac{11}{5}) = \frac{65}{20} + (-\frac{44}{20}) = \frac{21}{20} = 1\frac{1}{20}$

Another solution :

Since L.C.M. of the denominators 4 and 5 is 20 , then :

$3\frac{1}{4} + (-2\frac{1}{5}) = 3\frac{5}{20} + (-2\frac{4}{20}) = 1\frac{1}{20}$

TRY 1 by yourself

Add each of the following :

1. $\frac{1}{5} + \frac{2}{5}$
2. $\frac{2}{5} + \frac{1}{3}$
3. $\frac{1}{2} + (-\frac{5}{6})$
4. $-\frac{3}{4} + \frac{1}{5}$
5. $\frac{8}{12} + (-\frac{15}{18})$

UNIT 1

Properties of the addition operation in $\mathbb{Q}$

1 Closure property :

The sum of any two rational numbers is a rational number.

i.e. $\mathbb{Q}$ is closed under addition operation.

For example:

The sum of the two rational numbers $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{5}{6}$ which is a rational number too.

2 Commutative property :

If a and b are two rational numbers , then a + b = b + a

For example:

$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20}$, $\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20}$

i.e. $\frac{3}{4} + \frac{2}{5} = \frac{2}{5} + \frac{3}{4}$

3 Associative property :

If a , b and c are three rational numbers , then

(a + b) + c = a + (b + c)

For example:

$(\frac{3}{7} + \frac{2}{7}) + \frac{1}{7} = \frac{5}{7} + \frac{1}{7} = \frac{6}{7}$, $\frac{3}{7} + (\frac{2}{7} + \frac{1}{7}) = \frac{3}{7} + \frac{3}{7} = \frac{6}{7}$

i.e. $(\frac{3}{7} + \frac{2}{7}) + \frac{1}{7} = \frac{3}{7} + (\frac{2}{7} + \frac{1}{7})$

4 The existence of identity element (Neutral element) property in addition :

If a is a rational number , then a + zero = zero + a = a

i.e. When we add zero to any rational number the value of this number does not change.
Then we say : zero is the identity element in addition operation in $\mathbb{Q}$

For example: $\frac{1}{2} + 0 = 0 + \frac{1}{2} = \frac{1}{2}$

5 The existence of additive inverse property :

For every rational number a there exist an additive inverse to it that is – a where a + (– a) = zero (the identity element in addition)

For example:

The additive inverse of the number $\frac{3}{4}$ is $-\frac{3}{4}$

and vice versa the additive inverse of $-\frac{3}{4}$ is $\frac{3}{4}$

because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4}$ = zero (the identity element in addition).

Notice that :
Zero is its own additive inverse.

Example 2 Use the addition properties in $\mathbb{Q}$ to find the result of :

$\frac{6}{35} + (-\frac{5}{11}) + \frac{19}{35} + \frac{10}{22}$

Solution

Since $\frac{10}{22} = \frac{10 \div 2}{22 \div 2} = \frac{5}{11}$

Therefore $\frac{6}{35} + (-\frac{5}{11}) + \frac{19}{35} + \frac{10}{22} = \frac{6}{35} + (-\frac{5}{11}) + \frac{19}{35} + \frac{5}{11}$

$= (\frac{6}{35} + \frac{19}{35}) + (-\frac{5}{11} + \frac{5}{11})$ (commutative and associative properties)

$= \frac{25}{35}$ + zero (the additive inverse)

$= \frac{25}{35}$ (the identity element)

$= \frac{5}{7}$ (the result in its simplest form)

TRY 2 by yourself

Use the addition properties in $\mathbb{Q}$ to find the result of : $\frac{4}{5} + (-\frac{3}{7}) + \frac{1}{5} + \frac{3}{7}$

Second : Subtraction operation

Since each rational number has an additive inverse , then the subtraction operation is always possible in $\mathbb{Q}$ and it is defined as follows :

Definition :

If a and b are two rational numbers , then a – b = a + (– b)

i.e. The subtraction operation in $\mathbb{Q}$ is defined as adding the minuend (a) to the additive inverse of the subtrahend (b)

UNIT 1

Example 3 Find the result of each of the following in its simplest form :

1 $\frac{5}{8}-\frac{3}{8}$ | 2 $\frac{3}{4}-\frac{5}{6}$ | 3 $\frac{5}{7}-1$

4 $-\frac{2}{5}-\frac{3}{5}$ | 5 $7\frac{2}{5}-3\frac{1}{4}$

Solution

1 $\frac{5}{8}-\frac{3}{8}=\frac{5}{8}+(-\frac{3}{8})=\frac{2}{8}=\frac{1}{4}$

2 Since L.C.M. of denominators is 12

Therefore

$$\frac{3}{4}-\frac{5}{6}=\frac{3\times3}{4\times3}+(-\frac{5\times2}{6\times2})=\frac{9}{12}+(-\frac{10}{12})=-\frac{1}{12}$$

3 $\frac{5}{7}-1=\frac{5}{7}+(-1)=\frac{5}{7}+(-\frac{7}{7})=-\frac{2}{7}$

4 $-\frac{2}{5}-\frac{3}{5}=-\frac{2}{5}+(-\frac{3}{5})=-\frac{5}{5}=-1$

5 Since : $7\frac{2}{5}=\frac{37}{5}$ and $3\frac{1}{4}=\frac{13}{4}$ and since :

L.C.M. of denominators = 20 , then :

$$7\frac{2}{5}-3\frac{1}{4}=\frac{37\times4}{5\times4}+(-\frac{13\times5}{4\times5})=\frac{148}{20}+(-\frac{65}{20})=\frac{83}{20}$$

Another solution : Since L.C.M. of denominators is 20

therefore $7\frac{2}{5}-3\frac{1}{4}=7\frac{2\times4}{5\times4}+(-3\frac{1\times5}{4\times5})=7\frac{8}{20}+(-3\frac{5}{20})=4\frac{3}{20}$

Notice that :
We can use the number line to find the result of the subtraction after transform it to addition.

Remark

We can do without the step of converting subtraction operation into addition operation as follows :

- $\frac{5}{8}-\frac{3}{8}=\frac{5-3}{8}=\frac{2}{8}=\frac{1}{4}$
- $\frac{3}{4}-\frac{5}{6}=\frac{9}{12}-\frac{10}{12}=\frac{9-10}{12}=-\frac{1}{12}$

TRY 3 by yourself

Find each of the following in its simplest form :

1 $\frac{3}{5}-\frac{2}{5}$ | 2 $\frac{7}{9}-\frac{4}{9}$

3 $\frac{3}{4}-\frac{2}{3}$ | 4 $4\frac{1}{5}-3\frac{1}{8}$

Remarks

- $\mathbb{Q}$ is closed under subtraction operation.
  i.e. the result of subtracting any two rational numbers is a rational number.
- The subtraction operation in $\mathbb{Q}$ is not commutative and not associative.
- There is no identity element with respect to subtraction in $\mathbb{Q}$ and hence there is no inverses for the numbers with respect to subtraction in $\mathbb{Q}$

Example 4 If $a = \frac{3}{4}$, $b = -\frac{5}{2}$ and $c = \frac{1}{2}$, find the numerical value of each of the following :

1. $a - b$
2. $(a + b) - c$

Solution

1. $a - b = \frac{3}{4} - \left(-\frac{5}{2}\right) = \frac{3}{4} + \frac{5}{2} = \frac{3}{4} + \frac{10}{4} = \frac{13}{4}$ «from the subtraction operation definition»
2. $(a + b) - c = \left[\frac{3}{4} + \left(-\frac{5}{2}\right)\right] - \frac{1}{2} = \left[\frac{3}{4} + \left(-\frac{10}{4}\right)\right] - \frac{1}{2} = -\frac{7}{4} - \frac{1}{2}$
   $= -\frac{7}{4} - \frac{2}{4} = -\frac{9}{4}$

Answers of try by yourself

1 (1) $\frac{3}{5}$ (2) $\frac{11}{15}$ (3) $-\frac{1}{3}$ (4) $-\frac{11}{20}$ (5) $-\frac{1}{6}$

2 1

3 (1) $\frac{1}{5}$ (2) $\frac{1}{3}$ (3) $\frac{1}{12}$ (4) $\frac{43}{40}$

LESSON 4

Multiplying and Dividing Rational Numbers

Prelude

Before studying the concept of multiplication and division operation in $\mathbb{Q}$ we have to know the sign's rule :

Sign's rule in multiplication

$(+) \times (+) = (+)$, $(-) \times (-) = (+)$

$(+) \times (-) = (-)$, $(-) \times (+) = (-)$

For example:

- $3 \times 4 = 12$
- $(-2) \times (-3) = 6$
- $2 \times (-5) = -10$
- $(-4) \times 2 = -8$

Sign's rule in division

$(+) \div (+) = (+)$, $(-) \div (-) = (+)$

$(+) \div (-) = (-)$, $(-) \div (+) = (-)$

For example:

- $8 \div 2 = 4$
- $(-50) \div (-5) = 10$
- $14 \div (-7) = -2$
- $(-20) \div 4 = -5$

First : Multiplication operation

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

i.e. To multiply two rational numbers , multiply their numerators to get the numerator of the product , and multiply their denominators to get the denominator of the product.

For example: $\frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20}$, $-\frac{2}{3} \times \frac{5}{7} = -\frac{2 \times 5}{3 \times 7} = -\frac{10}{21}$

Example 1 Find the result of each of the following in its simplest form :

1. $\frac{3}{6} \times \frac{2}{5}$
2. $-\frac{3}{4} \times \frac{2}{9}$
3. $\frac{2}{4} \times (-2)$
4. $-4\frac{2}{7} \times (-3\frac{1}{6})$

Look at the activity at the end of the book using the Excel program.

Solution

1. $\frac{3}{6} \times \frac{2}{5} = \frac{3 \times 2}{6 \times 5} = \frac{6}{30} = \frac{1}{5}$

Notice that : After multiplying the numbers we should put the product in its simplest form.

2. $-\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}} = -\frac{1}{2} \times \frac{1}{3}$

$= -\frac{1}{6}$

Notice that : When we multiply we can reduce the numerator of one of the numbers with the denominator of the other.

3. $\frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times (-2) = \frac{1}{2} \times (-2) = \frac{1}{{}_{1}\cancel{2}} \times (-\frac{\cancel{2}^{1}}{1})$

$= -1$

Notice that : It is better to put the rational numbers in the simplest form to make the operation easier.

4. $-4\frac{2}{7} \times (-3\frac{1}{6}) = -\frac{\cancel{30}^{5}}{7} \times (-\frac{19}{\cancel{6}_{1}})$

$= -\frac{5}{7} \times (-\frac{19}{1})$

$= \frac{95}{7}$

Notice that : We should convert the mixed number to an improper fraction before carrying out the multiplication operation.

TRY 1 *by yourself*

Find the result of each of the following in its simplest form :

1. $\frac{3}{2} \times \frac{5}{9}$
2. $\frac{8}{5} \times (-\frac{4}{9})$
3. $-5 \times \frac{3}{10}$
4. $-4\frac{1}{2} \times (-\frac{5}{9})$

UNIT 1

Properties of the multiplication operation in $\mathbb{Q}$

1 Closure property :

The product of any two rational numbers is a rational number.

i.e. $\mathbb{Q}$ is closed under multiplication operation.

For example:

The product of the two rational numbers $\frac{3}{5}$ and $\frac{1}{4}$ is $\frac{3}{20}$ which is a rational number too.

2 Commutative property :

If a and b are two rational numbers , then : $a \times b = b \times a$

For example: $\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$, $\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$ **i.e.** $\frac{2}{7} \times \frac{3}{5} = \frac{3}{5} \times \frac{2}{7}$

3 Associative property :

If a , b and c are three rational numbers , then : $(a \times b) \times c = a \times (b \times c)$

For example: $\left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{7}{5} = \frac{1}{6} \times \frac{7}{5} = \frac{7}{30}$, $\frac{1}{2} \times \left(\frac{1}{3} \times \frac{7}{5}\right) = \frac{1}{2} \times \frac{7}{15} = \frac{7}{30}$

i.e. $\left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{7}{5} = \frac{1}{2} \times \left(\frac{1}{3} \times \frac{7}{5}\right)$

4 The existence of multiplicative identity (neutral) element property :

If a is a rational number , then : $a \times 1 = 1 \times a = a$

i.e. As multiplying any rational number by 1 the value of this number does not change.

Then we say : the number 1 is the multiplicative identity (neutral) in $\mathbb{Q}$

For example: $\frac{2}{3} \times 1 = 1 \times \frac{2}{3} = \frac{2}{3}$, $-\frac{3}{7} \times 1 = 1 \times -\frac{3}{7} = -\frac{3}{7}$

5 The existence of multiplicative inverse of the rational number property :

For every rational number $\frac{a}{b}$ except zero there is a multiplicative inverse that is the rational number $\frac{b}{a}$ where $\frac{a}{b} \times \frac{b}{a} = 1$ (the multiplicative identity)

For example:
- The multiplicative inverse of the number $\frac{3}{2}$ is $\frac{2}{3}$ and vice versa the multiplicative inverse of the number $\frac{2}{3}$ is $\frac{3}{2}$
- The multiplicative inverse of the number $-\frac{3}{4}$ is $-\frac{4}{3}$ and vice versa the multiplicative inverse of the number $-\frac{4}{3}$ is $-\frac{3}{4}$

- The multiplicative inverse of the number $\frac{1}{5}$ is 5
  and vice versa the multiplicative inverse of the number 5 is $\frac{1}{5}$

Remarks

- The multiplicative inverse of the rational number is called the reciprocal of the rational number.
- Zero has no multiplicative inverse because : $\frac{1}{\text{zero}}$ is meaningless **i.e.** (undefined)
- The multiplicative inverse of the number 1 is itself and the multiplicative inverse of the number -1 is itself also.
- Multiplying any rational number by zero equals zero.

For example: $0 \times \frac{1}{2} = 0$, $-\frac{5}{8} \times 0 = 0$

6 Property of distributing multiplication over addition and subtraction :

If a, b and c are three rational numbers, then :

1. $a \times (b + c) = a \times b + a \times c$, $(b + c) \times a = b \times a + c \times a$
   i.e. Multiplication is distributed over addition in $\mathbb{Q}$ from right and from left.
2. $a \times (b - c) = a \times b - a \times c$, $(b - c) \times a = b \times a - c \times a$
   i.e. Multiplication is distributed over subtraction in $\mathbb{Q}$ from right and from left.

Example 2 **Use the distributing property to find the value of each of the following :**

1. $\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7}$
2. $\frac{9}{17} \times 21 - \frac{9}{17} \times 4$
3. $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$
4. $\frac{7}{12} \times 5 + \frac{49}{12} - \frac{7}{12} \times 11$

Solution

1. $\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7} = \frac{5}{11}\left(\frac{6}{7} + \frac{1}{7}\right)$ (Distributing multiplication over addition)
   $= \frac{5}{11} \times \frac{7}{7} = \frac{5}{11} \times 1 = \frac{5}{11}$
2. $\frac{9}{17} \times 21 - \frac{9}{17} \times 4 = \frac{9}{17}(21 - 4)$
   $= \frac{9}{17} \times 17 = 9$
3. $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25} = \frac{22}{25}\left(\frac{6}{11} + \frac{5}{11} - 1\right)$
   $= \frac{22}{25}\left(\frac{11}{11} - 1\right) = \frac{22}{25} \times (1 - 1)$
   $= \frac{22}{25} \times \text{zero} = \text{zero}$

4 $\frac{7}{12}\times 5+\frac{49}{12}-\frac{7}{12}\times 11=\frac{7}{12}\times 5-\frac{7}{12}\times 11+\frac{49}{12}$ (Commutative property)

$$=\frac{7}{12}(5-11)+\frac{49}{12}=\frac{7}{12}(-6)+\frac{49}{12}$$

$$=-\frac{42}{12}+\frac{49}{12}=\frac{7}{12}$$

Another solution :

$$\frac{7}{12}\times 5+\frac{49}{12}-\frac{7}{12}\times 11=\frac{7}{12}\times 5+\frac{7}{12}\times 7-\frac{7}{12}\times 11$$

$$=\frac{7}{12}(5+7-11)=\frac{7}{12}\times 1=\frac{7}{12}$$

TRY 2 by yourself

Use the distributive property to find each of the following :

1. $\frac{5}{7}\times\frac{2}{3}+\frac{5}{7}\times\frac{1}{3}$
2. $11\times\frac{3}{10}-\frac{3}{10}$

Second : Division operation

Since every rational number (except zero) has a multiplicative inverse , then we can define the division operation in $\mathbb{Q}$ as follows :

Definition :

If $\frac{a}{b}$, $\frac{c}{d}$ are two rational numbers , $\frac{c}{d}\neq$ zero , then $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$

For example:
- $\frac{2}{3}\div\frac{7}{5}=\frac{2}{3}\times\frac{5}{7}=\frac{10}{21}$
- $-\frac{2}{5}\div\frac{6}{5}=-\frac{\cancel{2}^{1}}{\cancel{5}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{6}_{3}}=-\frac{1}{3}$

Remarks

- Since division by zero is impossible in $\mathbb{Q}$, therefore $\mathbb{Q}$ is not closed with respect to division operation.
- Division operation in $\mathbb{Q}$ is not commutative and not associative.
- There is no identity element in division operation in $\mathbb{Q}$ and hence there are no inverses numbers with respect to division operation in $\mathbb{Q}$

Example 3 Find the result of each of the following in its simplest form :

1. $-\frac{2}{3} \div \frac{5}{3}$
2. $\frac{3}{7} \div (-8)$
3. $2\frac{1}{5} \div 5\frac{1}{2}$
4. $0.2 \div \frac{1}{5}$
5. $\left(\frac{2}{7} + \frac{3}{7}\right) \div \frac{10}{7}$
6. $\left(\frac{5}{6} - \frac{3}{4}\right) \div \left(\frac{7}{12} - \frac{5}{9}\right)$

Look at the activity at the end of the book using the Excel program.

Solution

1. $-\frac{2}{3} \div \frac{5}{3} = -\frac{2}{\cancel{3}_1} \times \frac{\cancel{3}^1}{5} = -\frac{2}{5}$
2. $\frac{3}{7} \div (-8) = \frac{3}{7} \times \left(-\frac{1}{8}\right) = -\frac{3}{56}$
3. $2\frac{1}{5} \div 5\frac{1}{2} = \frac{11}{5} \div \frac{11}{2} = \frac{\cancel{11}^1}{5} \times \frac{2}{\cancel{11}_1} = \frac{2}{5}$
4. $0.2 \div \frac{1}{5} = \frac{2}{10} \div \frac{1}{5} = \frac{2}{10} \times \frac{5}{1} = \frac{10}{10} = 1$
5. $\left(\frac{2}{7} + \frac{3}{7}\right) \div \frac{10}{7} = \frac{5}{7} \div \frac{10}{7} = \frac{\cancel{5}^1}{\cancel{7}_1} \times \frac{\cancel{7}^1}{\cancel{10}_2} = \frac{1}{2}$
6. $\left(\frac{5}{6} - \frac{3}{4}\right) \div \left(\frac{7}{12} - \frac{5}{9}\right) = \left(\frac{10}{12} - \frac{9}{12}\right) \div \left(\frac{21}{36} - \frac{20}{36}\right)$

$= \frac{1}{12} \div \frac{1}{36} = \frac{1}{\cancel{12}_1} \times \frac{\cancel{36}^3}{1} = 3$

TRY 3 by yourself

Find the result of each of the following in its simplest form :

1. $\frac{3}{7} \div \frac{9}{14}$
2. $\frac{3}{4} \div \left(-\frac{15}{2}\right)$
3. $2\frac{1}{3} \div \left(-\frac{7}{3}\right)$
4. $-\frac{5}{6} \div 10$

UNIT 1

Example 4 If $x = -\frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, find the numerical value of each of the following :

1. $\frac{y}{z}$
2. $\frac{xy}{z}$
3. $\frac{x}{y} - \frac{y}{z}$

Solution

1. $\frac{y}{z} = \frac{3}{4} \div (-3) = \frac{\cancel{3}^{1}}{4} \times \left(-\frac{1}{\cancel{3}_{1}}\right) = -\frac{1}{4}$

2. $\frac{xy}{z} = \left(-\frac{1}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{4}\right) \div (-3) = -\frac{1}{4} \div (-3)$

$= -\frac{1}{4} \times \left(-\frac{1}{3}\right) = \frac{1}{12}$

Notice that : $xy = x \times y$

3. $\frac{x}{y} = -\frac{1}{3} \div \frac{3}{4} = -\frac{1}{3} \times \frac{4}{3} = -\frac{4}{9}$, $\frac{y}{z} = -\frac{1}{4}$

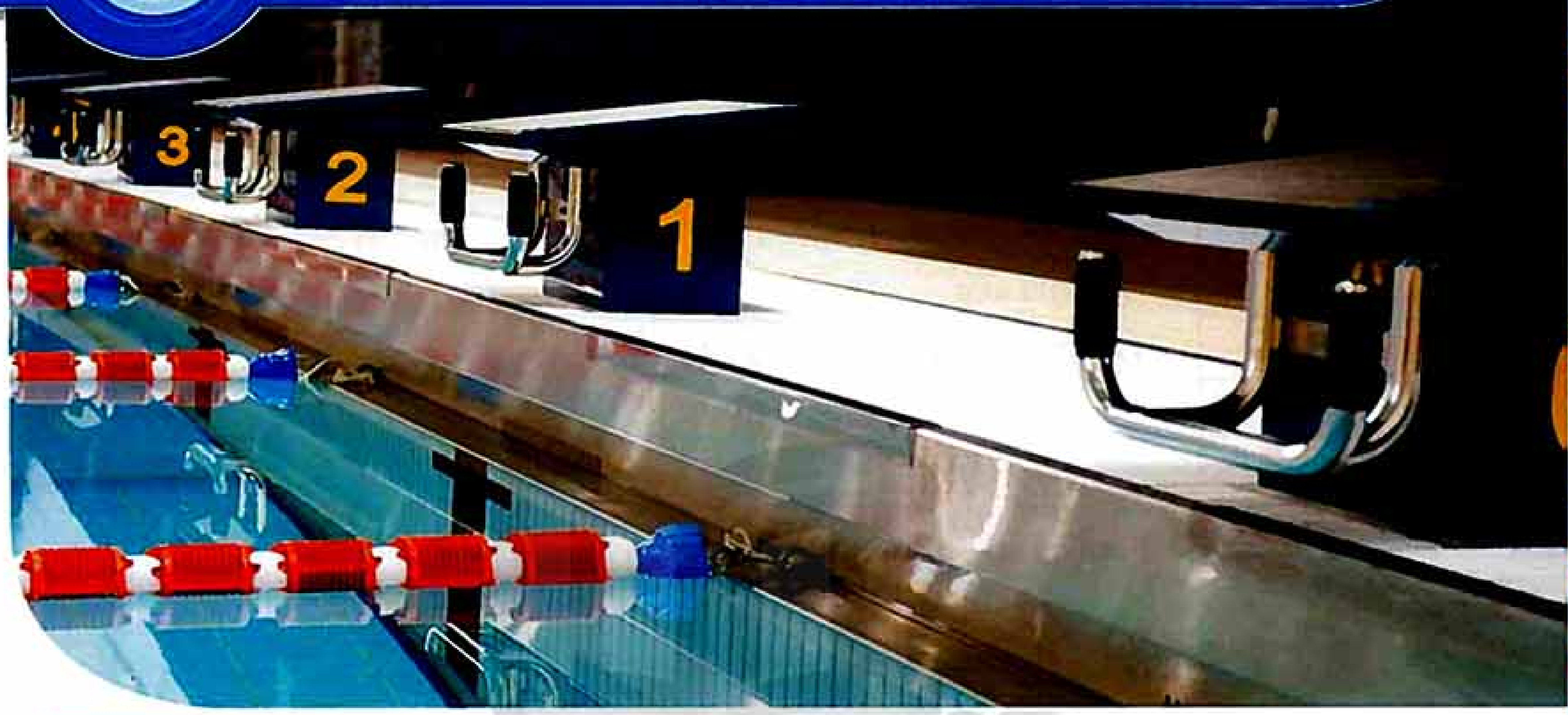
$\frac{x}{y} - \frac{y}{z} = -\frac{4}{9} - \left(-\frac{1}{4}\right) = -\frac{16}{36} + \frac{9}{36} = -\frac{7}{36}$

Answers of try by yourself

1. (1) $\frac{5}{6}$ (2) $-\frac{32}{45}$ (3) $-\frac{3}{2}$ (4) $\frac{5}{2}$
2. (1) $\frac{5}{7}$ (2) 3
3. (1) $\frac{2}{3}$ (2) $-\frac{1}{10}$ (3) -1 (4) $-\frac{1}{12}$

LESSON

5 Applications on Rational Numbers



The distance between two numbers

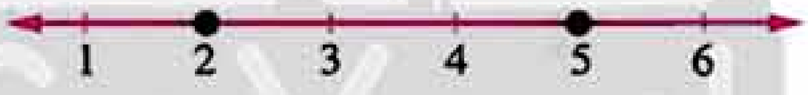
It is possible to express the distance between the two numbers x , y on the number line by using the absolute value as follows :

$|x - y|$ or $|y - x|$

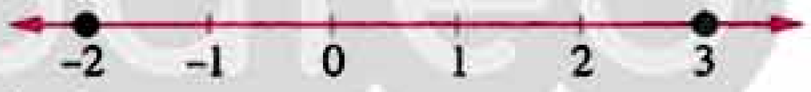
Notice that :
$|x - y| = |y - x|$

For example:

- The distance between 2 and 5 $= |2 - 5| = |-3|$
  $= 3$ length units.



- The distance between -2 and 3 $= |-2 - 3|$
  $= |-5| = 5$ length units.



- The distance between -1 and -5 $= |-1 - (-5)|$
  $= |-1 + 5|$
  $= |4| = 4$ length units.

Example 1 **Find a rational number lying at the middle of the way between 3 and 7**

Solution

- If we notice the number line in the opposite figure we find :
  The number that lies at the midpoint of the distance between 3 and 7 is 5
  , from that we can deduce the following rule :

Notice that :
There is a unique rational number lying at the middle of the way between any two rational numbers

The number that lies at the midpoint of the way between any two numbers
= the smaller number $+$ $\frac{1}{2}$ the distance between the two numbers
or = the greater number $-$ $\frac{1}{2}$ the distance between the two numbers

Where the distance between the two numbers 3 and 7 is $|3-7| = |-4|$ $= 4$ length unit.

i.e. The required number is : $3 + \frac{1}{\cancel{2}_1} \times \cancel{4}^{2} = 3 + 2 = 5$

or $7 - \frac{1}{\cancel{2}_1} \times \cancel{4}^{2} = 7 - 2 = 5$

Example 2 **Find a rational number in half-way between $\frac{2}{5}$ and $\frac{3}{7}$**

Solution

Since L.C.M. of the denominators = 35

Therefore , $\frac{2}{5} = \frac{2 \times 7}{5 \times 7} = \frac{14}{35}$, $\frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35}$

and since $\frac{14}{35} < \frac{15}{35}$

Therefore , the required number is :

$$\frac{14}{35} + \frac{1}{2}\left|\frac{15}{35} - \frac{14}{35}\right| = \frac{14}{35} + \frac{1}{2} \times \frac{1}{35} = \frac{14}{35} + \frac{1}{70} = \frac{28}{70} + \frac{1}{70} = \frac{29}{70}$$

TRY by yourself 1

Find a rational number in half-way between $\frac{5}{6}$ and $\frac{3}{8}$

Example 3 **Find a rational number lying at one third of the way between 2 and 8**

1 From the side of the smaller number.

2 From the side of the greater number.

Solution

By observing the opposite number line we find the following :

The number that lies at one third of the way between two numbers

1 From the side of the smaller number = the smaller number + $\frac{1}{3}$ the distance between the two numbers.

2 From the side of the greater number = the greater number $-$ $\frac{1}{3}$ the distance between the two numbers.

Therefore

1. The number that lies at one third of the way between 2 and 8 from the side of 2

$= 2 + \frac{1}{3}|8-2| = 2 + \frac{1}{\cancel{3}_1} \times \cancel{6}^2 = 4$

2. The number that lies at one third of the way between 2 and 8 from the side of 8

$= 8 - \frac{1}{3}|8-2| = 8 - \frac{1}{\cancel{3}_1} \times \cancel{6}^2 = 6$

Example 4 **Find a rational number lying at one fourth of the way between :**

$-\frac{1}{6}$ and $-\frac{1}{3}$ from the side of the smaller number.

Solution

Since L.C.M. of the denominators is 6

$-\frac{1}{3} = -\frac{2}{6}$

The greater number $= -\frac{1}{6}$,

The smaller number $= -\frac{2}{6}$

The distance between the two numbers $= \left|-\frac{1}{6} - \left(-\frac{2}{6}\right)\right| = \left|\frac{1}{6}\right| = \frac{1}{6}$

The required number is :

The smaller number + $\frac{1}{4}$ the distance between the two numbers

$= -\frac{2}{6} + \frac{1}{4} \times \frac{1}{6} = -\frac{2}{6} + \frac{1}{24} = -\frac{8}{24} + \frac{1}{24} = -\frac{7}{24}$

TRY 2 by yourself

Find a rational number lying at one fifth of the way between :

$\frac{2}{5}$ and $\frac{4}{7}$ from the side of the greater number.

Answers of try by yourself

1 $\frac{29}{48}$ 2 $\frac{94}{175}$

UNIT 2

Algebra

Lessons of the unit :

1. Algebraic terms and algebraic expressions.
2. Like algebraic terms.
3. Adding and subtracting algebraic expressions.
4. Multiplying and dividing algebraic terms.
5. Multiplying a monomial by an algebraic expression.
6. Multiplying a binomial by an algebraic expression.
7. Dividing an algebraic expression by a monomial.
8. Dividing an algebraic expression by another one.
9. Factorization by identifying the highest common factor (H.C.F.).

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the algebraic term and the algebraic expression and their degrees.
- perform the operations on the like algebraic terms.
- reduce the algebraic expression.
- multiply a monomial by an algebraic expression.
- perform the operations on the algebraic expressions.
- multiply two binomials by inspection.
- divide an algebraic expression by a monomial.
- divide an algebraic expression by another one.
- factorize the algebraic expression by identifying the highest common factor.
- solve different problems on the operations on the algebraic terms and the algebraic expressions.
- understand the role of mathematics in solving the real life problems.

Use your smart phone or tablet to scan the QR Code and enjoy watching videos.

Al Khowarezmy

Mohamed Ibn Moussa Al Khowarezmy (781 A.D. - 847 A.D.)

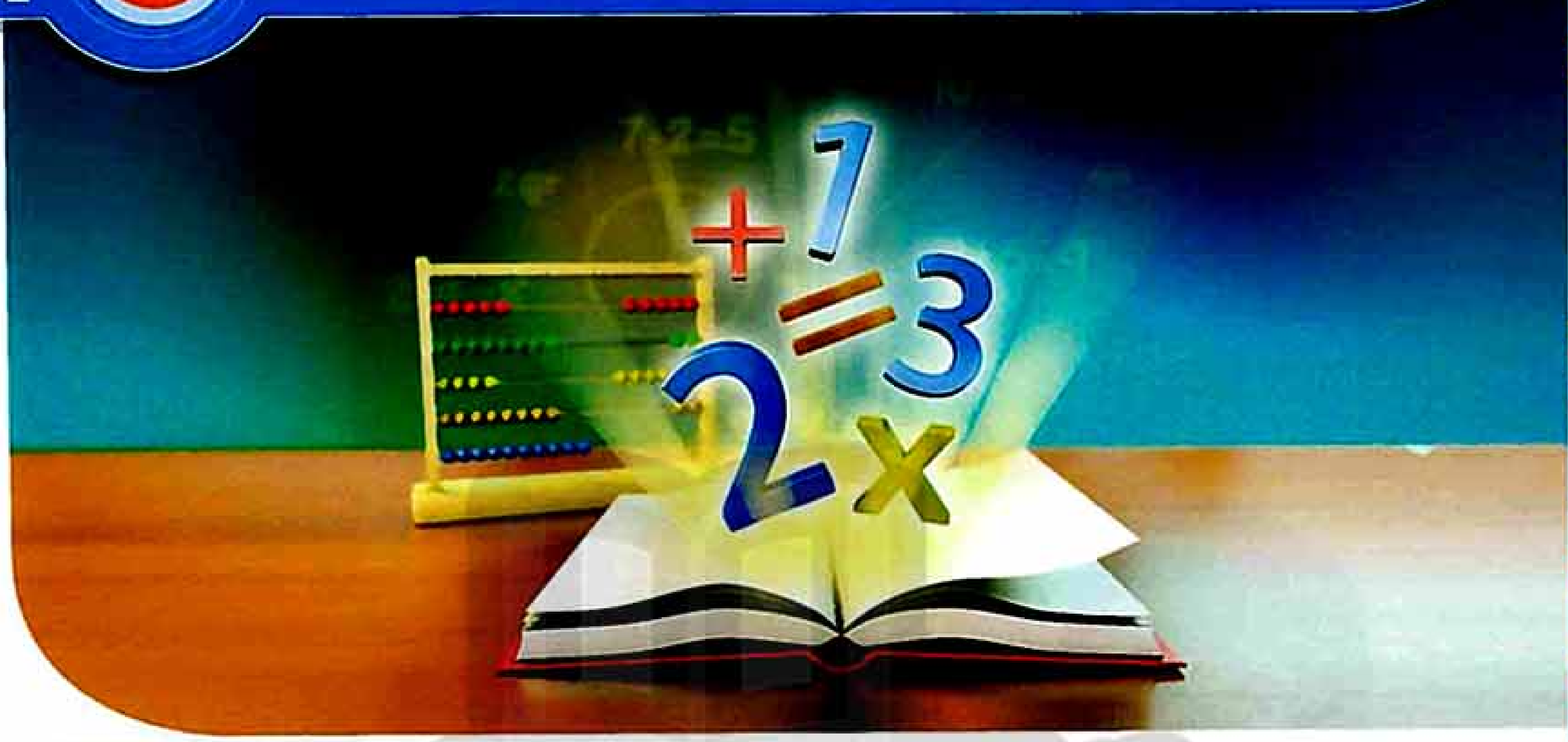
(the father of algebra)

A muslim Iraqi scientist (781 A.D. – 847 A.D.)

He is the first one to use algebra.

Thanks to Al Khowarezmy , the world knew the use of the Arabian digits which changed our concept of numbers. He also introduced the concept of zero.

LESSON 1 Algebraic Terms and Algebraic Expressions



Introduction : variable and constant

- The variable is a letter as : x or y or n or ... , which represents any number in a specified set of numbers.

For example:

We can write 7 n to represent the multiples of 7

In this case , the letter n represents any number in the set of integers.

If we replace n by 5 , we obtain : $7n = 7 \times 5 = 35$ which is a multiple of 7

and if we replace n by 100

, we obtain : $7n = 7 \times 100 = 700$ which is a multiple of 7 , and so on.

- The constant is a number or letter represents only one number.

Algebraic terms and algebraic expressions

First : Algebraic term

The algebraic term is a number , a variable or the product of numbers and variables.

i.e. The algebraic term consists of the product of two factors or more.

* **In the previous example :**
  7 n is an algebraic term which consists of two factors : 7 and n
  7 is called numerical factor (coefficient) and n is called algebraic factor.
* Also , $-5xy$ is an algebraic term which consists of the factors : -5 , x and y
  -5 (numerical factor) , x (algebraic factor) and y (algebraic factor)

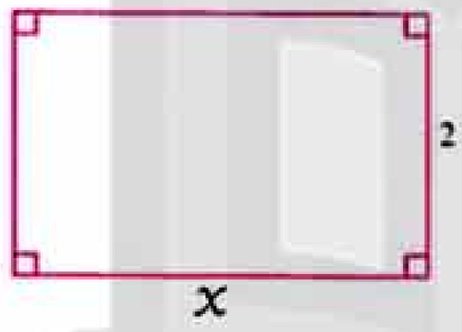
Second : Algebraic expression

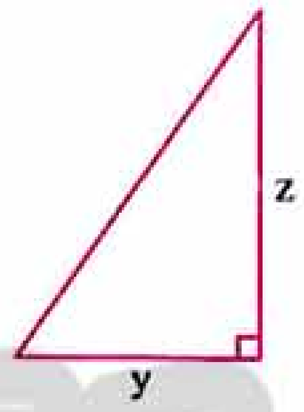
The algebraic expression consists of one or more terms connected by the sign + or –

For example:

- $5a + 3b$ is an algebraic expression which consists of the two terms : $5a$ and $3b$ "binomial"
- $5y^2 + 2xy - 3x$ is an algebraic expression which consists of three factors. "trinomial"
- $3x$ is an algebraic expression which consists of one factor.

Example 1 Write the algebraic term that represents the area of each of the following shapes :

1 

2 

Solution

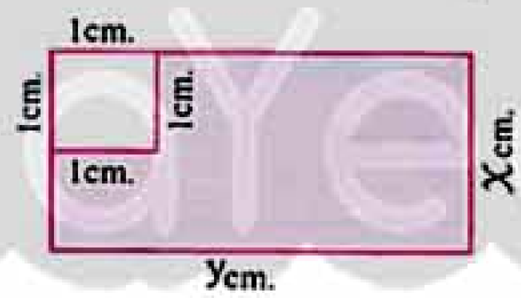
1 The area of the rectangle = length × width = $2x$

2 The area of the triangle = $\frac{1}{2}$ base length × height = $\frac{1}{2}yz$

Example 2 Write the algebraic expression that expresses each of the following :

1 The length of $\overline{AB}$

2 The area of the coloured part



Solution

1 The length of $\overline{AB}$ = AC + CB **i.e.** The length of $\overline{AB} = x + y$

It is an algebraic expression consisting of two terms.

2 The area of the coloured part = the area of the rectangle – the area of the square

$= (x \times y) - (1 \times 1)$

i.e. The area of the coloured part = $(xy - 1)$ cm^2

It is an algebraic expression consisting of two terms.

Remark

The algebraic term that has no algebraic factors is called the absolute term as the algebraic term 3 in the algebraic expression : $y^2 - 2y + 3$

UNIT 2

The degree of the algebraic term

The degree of the algebraic term is the sum of the indices of the algebraic factors in this term.

For example:

- The term 2 a is of the 1st degree because the index of a is 1
- The term $-7x^2$ is of the 2nd degree because the index of x is 2
- The term $-5xy$ is of the 2nd degree because the sum of indices of the two symbols x and y is 2
- The term $7m^2n$ is of the 3rd degree because the sum of indices of the two symbols m and n is 3

Remark

Any number is an algebraic term of zero degree.

For example:

The number -2 is an algebraic term of zero degree because it can be written in the form : $-2 \times x^0$ "where : $x^0 = 1$"

TRY 1 by yourself

Complete the following table :

The algebraic term	$5x$	$3xy$	$-5a^2$	$4x^2y$	$-2a^2b^2$	$15a^3b$	x	-4	$(-3)^2$
Its coefficient	....	....	....	....	....	....	....	....	....
Its degree	....	....	....	....	....	....	....	....	....

The degree of the algebraic expression

The degree of the algebraic expression is the highest degree of the terms forming it.

For example:

- The algebraic expression : $5x - 3$ is of the 1st degree because $5x$ is the term of the highest degree that is 1
- The algebraic expression : $7x^2 - 3x + 1$ is of the 2nd degree because $7x^2$ is the term of the highest degree that is 2
- The algebraic expression : $5ab - 2a^2b - b^2$ is of the 3rd degree because $-2a^2b$ is the term of the highest degree that is 3

Example 3 **Arrange the algebraic expression : $5x + 2x^3 - 4 - x^2$:**

1. According to the descending order of the indices of x
2. According to the ascending order of the indices of x

Solution

1. According to the descending order of the indices of x
   The expression $= 2x^3 - x^2 + 5x - 4$
2. According to the ascending order of the indices of x
   The expression $= -4 + 5x - x^2 + 2x^3$

Example 4 **State the degree of the algebraic expression : $2a^3b^2 - 7ab^3 + 5a^2b$, then arrange it :**

1. According to the descending order of the indices of a
2. According to the ascending order of the indices of b

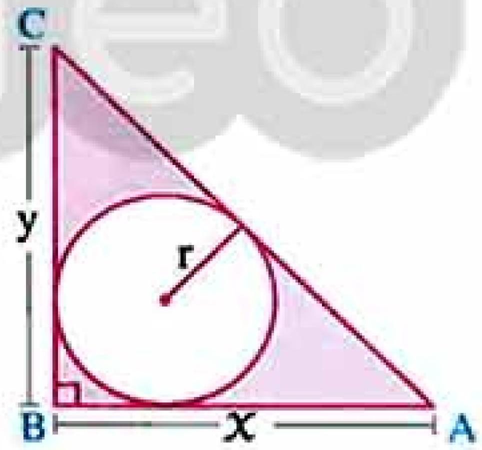
Solution

The expression is of the fifth degree because the term $2a^3b^2$ is the term of the highest degree that is 5

1. According to the descending order of the indices of a
   The expression $= 2a^3b^2 + 5a^2b - 7ab^3$
2. According to the ascending order of the indices of b
   The expression $= 5a^2b + 2a^3b^2 - 7ab^3$

Example 5 **From the opposite figure :**

Write the algebraic expression which represents the area of the coloured part , then state its degree (given that the area of the circle $= \pi r^2$)



Solution

The area of the coloured part = the area of Δ ABC – the area of the circle

$$= \frac{1}{2} x \times y - \pi r^2$$

Therefore the algebraic expression which represents the area of the coloured part

$= \frac{1}{2} x y - \pi r^2$

and it is of the 2^{nd} degree.

Notice that :
π expresses a number has an approximated value but does not express an algebraic symbol.

UNIT 2

TRY 2 by yourself

Complete the following table :

The algebraic expression	The number of its terms	Its name	Its degree
$-2a^2b^3$	....	....	....
$a^3 - 5a^2b^2 + 3b^2$	....	....	....
$\frac{1}{2}a + \frac{1}{4}b - 5$	....	....	....
$2x^2y + 5xy + 4y$	....	....	....
$1 - 7x^2y$	....	....	....
$3^2x^2 + 2^4x$	....	....	....

Answers of try by yourself

1 Coefficient of the algebraic term : 5 , 3 , −5 , 4 , −2 , 15 , 1 , −4 , 9

Degree of the algebraic term : 1 , 2 , 2 , 3 , 4 , 4 , 1 , zero , zero

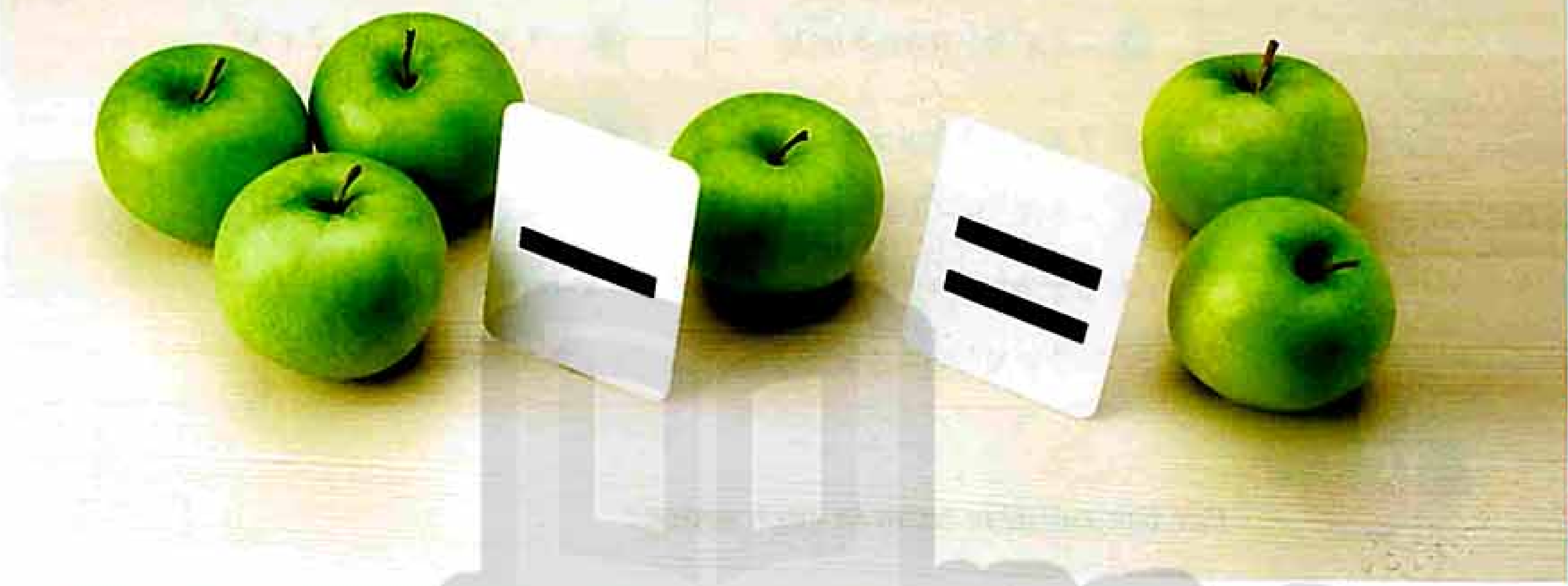
2 Number of terms of the algebraic expression : 1 , 3 , 3 , 3 , 2 , 2

Name of the algebraic expression : monomial , trinomial , trinomial , trinomial , binomial , binomial.

Degree of the algebraic expression : 5 , 4 , 1 , 3 , 3 , 2

LESSON 2

Like Algebraic Terms



The algebraic terms are said to be like if the algebraic symbols forming their factors are like and the indices of these symbols are equal.

Examples for like algebraic terms :

- $2a$, a and $-5a$
- $2x^2y$, $4yx^2$ and $-\frac{1}{2}x^2y$

Notice that :
$x^2y = yx^2$ «commutative property»

Examples for unlike algebraic terms :

- $2x$, $-3x^2$ and $7x^3$ are unlike algebraic terms **because their indices are different.**
- $4x^2$, $5xy$ and $-y^2$ are unlike algebraic terms **because their symbols are different.**

Adding and subtracting like terms

Adding or subtracting operation is performed as the following :

1. Add or subtract the numerical coefficients.
2. Use the sum or the difference as the coefficient of the result algebraic term.

Example 1 **Add :**

1. $5a$, $3a$, a , $6a$
2. $7ab^2$, $-2b^2a$, $-4b^2a$, ab^2

Solution

1. $5a + 3a + a + 6a = (5 + 3 + 1 + 6)a = 15a$
2. $7ab^2 + (-2b^2a) + (-4b^2a) + ab^2 = [7 + (-2) + (-4) + 1]ab^2 = 2ab^2$

UNIT 2

Example 2 **Subtract :**

1. $5xy$ from $7xy$
2. $2x^2y$ from $-5x^2y$
3. $-3a^2b^2$ from $5a^2b^2$
4. $-3x^3y$ from $-2yx^3$

Solution

1. $7xy - 5xy = (7-5)xy = 2xy$
2. $-5x^2y - 2x^2y = (-5-2)x^2y = -7x^2y$
3. $5a^2b^2 - (-3a^2b^2) = 5a^2b^2 + 3a^2b^2 = 8a^2b^2$
4. $-2yx^3 - (-3x^3y) = -2x^3y + 3x^3y = x^3y$

TRY 1 by yourself

Put the suitable term in each space :

1. $4x + 5x = \boxed{}$
2. $2x - 4x + x = \boxed{}$
3. $3x^2 + \boxed{} = 5x^2$
4. $7a^3 - \boxed{} = 2a^3$
5. $2b^4 + \boxed{} = b^4$
6. $3y^5 - \boxed{} = 5y^5$
7. $4x$ is less than $7x$ by $\boxed{}$
8. $7y$ is more than $-2y$ by $\boxed{}$

Reducing the algebraic expression

The algebraic expression is said to be in its simplest form if all its terms are unlike.

For example:

- The expression : $9x^2 - 3x + 1$ is in its simplest form because there are not like terms among its terms.
- The expression : $6x + 7y + 4x + 3y$ is not in its simplest form because there are like terms among its terms which are : $6x, 4x$ and $7y, 3y$

Reducing the algebraic expression means putting it in the simplest form. This will be carried out by adding like terms using the commutative and associative properties.

Example 3 **Reduce to the simplest form :**

1. $6x + 7y + 4x - 3y$
2. $6x^2 - 7x - 4x^2 + 5x - 3x + x^2$

Solution

1 $6x + 7y + 4x - 3y$

$= 6x + 4x + 7y - 3y$ (commutative property)

$= (6x + 4x) + (7y - 3y)$ (associative property)

$= 10x + 4y$

> **Notice that :**
> We can not add or subtract unlike terms.
> **For example:**
> $10x + 4y \neq 14xy$

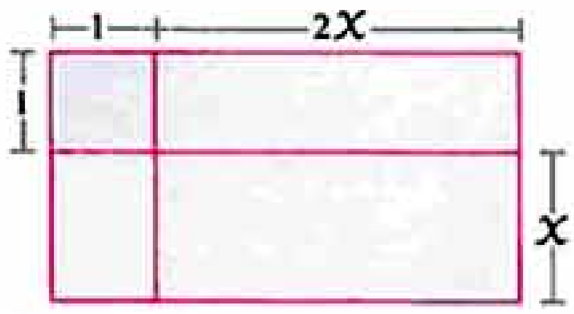
2 The expression $= (6x^2 - 4x^2 + x^2) + (-7x + 5x - 3x)$

(commutative and associative properties)

$= 3x^2 + (-5x) = 3x^2 - 5x$

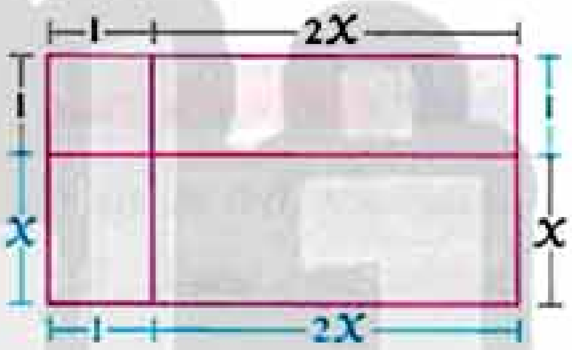
Example 4 **In the opposite figure :**

Write the algebraic expression that expresses the perimeter of the opposite figure.



Solution

We can deduce that the left lengths of the figure as in the opposite figure , then :



The perimeter of the figure

$= 2x + 1 + 1 + x + 1 + 2x + x + 1$

$= (2x + x + 2x + x) + (1 + 1 + 1 + 1)$

$= (6x + 4)$ length unit.

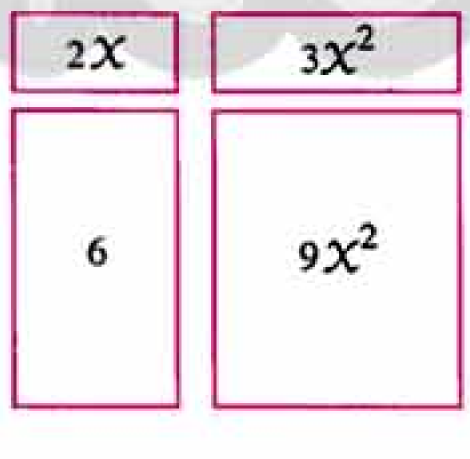
TRY 2 *by yourself*

1 ***Reduce the following expression to its simplest form :***

$a^2 + 3a - 4 + 4a^2 - 5a + 1$

2 ***In the opposite figure :***

Write the algebraic expression which expresses the sum of the areas of the rectangles which are shown in the opposite figure.



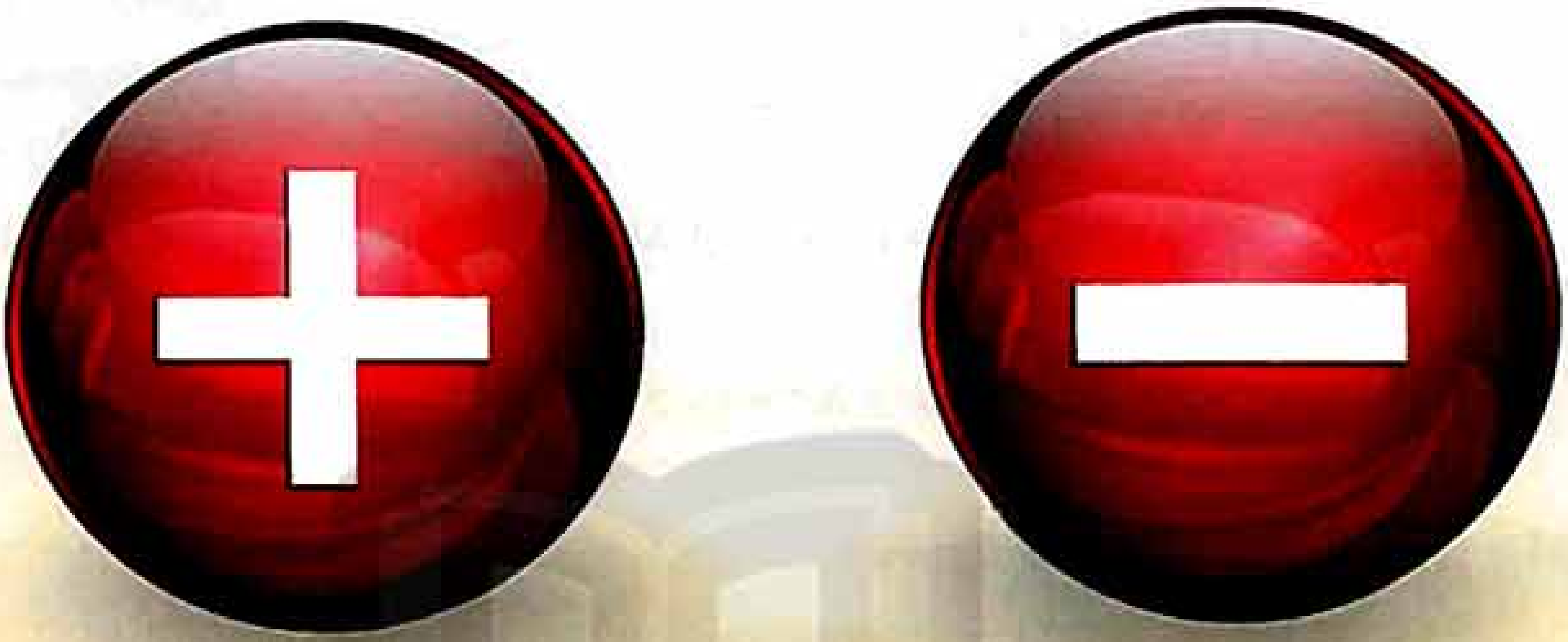
Answers of try by yourself

1 [1] $6x$ [2] $-x$ [3] $2x^2$ [4] $5a^3$ [5] $-b^4$ [6] $-2y^5$ [7] $3x$ [8] $9y$

2 [1] $5a^2 - 2a - 3$ [2] $12x^2 + 2x + 6$

LESSON

3 Adding and Subtracting Algebraic Expressions



First : Adding algebraic expressions

There are two methods for adding algebraic expressions as shown in the following example.

Example 1 **Add the two expressions :** $5a - 7b + 3$ **and** $2b - 1 - a$

Solution

The horizontal method :

In this method , we use the commutative and associative properties :

The sum $= (5a - 7b + 3) + (2b - 1 - a)$

$= (5a - a) + (-7b + 2b) + (3 - 1)$

(commutative and associative properties)

$= 4a - 5b + 2$

The vertical method :

In this method , we use the commutative property to arrange the two expressions such that the like terms lie under each other as follows :

The first expression	: $5a - 7b + 3$
The second expression	: $-a + 2b - 1$
The sum	$= 4a - 5b + 2$

Example 2 **Add the following expressions :**

$3x^3-4x^2+2x-1$, $5x^2-2x^3+3$ **and** $2-3x+x^2$

Solution

It is better to rearrange each expression ascendingly or descendingly according to the indices of the symbol x such that we leave a space under the terms that have no like terms to them.

The first expression : $3x^3 \quad -4x^2+2x \quad -1$

The second expression : $-2x^3 \quad +5x^2 \qquad +3$

The third expression : $\qquad + \quad x^2-3x \quad +2$

The sum $= \quad x^3 \quad +2x^2- \quad x \quad +4$

Example 3 **Add :** $4x^2-3xy+y^2$ **and** $3xy-3x^2+2y^2$

, then find the numerical value of the result when : $x=-2$ **and** $y=1$

Solution

$$\begin{array}{r} 4x^2-3xy+y^2 \\ -3x^2+3xy+2y^2 \\ \hline \end{array}$$

The sum $= \quad x^2 \quad +\text{zero} \quad +3y^2 = x^2+3y^2$

The numerical value $=(-2)^2+3\times 1^2=4+3=7$

TRY 1 by yourself

Add : $3x^2-5+2x$, $x+5x^2+7$ ***and*** $-4x^2-3$

, then find the numerical value of the result when : $x=2$

The additive inverse of the algebraic expression

- The additive inverse of the algebraic expression is that whose terms are the additive inverses of the terms of the first expression.
- The sum of any algebraic expression and its additive inverse is equal to zero.

For example:

The algebraic expression : $x^2 \quad - \quad 2x \quad +3$

Its additive inverse : $-x^2 \quad + \quad 2x \quad -3$

The sum $= \quad 0 \quad + \quad 0 \quad +0=0$

UNIT 2

Second : Subtracting algebraic expressions

We have two methods to subtract algebraic expressions as we studied before in addition. That will be shown in the following example.

Example 4 **Subtract :** $5x - 3y + 2z$ **from** $2y - z + 7x$

Solution

The horizontal method :

In this method , we put the subtraction operation in the form :

The remainder = (the minuend) – (the subtrahend)

After removing brackets , we reduce the like terms.

Therefore , the remainder $= (2y - z + 7x) - (5x - 3y + 2z)$

$= 2y - z + 7x - 5x + 3y - 2z$

$= (7x - 5x) + (2y + 3y) + (-z - 2z)$

$= 2x + 5y - 3z$

The vertical method :

In this method , we rearrange the terms of the subtrahend down the terms of the minuend , then we add the minuend to the additive inverse of the subtrahend.

The minuend : $2y \quad -z \quad +7x$

The subtrahend : $\overset{+}{-}3y \overset{-}{+} 2z \overset{-}{+} 5x$

The remainder = $5y - 3z + 2x$

Notice that :
We change the signs of the subtrahend terms to get its additive inverse.

Remember that :

- Subtract a from b , means b – a
- What is the increase of a than b ? means a – b
- What is the decrease of a than b ? means b – a
- What is the expression which should be added to a to get b ? means b – a
- What is the expression which should be subtracted from a to get b ? means a – b

Example 5 What is the expression that should be added to $8 - 3a^2 + 2a^3$ to get $5 + 4a^3 - 7a$?

Solution

$$\begin{array}{lcrrrr} \text{The minuend} & : & 4a^3 & & -7a & +5 \\ \text{The subtrahend} & : & \overset{-}{+2a^3} & \overset{+}{-3a^2} & & \overset{-}{+8} \\ \hline \text{The remainder} & = & 2a^3 & +3a^2 & -7a & -3 \end{array}$$

Notice that :

We arranged the terms of the minuend and the terms of the subtrahend descendingly according to the indices of a , and we left spaces up and down the terms which have no like terms.

TRY 2 by yourself

What is the expression which should be subtracted from :

$-x^2 + 2x - 1$ to get $3x^2 - 5$?

Example 6 What is the increase of $3a^2 - 4b^2 + 2ab$ than the sum of $2a^2 - 3ab + b^2$ and $2b^2 + a^2 + ab$?

Solution

$$\begin{array}{lrrr} & 2a^2 & -3ab & +b^2 \\ & a^2 & +ab & +2b^2 \\ \hline \text{The sum} = & 3a^2 & -2ab & +3b^2 \end{array}$$

To get the increase , subtract the resulting sum from the given expression :

$$\begin{array}{lrrr} & 3a^2 & +2ab & -4b^2 \\ & \overset{-}{3a^2} & \overset{+}{-2ab} & \overset{-}{+3b^2} \\ \hline \text{The increase} = & & 4ab & -7b^2 \end{array}$$

TRY 3 by yourself

What is the decrease of $7 - 5a + a^2$ than $3a^2 - 5a - 2$?

Answers of try by yourself

1. $4x^2 + 3x - 1$, 21
2. $-4x^2 + 2x + 4$
3. $2a^2 - 9$

LESSON 4 Multiplying and Dividing Algebraic Terms

Multiplying the like bases

- **We know that :**

$2^3 = 2 \times 2 \times 2 = 8$, $2^2 = 2 \times 2 = 4$, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

- And we know that : $8 \times 4 = 32$ **i.e.** $2^3 \times 2^2 = 2^5$ "Note the addition of the indices"

Generally

When multiplying the like bases , we add their indices.

i.e. If a is a rational number , m and n are two positive integers , then :

$a^m \times a^n = a^{m+n}$

Look at the activity at the end of the book using the Excel program.

Dividing the like bases

- **We know that :**

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, $2^3 = 2 \times 2 \times 2 = 8$, $2^2 = 2 \times 2 = 4$

- And we know that : $\frac{32}{8} = 4$ **i.e.** $\frac{2^5}{2^3} = 2^2$ "Note the subtraction of the indices"

Generally

When dividing the like bases , subtract their indices.

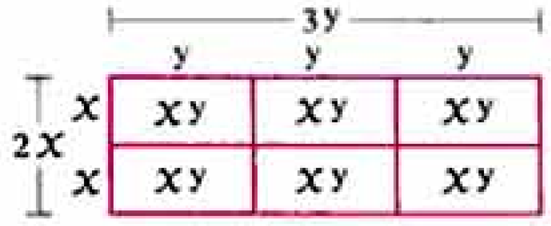
i.e. If a is a rational number not equal to zero , m and n are two positive integers where $m \geq n$, then : $a^m \div a^n = a^{m-n}$

Look at the activity at the end of the book using the Excel program.

First : Multiplying the algebraic terms

In the opposite figure :

We can calculate the area of the rectangle by two different methods :



1 The area of the rectangle = length × width = $3\,y \times 2\,x$

2 The area of the rectangle = the sum of areas of the small rectangles into which the rectangle is divided = $x\,y + x\,y + x\,y + x\,y + x\,y + x\,y = 6\,x\,y$

i.e. $3\,y \times 2\,x = 6\,x\,y$

From the previous, we deduce that :

When multiplying the algebraic terms , follow the following :

1 Multiply the coefficients using the signs rule.

2 Multiply the symbols by adding the indices of symbols which have like bases.

For example:

- $2\,a \times 5\,b = (2 \times 5) \times (a \times b) = 10\,a\,b$
- $(5\,x^2) \times (3\,x) = (5 \times 3) \times (x^2 \times x) = 15\,x^3$

Remark

After training , you can give the result directly.

Example 1 **Find the result of each of the following :**

1 $5\,a^3\,b \times 3\,a\,b$

2 $\frac{3}{4}\,a^2 \times \frac{4}{3}\,a$

3 $\frac{2}{5}\,x^2 \times (-15\,x^3)$

Solution

1 $5\,a^3\,b \times 3\,a\,b = 15\,a^4\,b^2$

2 $\frac{3}{4}\,a^2 \times \frac{4}{3}\,a = a^3$

3 $\frac{2}{5}\,x^2 \times (-15\,x^3) = -6\,x^5$

TRY 1 by yourself

Complete each of the following :

1 $2\,a \times (-3\,a\,b) =$ ....

2 $-2\,x^2\,y \times 3\,x\,y^2 =$ ....

3 $-4\,\ell\,m^2 \times \frac{1}{2}\,\ell^2\,m^2 =$ ....

4 $\frac{2}{3}\,m^2\,n \times \frac{9}{4}\,n =$ ....

UNIT 2

Second : Dividing the algebraic terms

When dividing an algebraic term by another algebraic term , follow the following :

1. Divide the coefficients using the signs rule.
2. Divide the symbols taking care that the indices of like bases should be subtracted. (subtracting the indices of the divisor from the indices of the dividend)

Example 2 **Find the quotient of each of the following :**

1. $12\,a^3$ **by** $3\,a$
2. $21\,x$ **by** (-3)
3. $-15\,x^2 y^3$ **by** $5\,x y^2$
4. $-24\,a^5 b^3 c^2$ **by** $(-8\,a^2 b)$

Solution

1. $12\,a^3 \div 3\,a = 4\,a^{3-1} = 4\,a^2$
2. $21\,x \div (-3) = -7\,x$
3. $-15\,x^2 y^3 \div 5\,x y^2 = -3\,x^{2-1} y^{3-2}$
   $= -3\,x y$
4. $-24\,a^5 b^3 c^2 \div (-8\,a^2 b) = 3\,a^{5-2} b^{3-1} c^2$
   $= 3\,a^3 b^2 c^2$

Notice that :
We can express dividing a term by another in the form of a fraction.
i.e. We write $\frac{12\,a^3}{3\,a} = 4\,a^2$

Remarks

1. The quotient of two equal factors is 1
   Hence , we can cancel the equal factors in division operation.
   For example: $\frac{-15\,a^5 b^3 c^2}{3\,a^5 b^3 c} = -5\,c$
2. Dividing any term by zero is meaningless. Thus all the problems that we will deal with which contains symbols , the divisor is not equal to zero.

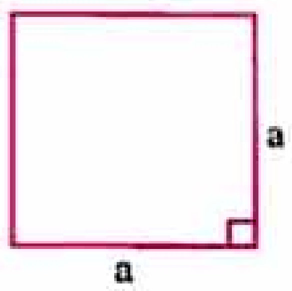
TRY 2 by yourself

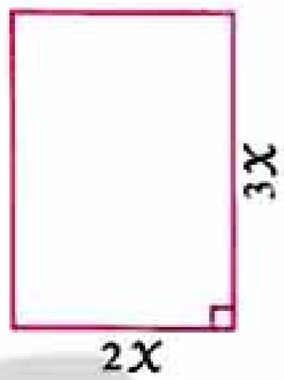
Complete each of the following :

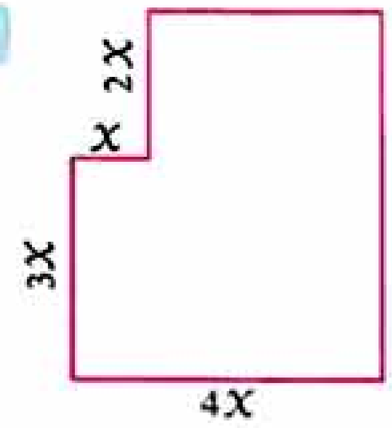
1. $5\,a^6 \div 5\,a = \dots\dots$
2. $\frac{6\,a^2 b^3}{-3\,a b^2} = \dots\dots$
3. $\frac{-12\,x^3 y^2}{4\,x^2 y^2} = \dots\dots$
4. $-8\,x^5 y^3 \div (-y^3 x^4) = \dots\dots$
5. $10\,x^5 \div \dots\dots = 2\,x^3$
6. $48\,a^4 b^7 = 12\,a^2 b^2 \times \dots\dots$

Applications on multiplying and dividing algebraic terms

Example 3 **Calculate the perimeter and the area of each figure of the following :**

1


2


3


Solution

1 • The perimeter of the square = side length × 4

$= a \times 4 = 4a$

• The area of the square = side length × itself

$= a \times a = a^2$

2 • The perimeter of the rectangle = (length + width) × 2

$= (3x + 2x) \times 2$

$= (5x) \times 2 = 10x$

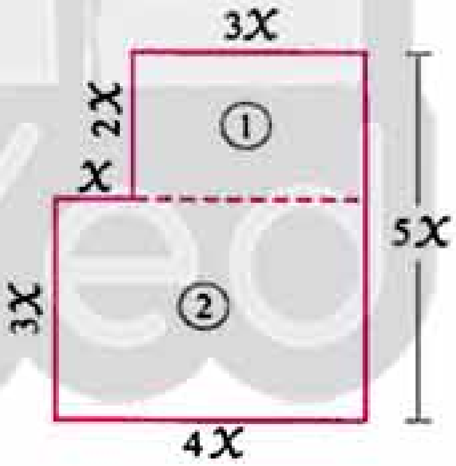
• The area of the rectangle = length × width

$= (3x) \times (2x) = 6x^2$

3 • The perimeter of the figure

$= 5x + 4x + 3x + x + 2x + 3x = 18x$

• To find the area of the figure , we can divide it into two parts as shown in the figure , then : we find the sum of areas of the two parts.



Therefore ,

the area of the figure = the area of part (1) + the area of part (2)

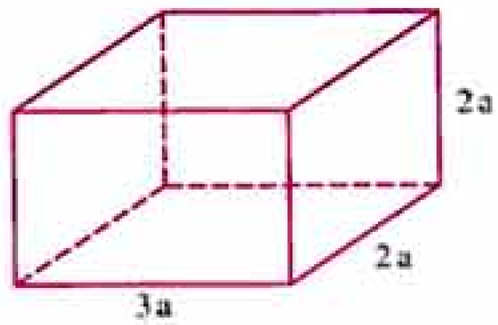
$= (3x \times 2x) + (4x \times 3x)$

$= 6x^2 + 12x^2 = 18x^2$

Try to solve number 3 using a different way of dividing the figure.

UNIT 2

Example 4 **Calculate the total surface area and the volume of the opposite solid.**



Solution

- The total surface area of the cuboid

= its lateral area + (2 × the area of the base)

$= 2\ (2\ a + 3\ a) \times 2\ a + 2 \times 2\ a \times 3\ a$

$= 10\ a \times 2\ a + 12\ a^2 = 20\ a^2 + 12\ a^2$

$= 32\ a^2$

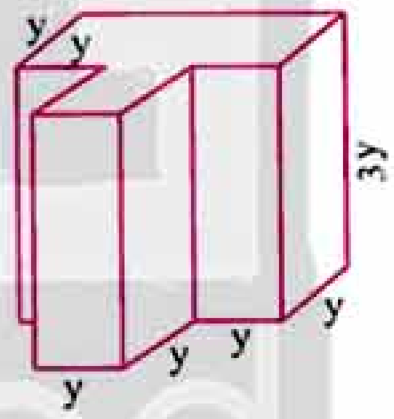
Notice that :
The lateral area of the cuboid
= perimeter of base × height

- The volume of the cuboid = length × width × height

$= 3\ a \times 2\ a \times 2\ a$

$= 12\ a^3$

Example 5 **Find the volume of the opposite solid.**



Solution

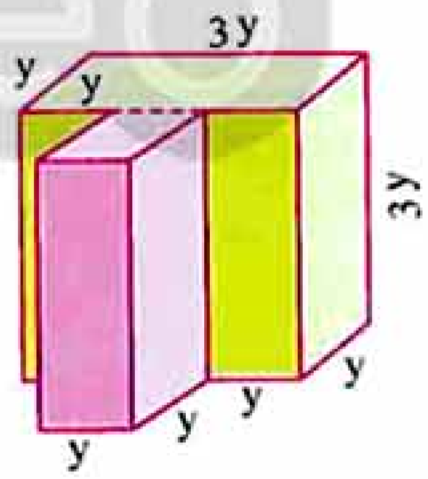
The solid is formed from two cuboids.

Therefore ,

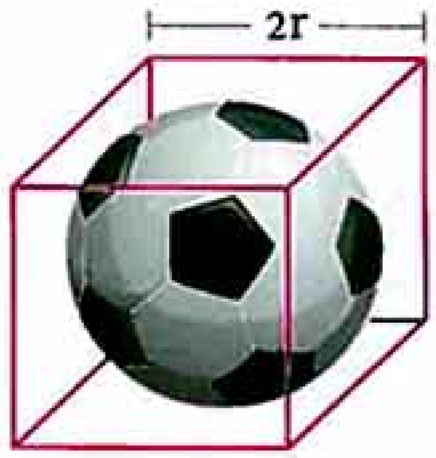
the volume of the solid $= y \times y \times 3\ y + 3\ y \times y \times 3\ y$

$= 3\ y^3 + 9\ y^3$

$= 12\ y^3$



Example 6 A sphere is put inside a cube as shown in the figure to touch all its six faces internally. Find the ratio between the volume of the sphere and that of the cube (Consider : $\pi \simeq \frac{22}{7}$)



(The volume of the sphere = $\frac{4}{3}\pi r^3$)

Solution The diameter length of the sphere = the edge length of the cube = 2 r

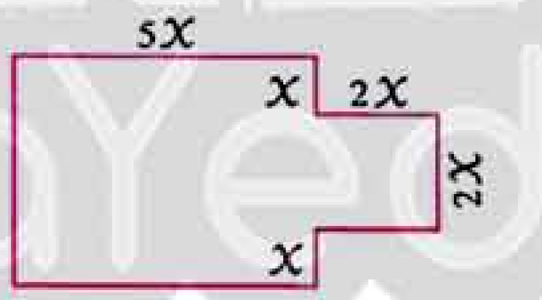
Therefore , the ratio between the sphere volume and the cube volume

$$= \frac{\text{The volume of the sphere}}{\text{The volume of the cube}} = \frac{\frac{4}{3}\pi r^3}{2r \times 2r \times 2r} = \frac{\frac{4}{3}\pi \cancel{r^3}}{8\cancel{r^3}}$$

$$= \left(\frac{\overset{1}{\cancel{4}}}{3} \times \frac{1}{\underset{2}{\cancel{8}}}\right)\pi = \frac{1}{6}\pi \simeq \frac{1}{6} \times \frac{22}{7} \simeq \frac{11}{21}$$

TRY 3 by yourself

Calculate the perimeter and the area of the opposite figure.



Answers of try by yourself

1 [1] $-6a^2b$ [2] $-6x^3y^3$
[3] $-2\ell^3m^4$ [4] $\frac{2}{3}m^2n^2$

2 [1] a^5 [2] $-2ab$
[3] $-3x$ [4] $8x$
[5] $5x^2$ [6] $4a^2b^5$

3 The perimeter = $22x$, the area = $24x^2$

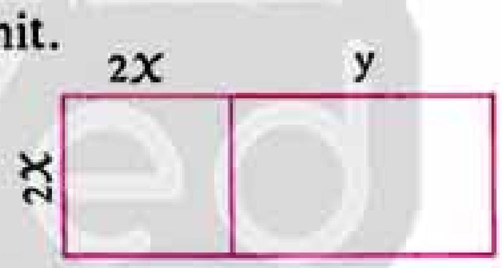
LESSON 5

Multiplying a Monomial by an Algebraic Expression

Prelude example :

In the opposite figure :

A square of side length $2x$ length unit and a rectangle of dimensions $2x$ length unit and y length unit , then the sum of areas of the square and the rectangle $= (2x \times 2x) + (2x \times y) = (4x^2 + 2xy)$ square unit.



If we stick the square and the rectangle as in the opposite figure , then we get a new rectangle of dimensions $2x$ length unit and $(2x + y)$ length unit , then its area $= 2x(2x + y)$ square unit

Since the area is the same in the two cases ,

Therefore $2x(2x + y) = (2x \times 2x) + (2x \times y) = 4x^2 + 2xy$

This is the result of multiplying the monomial $2x$ by the algebraic expression $2x + y$

Multiplying a monomial by an algebraic expression

To multiply a monomial by an algebraic expression , we have to multiply this monomial by each term of the algebraic expression using the distribution property.

For example:

$$2x(3x + 5y) = (2x \times 3x) + (2x \times 5y) \quad \text{(distribution property)}$$
$$= 6x^2 + 10xy$$

We can find the product by using the vertical method as follows :

$$\begin{array}{r} 3x + 5y \\ \times\ 2x \\ \hline \end{array}$$

The product $= 6x^2 + 10xy$

Example 1 **Find the product of each of the following :**

1. $b(-2a + a^2 b)$
2. $-3ab(5a - 2b + 3)$
3. $(a^2 - ab - 2b^2) \times 4ab$

Solution

1. $b(-2a + a^2 b) = -2ab + a^2 b^2$
2. $-3ab(5a - 2b + 3) = -15a^2 b + 6ab^2 - 9ab$
3. $(a^2 - ab - 2b^2) \times 4ab = 4a^3 b - 4a^2 b^2 - 8ab^3$

Try to solve the example by another method

Example 2 **Simplify :**

$2a(a + 4b) - 3b(a - 3b) - (2a^2 + 8b^2)$

, then find the numerical value of the result when : $a = 1$ and $b = -2$

Solution

The expression $= 2a^2 + 8ab - 3ab + 9b^2 - 2a^2 - 8b^2$
$= b^2 + 5ab$

The numerical value $= (-2)^2 + 5 \times 1 \times (-2)$
$= 4 - 10 = -6$

TRY by yourself

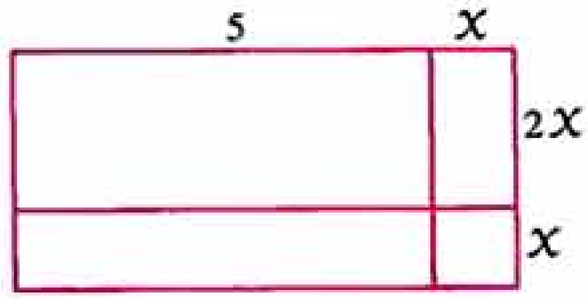
1. ***Find the product of each of the following :***
   1. $3a(2a - 4b)$
   2. $-2x(3xy - 5x)$
2. ***Simplify the following :*** $2x(3x - 2) + 3x(x + 1)$

UNIT 2

Example 3 **In the opposite figure :**

A rectangle divided into three rectangles and a square.

Find the area of the whole figure.



Solution

The length of the whole rectangle $= x + 5$

, its width $= 2x + x = 3x$

Therefore , its area = length × width

$= (x + 5) \times 3x = 3x^2 + 15x$

Example 4 **Find the area of the coloured part in each of the following figures :**

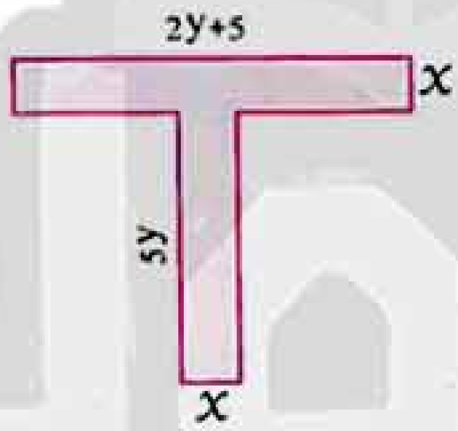


fig. (1)

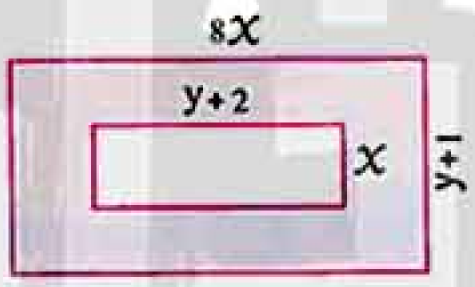


fig. (2)

Solution

- The area of the coloured part in fig. (1) $= x(2y + 5) + x \times 5y$
  $= 2xy + 5x + 5xy$
  $= 7xy + 5x$
- The area of the coloured part in fig. (2) $= 8x(y + 1) - x(y + 2)$
  $= 8xy + 8x - xy - 2x$
  $= 7xy + 6x$

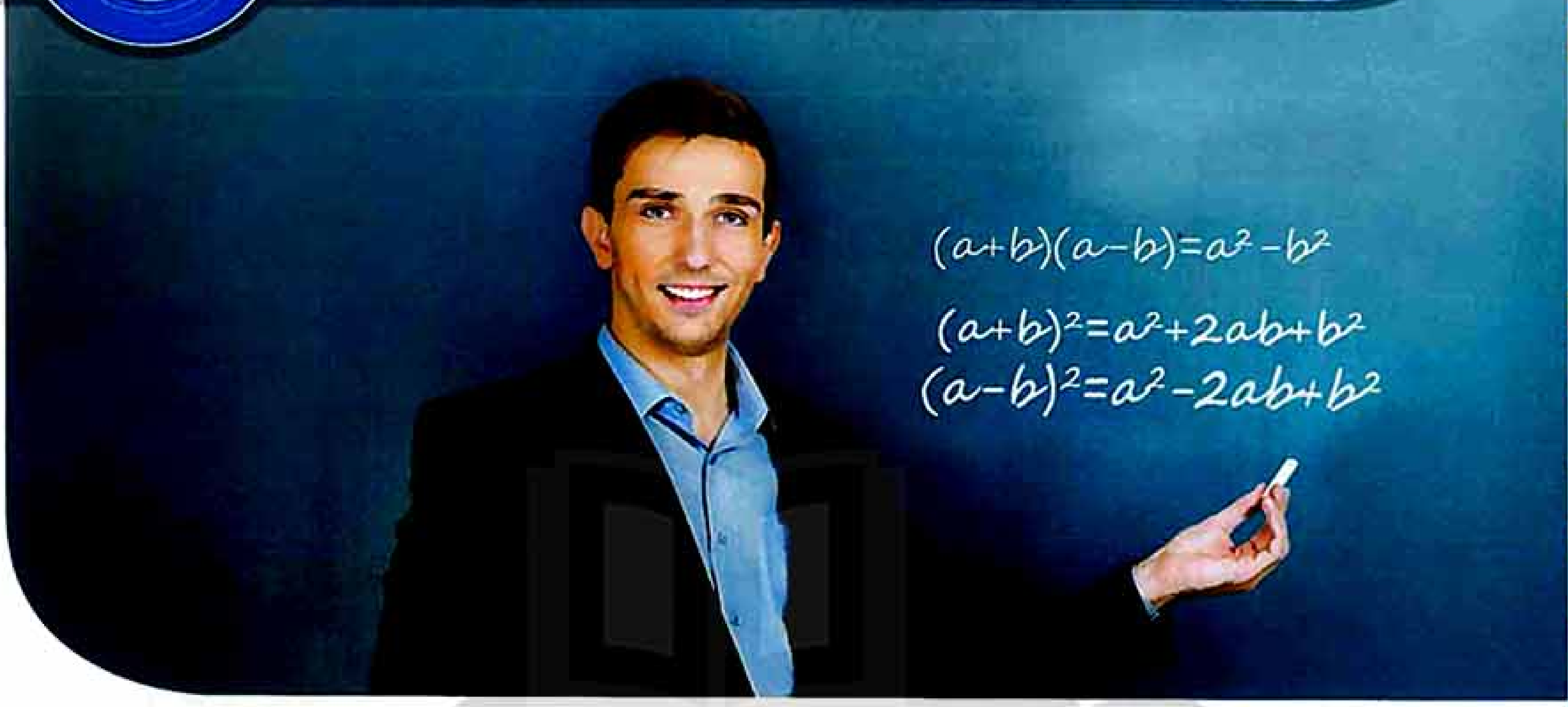
Answers of try by yourself

1 (1) $6a^2 - 12ab$ (2) $-6x^2y + 10x^2$

2 $9x^2 - x$

LESSON 6

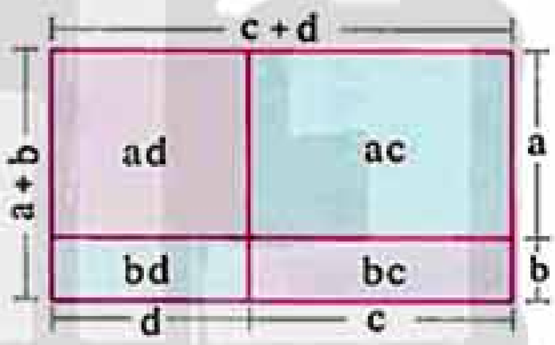
Multiplying a Binomial by an Algebraic Expression



Multiplying two binomials

In the opposite figure , a rectangle of dimensions (a + b) cm. and (c + d) cm. , its area = (a + b) (c + d) cm^2. (1)

We can get its area using another method by dividing it into 4 parts as shown in the figure.



Therefore , the area of the rectangle is the sum of the areas of the four parts.

i.e. The area of the rectangle = ac + ad + bc + bd (2)

From (1) and (2) , we deduce that :

(a + b) (c + d) = ac + ad + bc + bd

From the previous, we notice that

$(a + b)(c + d) = (a \times c) + (a \times d) + (b \times c) + (b \times d)$

first × first	+	first × second	+	second × first	+	second × second

We can find the product of two binomials using one of the two explained methods in the next example.

UNIT 2

Example 1 Find the product of : $(x+5)(2x-3)$

Solution

The horizontal method :

$(x+5)(2x-3) = x(2x-3) + 5(2x-3)$

$= 2x^2 \boxed{-3x+10x} - 15$

$= 2x^2 + 7x - 15$

Notice that : Simplifying of the product to the simplest form by adding the two like terms $-3x$ and $10x$

The vertical method :

- Put one of the two expressions under the other as shown :

$$\begin{array}{r} x+5 \\ 2x-3 \\ \hline 2x^2+10x \\ -3x-15 \\ \hline 2x^2+7x-15 \end{array}$$

- Multiply $2x$ by $(x+5)$ ⟶ $2x^2+10x$
- Multiply -3 by $(x+5)$ ⟶ $-3x-15$
- By adding , we get ⟶ $2x^2+7x-15$

Notice that : $-3x$ must be under $10x$ because they are like terms.

TRY 1 by yourself

Complete the following :

1. $(3x+7)(2x-3) =$ .... $-9x+$ .... -21

$=$ .... $+5x-$ ....

2.

$$\begin{array}{l} 2x+3y \\ x-y \\ \hline 2x^2+ \text{..........} \\ \quad - \text{..........} -3y^2 \\ \hline \text{..........}+\text{..........}-\text{..........} \end{array}$$

Multiplying by inspection

In the previous example , we found that :
$(x+5)(2x-3) = 2x^2+7x-15$

Notice that :
- The two terms 5 and $2x$ are called the means.
- The two terms x and -3 are called the extremes.

Noticing the product, we get :

- The first term $(2x^2)$ = the 1^{st} term in the 1^{st} expression $(x) \times$ the 1^{st} term in the 2^{nd} expression $(2x)$
- The middle term $(7x)$ = the product of means $(10x)$ + the product of extremes $(-3x)$
- The 3^{rd} term (-15) = the 2^{nd} term in the 1^{st} expression $(5) \times$ the 2^{nd} term in the 2^{nd} expression (-3)

Example 2 **Find by inspection the product of each of the following :**

1. $(2a+3)(5a+1)$
2. $(3x+4)(2x-5)$
3. $(5a-2b)(7a-3b)$
4. $(4x-3y)(3y+x)$

Solution

1. $(2a+3)(5a+1) =$ (The first × The first) + (Product of means) + (Product of extremes) + (The second × The second)

$= (2a \times 5a) + (3 \times 5a + 2a \times 1) + 3 \times 1$

$= 10a^2 + (15a + 2a) + 3$

$= 10a^2 + 17a + 3$

By more training , we will not write the previous steps.

2. $(3x+4)(2x-5) = 6x^2 - 7x - 20$
3. $(5a-2b)(7a-3b) = 35a^2 - 29ab + 6b^2$
4. $(4x-3y)(3y+x) = (4x-3y)(x+3y)$

$= 4x^2 + 9xy - 9y^2$

Notice that : We rearranged the two terms of the 2^{nd} expression before multiplying.

TRY 2 by yourself

Complete the missing terms in each of the following :

1. $(2a+1)(5a+3) = 10a^2 +$ .... + ...
2. $(3x+4)(2x-1) =$ .... + ... -4

UNIT 2

Two special cases

I Expanding the square of an expression consisting of two terms :

Look at the activity at the end of the book using the Excel program.

1 $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

Generally

The square of an expression consisting of the sum of two terms =
The square of the first + 2 × The first × The second + The square of the second.

2 $(x-y)^2 = (x-y)(x-y) = x^2 - 2xy + y^2$

Generally

The square of an expression consisting of the difference between two terms =
The square of the first − 2 × The first × The second + The square of the second.

Example 3 Find the expansion of each of the following :

1 $(3a+5)^2$

2 $(2x-3y)^2$

Solution

1 $(3a+5)^2 = (3a)^2 + (2 \times 3a \times 5) + (5)^2$
$= 9a^2 + 30a + 25$

2 $(2x-3y)^2 = (2x)^2 - (2 \times 2x \times 3y) + (3y)^2$
$= 4x^2 - 12xy + 9y^2$

TRY 3 by yourself

Find the expansion of each of the following :

1 $(3m+2)^2$

2 $(5x-7y)^2$

II The product of the sum of two terms and the difference between them :

$(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$

Look at the activity at the end of the book using the Excel program.

Generally

The product of the sum of two terms and their difference =
the square of the first − the square of the second.

Example 4 Find the product of each of the following :

1. $(2\ell - 5)(2\ell + 5)$
2. $(5x + 3y)(5x - 3y)$
3. $(a^2 + 2b)(a^2 - 2b)$
4. $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right)$

Solution

1. $(2\ell - 5)(2\ell + 5) = (2\ell)^2 - (5)^2 = 4\ell^2 - 25$
2. $(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2 = 25x^2 - 9y^2$
3. $(a^2 + 2b)(a^2 - 2b) = (a^2)^2 - (2b)^2 = a^4 - 4b^2$
4. $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right) = \left(\frac{1}{3}a\right)^2 - \left(\frac{2}{5}b\right)^2 = \frac{1}{9}a^2 - \frac{4}{25}b^2$

TRY 4 by yourself

Find the product of each of the following :

1. $(2a + 3b)(2a - 3b)$
2. $(3a - 4b)(3a + 4b)$

Example 5 Put each of the following in the simplest form :

1. $(x + 4)^2 - (x + 2)(x + 6)$
2. $(x + 5)(x - 5) + (x - 5)^2$

Solution

1. $(x + 4)^2 - (x + 2)(x + 6) = (x^2 + 8x + 16) - (x^2 + 8x + 12)$
$= \cancel{x^2} + \cancel{8x} + 16 - \cancel{x^2} - \cancel{8x} - 12 = 4$
2. $(x + 5)(x - 5) + (x - 5)^2 = (x^2 - 25) + (x^2 - 10x + 25)$
$= x^2 - \cancel{25} + x^2 - 10x + \cancel{25} = 2x^2 - 10x$

Multiplying a binomial by an expression formed from more than two terms

As we studied before how to multiply two binomials , the operation of multiplication can be performed by one of two methods as shown in the following example and it is prefered to arrange the terms descendingly with respect to one of the given symbols.

UNIT 2

Example 6 **Find the product of :** $(x-3)(x^2+4x-7)$

Solution

The horizontal method :

$$(x-3)(x^2+4x-7) = x\,(x^2+4x-7) - 3\,(x^2+4x-7)$$
$$= x^3+4x^2-7x-3x^2-12x+21$$
$$= x^3+x^2-19x+21$$

The vertical method :

Multiplicand ⟶ x^2+4x-7

Multiplier ⟶ $x-3$

Multiply (x) by multiplicand ⟶ x^3+4x^2-7x

Multiply (– 3) by multiplicand ⟶ $-3x^2-12x+21$

i.e. The product is ⟶ $x^3+x^2-19x+21$

Notice that :

- Putting the expression x^2+4x-7 firstly because it consists of more terms than other.
- Putting the like terms under each other during the performing of multiplying operation

Remark

It is better to use the vertical method for multiplying algebraic expressions forming from more than two terms.

Example 7 **Find the product of :** $3a^3+a^2-4$ **by** $2a+3$

Solution

$$\begin{array}{rrrrr}
 & 3a^3 & +a^2 & & -4 \\
 & & & 2a & +3 \\
\hline
6a^4 & +2a^3 & & -8a & \\
 & 9a^3 & +3a^2 & & -12 \\
\hline
6a^4 & +11a^3 & +3a^2 & -8a & -12
\end{array}$$

Notice that :

We leave spaces up and down the terms which have no like terms to them.

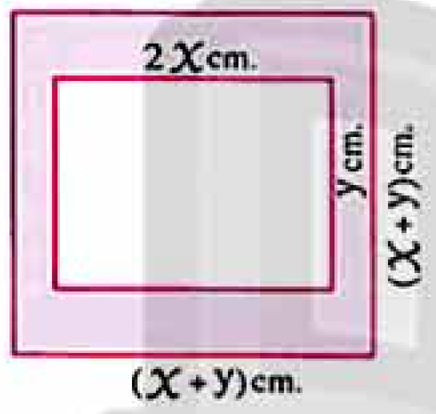
TRY 5 by yourself

Find the product of : $(-3x + x^2 + 3)(x - 2)$

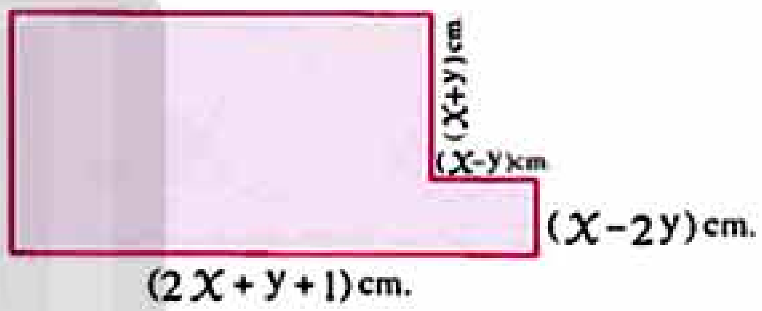
Applications on multiplying of algebraic expressions

Example 8 **Find the expression which expresses the area of the coloured part in each of the following figures :**

1



2



Solution

1 The area of the coloured part $= (x + y)^2 - 2xy$

$= x^2 + 2xy + y^2 - 2xy$

$= (x^2 + y^2)$ cm^2.

2 The area of the coloured part

$= (x + y + x - 2y)(2x + y + 1) - (x + y)(x - y)$

$= (2x - y)(2x + y + 1) - x^2 + y^2$

$= 4x^2 + 2xy + 2x - 2xy - y^2 - y - x^2 + y^2$

$= (3x^2 + 2x - y)$ cm^2.

Example 9 **Use the multiplication by inspection to find the value of each of the following easily :**

1 $(52)^2$ 2 $(195)^2$ 3 502×498

Solution

1 $(52)^2 = (2 + 50)^2 = 4 + 200 + 2500 = 2704$

2 $(195)^2 = (200 - 5)^2 = 40000 - 2000 + 25 = 38025$

3 $502 \times 498 = (500 + 2)(500 - 2) = (500)^2 - (2)^2 = 250000 - 4 = 249996$

UNIT 2

TRY 6 by yourself

Complete the following :

1. $(31)^2 = (30 + \cdots\cdots)^2 = 900 + \cdots\cdots + \cdots\cdots = \cdots\cdots$
2. $(89)^2 = (\cdots\cdots - 1)^2 = \cdots\cdots - \cdots\cdots + 1 = \cdots\cdots$
3. $42 \times 38 = (40 + \cdots\cdots)(\cdots\cdots - \cdots\cdots) = \cdots\cdots - \cdots\cdots = \cdots\cdots$

Answers of try by yourself

1 (1) $(3x+7)(2x-3) = 6x^2 - 9x + 14x - 21 = 6x^2 + 5x - 21$

(2)
$$\begin{array}{rrr} & 2x^2 \ (2x) & + 3y \\ \times & x & - y \\ \hline & 2x^2 & + 3xy \\ & - 2xy & - 3y^2 \\ \hline & 2x^2 + xy & - 3y^2 \end{array}$$

2 (1) $10a^2 + 11a + 3$ (2) $6x^2 + 5x - 4$

3 (1) $9m^2 + 12m + 4$ (2) $25x^2 - 70xy + 49y^2$

4 (1) $4a^2 - 9b^2$ (2) $9a^2 - 16b^2$

5 $x^3 - 5x^2 + 9x - 6$

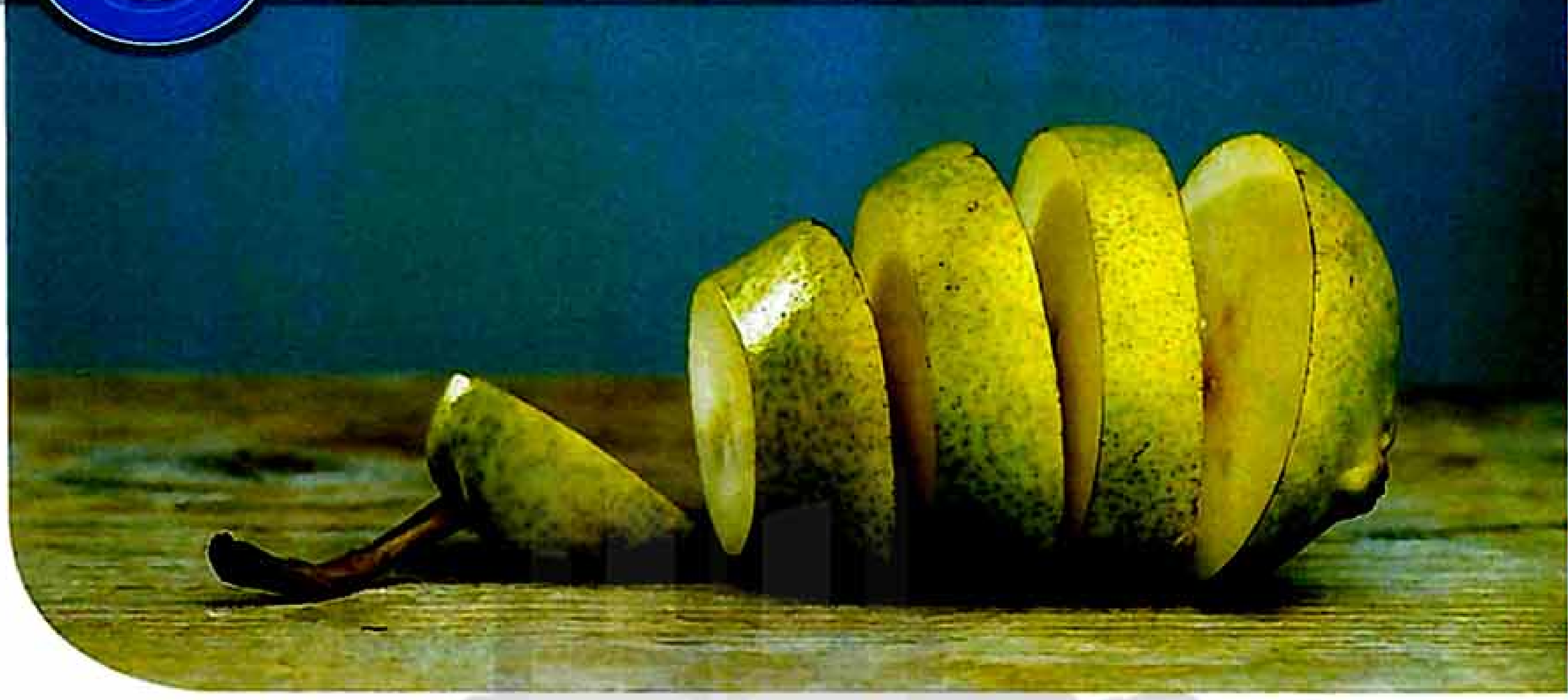
6 (1) $(31)^2 = (30+1)^2 = 900 + 60 + 1 = 961$

(2) $(89)^2 = (90-1)^2 = 8100 - 180 + 1 = 7921$

(3) $42 \times 38 = (40+2)(40-2)$
$= (40)^2 - (2)^2 = 1596$

LESSON 7

Dividing an Algebraic Expression by a Monomial



- We know that in fractions , we can consider : $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9}$

  and also $\frac{2+5}{9} = \frac{2}{9} + \frac{5}{9}$

- We can do the same in dividing an algebraic expression by an algebraic term , then we can write :

  $\frac{6x^2 + 2xy}{2x} = \frac{6x^2}{2x} + \frac{2xy}{2x}$ which equals $3x + y$

Generally

When we divide an algebraic expression by a monomial , we divide each term of the expression by this monomial.

Example 1 **Find the quotient in each of the following where $x \neq 0$ and $y \neq 0$:**

1. $\frac{21x^2 + 14x}{7x}$
2. $(16x^3y + 8x^2y^3 - 12x^2y)$ by $(-4x^2y)$

Solution

1. $\frac{21x^2 + 14x}{7x} = \frac{21x^2}{7x} + \frac{14x}{7x} = 3x + 2$
2. $(16x^3y + 8x^2y^3 - 12x^2y) \div (-4x^2y)$

   $= \frac{16x^3y}{-4x^2y} + \frac{8x^2y^3}{-4x^2y} + \frac{-12x^2y}{-4x^2y} = -4x - 2y^2 + 3$

Notice that :

We can check the answer by multiplying the divisor by the quotient to obtain the dividend.

Example 2 **Divide :** $\frac{3ab^2c - 5a^2bc + 2abc^2}{abc}$ where $abc \neq$ zero

, then find the absolute value of the result when : $a = 1$, $b = -2$ and $c = 3$

Solution

$\frac{3ab^2c - 5a^2bc + 2abc^2}{abc} = 3b - 5a + 2c$

The absolute value $= |3 \times (-2) - 5 \times 1 + 2 \times 3| = |-6 - 5 + 6| = |-5| = 5$

TRY by yourself

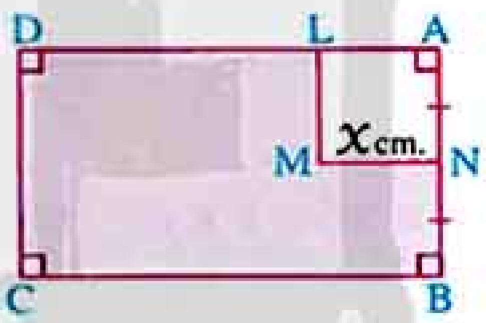
Find the quotient of each of the following where the symbols represent integers which aren't equal to zero :

1. $(12x^4 + 8x^2) \div 4x$
2. $(14x^3 - 21x^2 + 7x) \div (-7x)$
3. $\frac{10a^6b^4 - 8a^5b^3 + 2a^4b^2}{2a^4b}$

Example 3 **In the opposite figure :**

ABCD is a rectangle , ANML is a square , N is the midpoint of $\overline{AB}$ and $NM = x$ cm. If the area of the coloured part is $(x^2 + 10x)$ cm².

Find the length of $\overline{LD}$



Solution

The area of the square $= x \times x = x^2$ cm².

The area of the rectangle = the area of the square + the area of the coloured part

$= x^2 + x^2 + 10x = (2x^2 + 10x)$ cm².

Since , the width of the rectangle = twice of the length of $\overline{AN} = 2x$ cm.

Therefore , the length of the rectangle (AD) $= \frac{\text{the area of the rectangle}}{\text{the width of the rectangle}}$

$= (2x^2 + 10x) \div 2x$

$= (x + 5)$ cm.

Therefore , $LD = AD - AL = x + 5 - x = 5$ cm.

Answers of try by yourself

1. $3x^3 + 2x$
2. $-2x^2 + 3x - 1$
3. $5a^2b^3 - 4ab^2 + b$

LESSON **8**

Dividing an Algebraic Expression by Another One

Illustrative example :

Divide : $x^2 + x - 12$ by $x + 4$ where $x \neq -4$

To operate the division , we do the following steps :

1 Divide x^2 by x , then the result is x

2 Multiply x by $(x + 4)$, then we get $x^2 + 4x$

3 Subtract $x^2 + 4x$ from $x^2 + x - 12$, then we get $-3x - 12$

4 Repeat the previous steps (in order) till the difference will be equal to zero. Then the operation of division will be finished and the quotient $= x - 3$

$$
\begin{array}{l|rr}
x + 4 & x^2 + x & - 12 \\
\hline
x - 3 & \ominus x^2 \ominus + 4x & \\
 & -3x & - 12 \\
 & \oplus -3x & \oplus - 12 \\
\hline
 & 0 & 0
\end{array}
$$

Notice that :

The like terms should be written down in one column.

Remark

It is necessary to arrange each of the dividend and the divisor either in a descending order or in an ascending order according to the power of the given symbol (x)

(It is preferable to arrange in a descending order)

UNIT 2

Example 1 Find the quotient of dividing :

$5a - 10a^2 + 6a^3 + 3$ by $3 + 2a^2 - 4a$ where the divisor $\neq 0$

Solution

$$\begin{array}{r|l} 2a^2 - 4a + 3 & 6a^3 - 10a^2 + 5a + 3 \\ \hline 3a + 1 & 6a^3 - 12a^2 + 9a \\ & \overline{\quad 2a^2 - 4a + 3} \\ & 2a^2 - 4a + 3 \\ & \overline{\quad 00 \quad 00 \quad 00} \end{array}$$

Notice that : Each of the dividend and the divisor is in a descending order according to the power of "a".

i.e. The quotient $= 3a + 1$

Example 2 Find the quotient of dividing :

$x^3 + x + 10$ by $x + 2$ where $x \neq -2$

Solution

$$\begin{array}{r|l} x + 2 & x^3 + \qquad x + 10 \\ \hline x^2 - 2x + 5 & x^3 + 2x^2 \\ & \overline{-2x^2 + x + 10} \\ & -2x^2 - 4x \\ & \overline{\quad 5x + 10} \\ & 5x + 10 \\ & \overline{\quad 00 \quad 00} \end{array}$$

Notice that : There is no term with x^2 in the dividend , so we leave its place empty.

i.e. The quotient $= x^2 - 2x + 5$

Example 3 If $(x - 1)$ is one of the factors of $(x^2 + 5x - 6)$, then find the other factor.

Solution

The other factor is the quotient of dividing

$x^2 + 5x - 6$ by $(x - 1)$

i.e. The other factor is $(x + 6)$

$$\begin{array}{r|l} x - 1 & x^2 + 5x - 6 \\ \hline x + 6 & x^2 - x \\ & \overline{\quad 6x - 6} \\ & 6x - 6 \\ & \overline{\quad 00 \quad 00} \end{array}$$

Example 4 **If the expression $(2x^3 + 11x^2 + 12x + m)$ is divisible by $(x + 3)$, find the value of m**

Solution

Where the dividend is divisible by $(x + 3)$

Then, the remainder should vanish

i.e. $m + 9 = 0$

So, $m = -9$

$$\begin{array}{r|l} x+3 & 2x^3 + 11x^2 + 12x + m \\ \hline 2x^2 + 5x - 3 & \ominus 2x^3 \ominus 6x^2 \\ & \quad 5x^2 + 12x + m \\ & \ominus 5x^2 \ominus 15x \\ & \quad -3x + m \\ & \oplus -3x \oplus 9 \\ & \quad m + 9 \end{array}$$

Example 5 **A rectangle whose area is $(8x^2 + 6xy - 9y^2)$ cm². , if its width is $(4x - 3y)$ cm. , then find its length , and calculate its perimeter when $x = 2$ and $y = 1$**

Solution

The length of the rectangle = its area ÷ its width

$= (8x^2 + 6xy - 9y^2) \div (4x - 3y)$

$$\begin{array}{r|l} 4x - 3y & 8x^2 + 6xy - 9y^2 \\ \hline 2x + 3y & \ominus 8x^2 \oplus 6xy \\ & \quad 12xy - 9y^2 \\ & \ominus 12xy \oplus 9y^2 \\ & \quad 0\,0 \qquad 0\,0 \end{array}$$

i.e. The length of the rectangle = $(2x + 3y)$ cm.

when $x = 2$, $y = 1$, then :

The length of the rectangle = $2x + 3y = 2 \times 2 + 3 \times 1 = 7$ cm.

, the width of the rectangle = $4x - 3y = 4 \times 2 - 3 \times 1 = 5$ cm.

, So the perimeter of the rectangle = (length + width) × 2 = (7 + 5) × 2 = 24 cm.

TRY *by yourself*

Find the quotient of dividing the following expressions, where the divisors ≠ 0 :

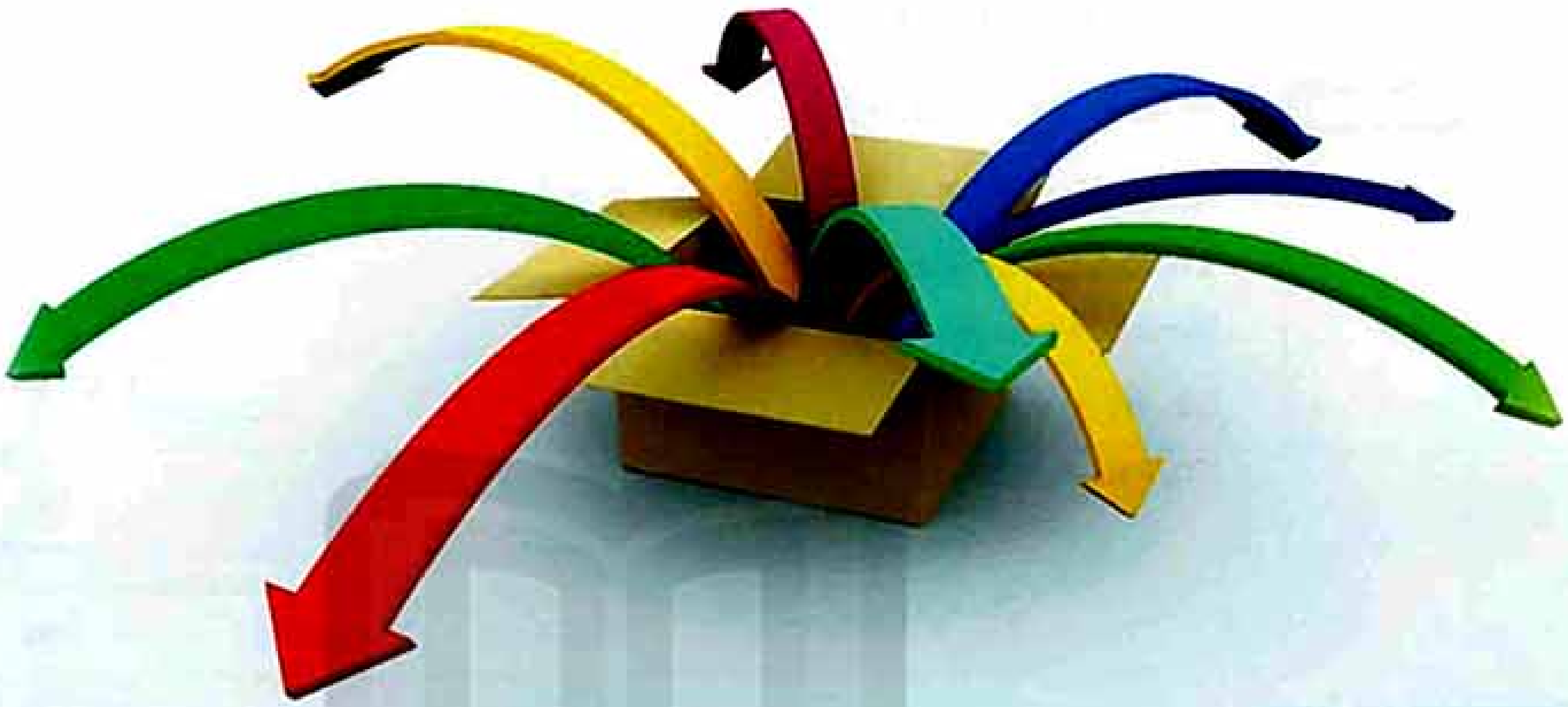
1. $14x^2 + 25x + 6$ ***by*** $2x + 3$
2. $2x^3 + x^2 - 19x + 10$ ***by*** $2x - 5$

Answers *of try by yourself*

1. $7x + 2$
2. $x^2 + 3x - 2$

LESSON

9 Factorization by Identifying the Highest Common Factor (H.C.F.)



Meaning of the factorization :

- **Factorization of a number is to write it as a product of two factors or more.**

For example:

- The number 24 can be factorized as the following :

$24 = 2 \times 12$ or $24 = 3 \times 8$ or $24 = 3 \times 2 \times 2 \times 2$ or ...

- Also the number 36 can be factorized as the following :

$36 = 3 \times 12$ or $36 = 6 \times 6$ or $36 = 2 \times 2 \times 3 \times 3$ or ...

- **Also factorization of an algebraic term is to write it as a product of two factors or more.**

For example:

- The algebraic term $4x$ can be factorized as the following :

$4x = 4 \times x$ or $4x = 2 \times 2x$ or ...

- Also the algebraic term $6x^2$ can be factorized as the following :

$6x^2 = 6 \times x^2$ or $6x^2 = 2x \times 3x$ or ...

Meaning of the common factor :

- **The common factor of two numbers is the number that divides each of the two numbers.**

For example:

3 is a common factor for the two numbers 24 and 36 because it divides each of them.

$(\frac{24}{3} = 8 , \frac{36}{3} = 12)$

- **Also the common factor of two algebraic terms is the algebraic term that divides each of the two terms.**

For example:

2 is a common factor for the two algebraic terms $4x$ and $6x^2$ $(\frac{4x}{2} = 2x , \frac{6x^2}{2} = 3x^2)$

, $2x$ is a common factor for the two algebraic terms $4x$ and $6x^2$ $(\frac{4x}{2x} = 2 , \frac{6x^2}{2x} = 3x)$

Meaning of the highest common factor :

- **The highest common factor for two numbers is the greatest number divides each of the two numbers , and denoted by (H.C.F.)**

For example:

12 is the highest common factor for the two numbers 24 and 36

- **The highest common factor for two algebraic terms is the greatest term divides each of the two terms , and also denoted by (H.C.F.)**

For example:

$2x$ is the highest common factor for the two terms $4x$ and $6x^2$

To find the highest common factor (H.C.F.) for some algebraic terms :

1. Find the highest common factor of the numerical coefficients of these terms.
2. Take each repeated letter in all terms with the smallest index.

For example:

The H.C.F. of the algebraic terms : $6x^2y$, $-8xy^3$, $4xyz$ is $2xy$

The method of factorization by identifying the highest common factor (H.C.F.) :

1. Find H.C.F. of the algebraic terms of the expression.
2. Put H.C.F. outside two brackets.
3. Divide each term of the algebraic expression by the H.C.F. and write the quotients inside the two brackets.

Example 1 **Factorize each of the following by identifying the highest common factor :**

1 $5a + 15b$	2 $10xy - 8xz$
3 $12x^2 - 4xy$	4 $21a^3b^2 - 7a^2b^2 - 35a^2b^3$

UNIT 2

Solution

1 Since H.C.F. = 5 Then $5a + 15b = 5(a + 3b)$

2 Since H.C.F. = $2x$ Then $10xy - 8xz = 2x(5y - 4z)$

3 Since H.C.F. = $4x$ Then $12x^2 - 4xy = 4x(3x - y)$

4 Since H.C.F. = $7a^2b^2$

Then $21a^3b^2 - 7a^2b^2 - 35a^2b^3 = 7a^2b^2(3a - 1 - 5b)$

Remark

Sometimes the H.C.F. is an algebraic expression consists of more than one algebraic term.

Example 2 Factorize each of the following by identifying the highest common factor :

1 $(x - y)(x + 3y) + 2x(x - y)$

2 $3a(c - d) + 4b(d - c)$

Solution

1 Since H.C.F. = $(x - y)$

Then $(x - y)(x + 3y) + 2x(x - y) = (x + 3y + 2x)(x - y)$

$= (3x + 3y)(x - y)$

$= 3(x + y)(x - y)$

2 Since $d - c = -c + d = -(c - d)$

Then $3a(c - d) + 4b(d - c) = 3a(c - d) - 4b(c - d)$

Since H.C.F. = $(c - d)$

Then $3a(c - d) - 4b(c - d) = (3a - 4b)(c - d)$

TRY 1 by yourself

Factorize each of the following by identifying the highest common factor :

1 $3x + 21y$

2 $2a^3 + 6a^2 - 4a$

3 $3x^2 + 15xz + 21xy^2$

4 $3x(a + b) - 7(a + b)$

Example 3 **Use factorization by identifying the highest common factor to find the result of each of the following :**

1 $57 \times 43 - 57 \times 33$

2 $(153)^2 - 153 \times 53$

3 $4\,(10)^2 + 24 \times 10 - 28 \times 10$

Solution

1 Since H.C.F. = 57

Then $57 \times 43 - 57 \times 33 = 57\,(43 - 33) = 57 \times 10 = 570$

2 Since H.C.F. = 153

Then $(153)^2 - 153 \times 53 = 153\,(153 - 53) = 153 \times 100 = 15300$

3 Since H.C.F. = 4×10

Then $4\,(10)^2 + 24 \times 10 - 28 \times 10 = 4 \times 10\,(10 + 6 - 7) = 40 \times 9 = 360$

TRY 2 by yourself

Use factorization by identifying the highest common factor to find the result of each of the following :

1 $47 \times 15 - 23 \times 15 + 76 \times 15$

2 $12 \times 75 + 13 \times 75 + (75)^2$

Example 4 **If $m - 2\,n = 10$, find using factorization by identifying H.C.F. the numerical value of the expression :** $3\,m\,(m - 2\,n) - 6\,n\,(m - 2\,n)$

Solution

Since H.C.F. = $3\,(m - 2\,n)$

Then $3\,m\,(m - 2\,n) - 6\,n\,(m - 2\,n) = 3\,(m - 2\,n)\,(m - 2\,n)$

$= 3 \times 10 \times 10 = 300$

Another solution :

Since $m - 2\,n = 10$

Then $3\,m\,(m - 2\,n) - 6\,n\,(m - 2\,n) = 3\,m \times 10 - 6\,n \times 10$

$= 30\,m - 60\,n = 30\,(m - 2\,n)$

$= 30 \times 10 = 300$

Answers *of try by yourself*

1 [1] $3\,(X + 7\,y)$ [2] $2\,a\,(a^2 + 3\,a - 2)$

[3] $3\,X\,(X + 5\,z + 7\,y^2)$ [4] $(a + b)\,(3\,X - 7)$

2 [1] $15\,(47 - 23 + 76) = 15 \times 100 = 1500$ [2] $75\,(12 + 13 + 75) = 75 \times 100 = 7500$

UNIT 3

Statistics

Lessons of the unit :

1. The arithmetic mean.
2. The median.
3. The mode.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of the central tendency.
- recognize the concept of the arithmetic mean.
- calculate the arithmetic mean of a set of values.
- recognize the concept of the median.
- find the median of a set of values.
- recognize the concept of the mode.
- find the mode of a set of values.
- solve different problems on the arithmetic mean, the median and the mode.
- appreciate the role of statistics in the practical life.

Use your smart phone or tablet to scan the QR Code and enjoy watching videos.

Gauss

Carl Friedrich Gauss

(1777 A.D. - 1855 A.D.)

A German mathematician who developed the methods, theories and applications of statistics.

LESSON 1

The Arithmetic Mean

Measures of central tendency

- At studying different phenomena , we find that the data of any phenomenon trend to the centering and accumulation around a certain value which is the mean of this phenomenon or measure of its tendency.

For example:

Heights of men accumulate around a certain number which is the mean of heights , also their weights and intelligence rates , and another different phenomena.

- Measures of central tendency (averages) are measures used to measure the position of centering of the data , and used to give an abbreviation description of the phenomenon which we study.
- There are many measures of central tendency , in this unit we will study three of these measures :

1. The arithmetic mean.
2. The median.
3. The mode.

The arithmetic mean

Definition :

The arithmetic mean of a set of values = $\dfrac{\text{Sum of these values}}{\text{Number of these values}}$

Example 1 If the number of studying hours daily of a student in 6 days are :
6 , 5 , 6 , 4 , 7 and 2
, what is the arithmetic mean of number of studying hours daily of this student ?

Solution

The arithmetic mean $= \dfrac{\text{Sum of number of studying hours}}{\text{Number of days}}$

$= \dfrac{6+5+6+4+7+2}{6}$

$= \dfrac{30}{6} = 5$ hours

From the previous exmaple , notice the following :

- Number of studying hours of this student spend daily during the six days is not constant. In other words changes from a day to another , and its total number during the six days is 30 hours.
- This student can preserve the total number of studying hours during the six days (30 hours) where he spends it in a constant way daily which is 5 hours every day.
  [$5+5+5+5+5+5 = 6+5+6+4+7+2 = 30$]

i.e.
The arithmetic mean of a set of values is the value if it replaced each value of the set of values , the sum of the new values equals the sum of the original values.

TRY 1 *by yourself*

Find the arithmetic mean of the values : 3 , 8 , 11 , 4 and 9

UNIT 3

Example 2 **If the arithmetic mean of the values : 5 , 7 , x and 9 is 6 , find the value of x**

Solution

Since the arithmetic mean $= \dfrac{\text{Sum of values}}{\text{Number of values}}$

, then $6 = \dfrac{5 + 7 + x + 9}{4}$

, then $6 = \dfrac{21 + x}{4}$

, then $x = 3$

Example 3 **If the arithmetic mean of the values : 2 a , a + 3 , 3 a – 2 , 11 – a and 3 is 13 , find the value of a**

Solution

Since the arithmetic mean $= \dfrac{\text{Sum of values}}{\text{Number of values}}$

, then $13 = \dfrac{(2\,a) + (a + 3) + (3\,a - 2) + (11 - a) + 3}{5}$

, then $13 = \dfrac{5\,a + 15}{5}$

, then $13 = \dfrac{\cancel{5}\,(a + 3)}{\cancel{5}}$

, then $13 = a + 3$, then $a = 10$

TRY 2 by yourself

If the arithmetic mean of the values :

k , 3 k , 5 and 7 is 4 , then find the value of k

Example 4 **Find the arithmetic mean of the two numbers 5 and 8 , then represent the three numbers on the number line. What do you notice ?**

Solution

The arithmetic mean $= \frac{5+8}{2} = 6\frac{1}{2}$

We notice that the number $6\frac{1}{2}$ lies at the middle of the distance between 5 and 8

Generally

The number that lies at the middle of the distance between two numbers is the number which represents the arithmetic mean of the two numbers.

TRY 3 *by yourself*

Find the rational number that lies at the middle of the distance between the two numbers :

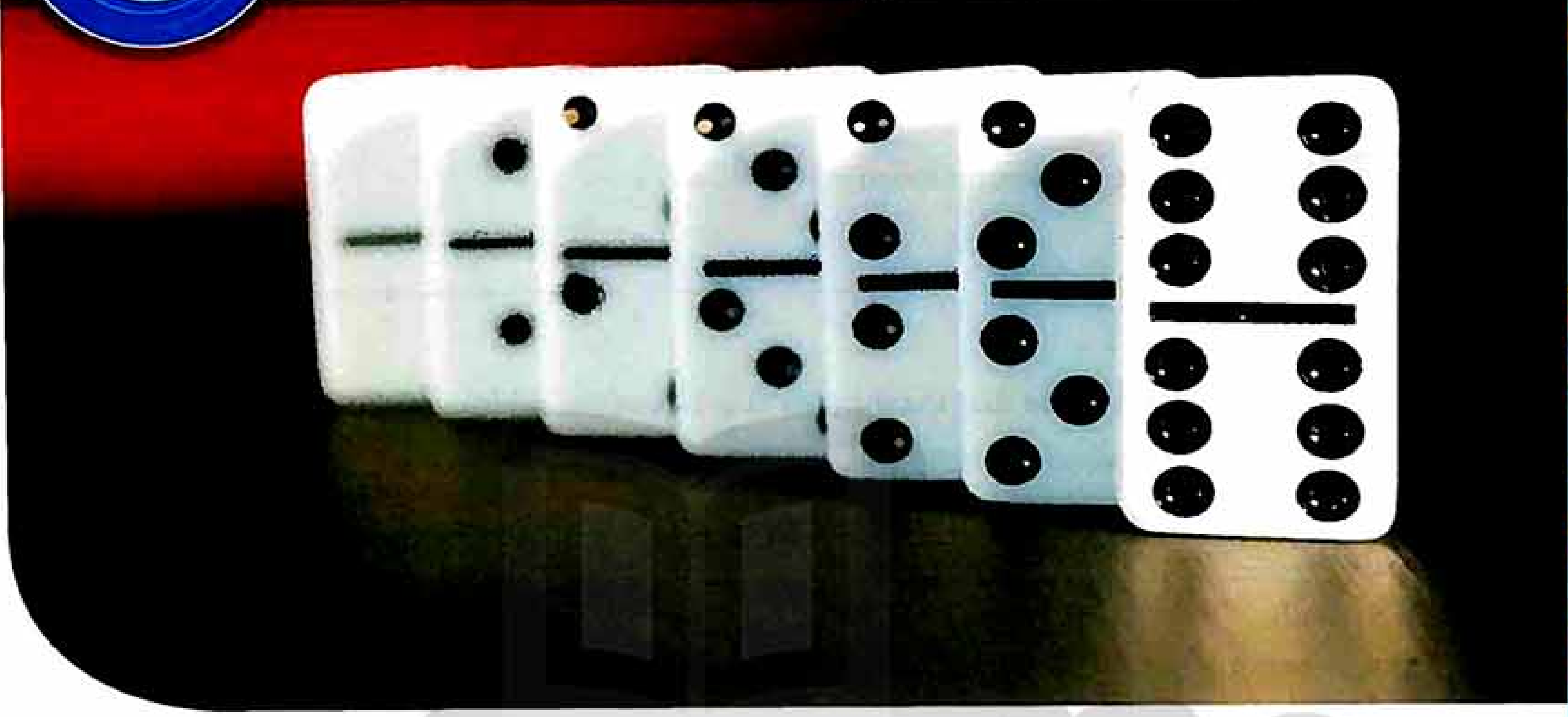
$\frac{1}{6}$ and $\frac{5}{6}$

Answers of try by yourself

1 7 2 1 3 $\frac{1}{2}$

LESSON

2 The Median



Definition :

The median of a set of values is the value which lies exactly in the middle of the set if it is arranged ascendingly or descendingly.

i.e. The median is the value which divides the set of values into two parts such that the number of values which are greater than it is equal to the number of values which are smaller than it.

First : Finding the median if the number of values is odd

If the number of values (n) is odd , then the median equals the value which lies in the middle of the values after arranging them , which is the value whose order is $\frac{n+1}{2}$

Example 1 The following are the lengths of 7 students from the students of first preparatory in centimetres : 142 , 150 , 160 , 155 , 140 , 145 and 158

What is the median length for these students ?

Solution

1. **Arrange the lengths ascendingly (or descendingly) as the following :**
   140 , 142 , 145 , 150 , 155 , 158 and 160
2. **Determine the order of the median :**
   where the number of values = 7 [odd number]
   , then the order of the median $= \frac{7+1}{2} = 4$

3 Find the median :

The median length is the fourth value and equals 150 cm. (notice that there are three values smaller than it and three values greater than it)

Second : Finding the median if the number of values is even

If the number of values (n) is even , then the median equals the arithmetic mean of the two values which lie in the middle of the values after arranging them , and the orders of these values are $\frac{n}{2}$, $\frac{n}{2} + 1$

Example 2 The following are the marks of 8 students in an exam of mathematics :
44 , 47 , 50 , 39 , 48 , 46 , 37 and 41

What is the median mark for these students ?

Solution

1 Arrange the marks ascendingly (or descendingly) as the following :

50 , 48 , 47 , 46 , 44 , 41 , 39 and 37

2 Determine the order of the median :

where the number of values = 8 [even number]

, then the order of the median is $\frac{8}{2}$ and $\frac{8}{2} + 1$

i.e. 4 and 5

3 Find the median :

The median mark is the arithmetic mean of the fourth and the fifth marks which are 46 and 44 , and they are the two marks which lie in the middle of the set of marks where there are three marks smaller than them and three marks greater than them.

i.e. The median mark $= \frac{46 + 44}{2} = 45$

Remarks

- The order of the median is always a positive integer.
- The value of the median may be a negative number or a fraction according to the given values.

UNIT 3

Conclusion :

To get the median, do as follows :

Arrange the values ascendingly or descendingly

then

If the number of values is odd, then :
The median is the value which is in the middle exactly.

For example:

- **If the values are :**

42 , 23 , 17 , 30 , 20

Then its ascending order is :

17 , 20 , 23 , 30 , 42

The median = 23

If the number of values is even, then :
The median

$$= \frac{\text{The sum of the two middle values}}{2}$$

For example:

- **If the values are :**

27 , 13 , 23 , 24 , 13 , 21

Then its ascending order is :

13 , 13 , 21 , 23 , 24 , 27

$$\text{The median} = \frac{21 + 23}{2} = 22$$

TRY by yourself

1 ***Find the median of the values :*** 5 , 11 , 7 , 14 , 10

2 ***Find the median of the values :*** 2 , 6 , 1 , 8 , 4 , 10

Answers of try by yourself

1 10 2 5

LESSON 3

The Mode

Definition :

The mode of a set of data is the most common data.

The mode is used as a measure of central tendency in the case of numerical data , also in the case of descriptive data.

Example 1 **Find the mode of each of the following :**

1. 5 , 8 , 7 , 5 , 6 , 8 , 5
2. very good , excellent , very good , pass , excellent , very good , pass , excellent , very good.

Solution

1. The most common value is 5 , then the mode = 5
2. The most common data is very good , then the mode is very good.

Example 2 **The following table shows the marks of 30 pupils in an examination :**

Mark	5	6	7	8	9	10
Number of pupils (frequency)	3	5	7	9	4	2

The opposite table is called simple frequency table.

Find the mode mark.

Solution

From the table , we find that the greatest number of pupils obtained a mark is 9 pupils and they obtained the mark 8 , then the mode = 8

UNIT 3

Remarks

- If all of the data are different , then these data have not a mode.

For example:

The mode of the values : 25 , 19 , 26 , 7 , 10 , 32 and 15 is not exist because all of the values are different. In other words , there is not a value of these repeated more than the others.

- Some of data have more than one mode.

For example:

For the set of values : 15 , 10 , 24 , 7 , 10 , 31 and 7 , there are two values repeated more than the others and they are : 10 and 7 (each one of them repeated twice).

i.e. This set of values has two modes which are : 10 and 7 and is called a set of two modes.

TRY by yourself

1 ***Complete :***

The mode of the values 6 , 8 , 8 , 5 , 6 , 8 is ………

2 ***The following table is the frequency table of ages in years of a group of friends :***

The age	9	10	11	12	13
Frequency	2	3	4	3	1

Find the mode.

Answers of try by yourself

1 8 2 11

Second Geometry

UNIT

4

Geometry and Measurement

Lessons of the unit :

1. Geometric concepts - The relations between the angles.
2. The relations between the angles "follow".
3. Congruence.
4. Congruent triangles.
5. Parallelism.
6. Geometric constructions.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of each : line segment – straight line – ray – angle.
- recognize the types of angles.
- recognize the complementary angles and the supplementary angles.
- recognize the relation between two vertically opposite angles.
- recognize the sum of measures of accumulative angles at a point.
- recognize the conditions of congruence of two polygons.
- recognize the cases of congruence of two triangles.
- solve different problems on congruence of two triangles.
- prove that two lines are parallel.
- construct a perpendicular to a straight line from a point does not belong to it.
- construct a perpendicular to a straight line from a point belongs to it.
- construct an axis of symmetry of a line segment.
- bisect an angle of a given measure.
- construct an angle to be congruent to a given angle.
- draw a straight line parallel to another straight line.

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Euclid

Euclid
(325 B.C. - 265 B.C.)

Euclid is a Greek mathematician scientist. He lived in Alexandria. He is considered the father of Geometry. He said that "What is made without evidence can be refused without evidence". He put some definitions such that :

- The point is what has no part.
- The straight line is length without width.

And from his axioms :

- A straight line can be drawn by joining any two points.
- A straight line segment can be extended infinitely in a straight line.
- All right angles are equal.

LESSON 1

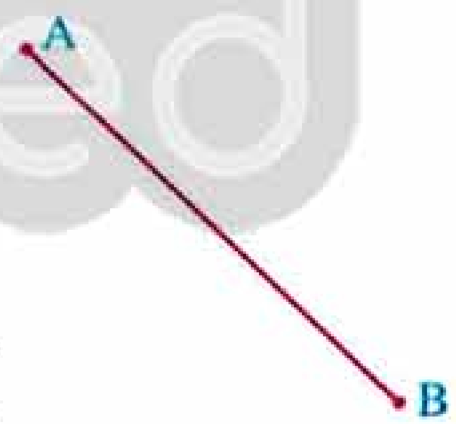
Geometric Concepts - The Relations between the Angles

Geometric concepts

1 The line segment

The line segment is a set of points consisting of two distinct points and all points between them when we join them by a ruler.

- A line segment has two end points , and the symbol — on top of two letters is used to denote the line segment.
  The opposite figure represents the line segment whose end points A and B , and is denoted by $\overline{AB}$ or $\overline{BA}$

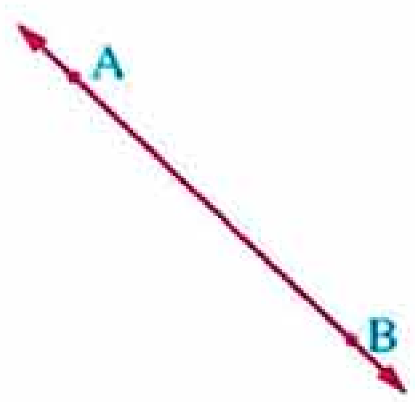


- A line segment has a length and this is the number which refers to the distance between its end points.
  If the length of the line segment whose end points are A and B is 4 cm.
  , then we write the length of $\overline{AB}$ = 4 cm. or AB = 4 cm. or BA = 4 cm.

2 The straight line

The straight line is a line segment extended from both directions infinitely.

- A straight line does not have a starting point and an end point , and the symbol $\leftrightarrow$ on top of two letters is used to denote the straight line , where the two arrows show that the line can be extended without limit on both sides.
  The opposite figure represents the straight line which passes through the two points A and B , and is denoted by $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$



- The straight line is extended without limit on both sides , then it has no length.
- Through any two distinct points exactly one straight line can be drawn.

3 The ray

The ray is a line segment extended from only one of its terminals without limit.

- A ray has a starting point and it has no end point and the symbol $\rightarrow$ on top of the starting point and any other point on the ray is used to denote the ray.
- If the line segment $\overline{AB}$ is extended from its terminal B without limit in a straight line , we will get the ray $\overrightarrow{AB}$ which starts at A and passes through the point B , and it is denoted by the symbol $\overrightarrow{AB}$

- If the line segment $\overline{AB}$ is extended from its terminal A without limit in a straight line , we will get the ray $\overrightarrow{BA}$ which starts at B and passes through the point A , and it is denoted by the symbol $\overrightarrow{BA}$

Notice that : $\overrightarrow{AB} \neq \overrightarrow{BA}$

- A ray extends from one of its terminals without limit , so it has no length.

Remarks

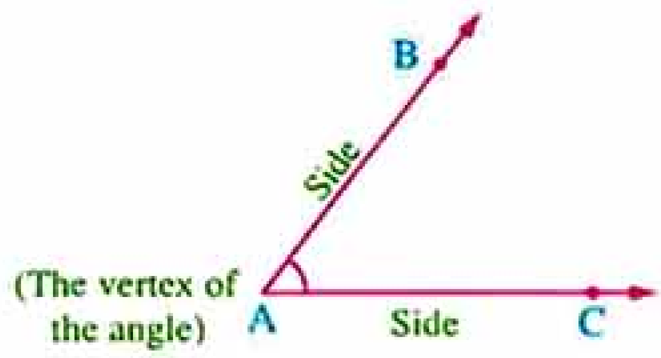
- Each of the line segment , the straight line and the ray is an infinite set of points.
- $\overline{AB} \subset \overrightarrow{AB}$, $\overrightarrow{AB} \subset \overleftrightarrow{AB}$ **i.e.** $\overline{AB} \subset \overrightarrow{AB} \subset \overleftrightarrow{AB}$

4 The angle

The angle is the union of two rays with the same starting point , and this point is called the vertex of the angle , and the two rays are called the two sides of the angle.

For example :

In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two rays having the same starting point A , then :

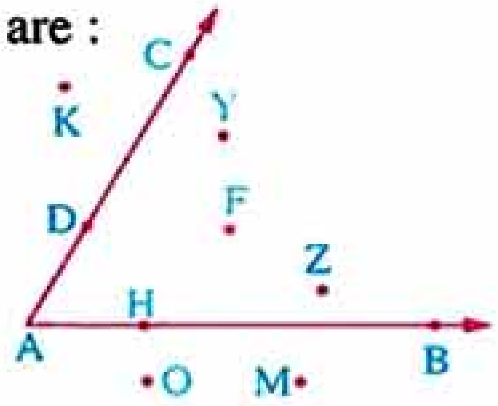
$\overrightarrow{AB} \cup \overrightarrow{AC}$ = the angle CAB

* A is the vertex of the angle CAB
* $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the two sides of the angle CAB

- The symbol $\angle$ is used to denote an angle.

UNIT 4

- An angle can be named by the letters of three points , one letter from one side , one letter from the other side , and the third is the vertex of the angle where the middle letter is the letter of the vertex , then we write : ∠ CAB or ∠ BAC , it can also be named by only of the vertex , then we write : ∠ A if no other angle shares the same vertex.
- The angle divides the plane in which it lies to three sets of points that are :

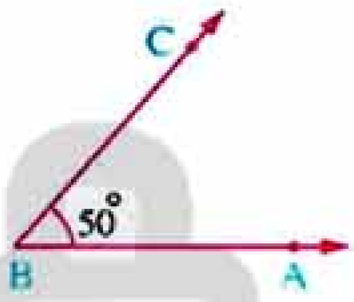


1. The set of "points of the angle" as : B , C , H , ...
2. The set of "interior points of the angle" as : F , Y , Z , ...
3. The set of "exterior points of the angle" as : M , K , O , ...

Measurement of the angle

The measurement of the angle is the number expressing the amount of happened divergence between the two sides.

- A protractor is used to measure an angle , and the angle is measured using degree unit which is denoted by (°) and the opposite figure represents an angle of measure 50° , then we write : m (∠ ABC) = 50°

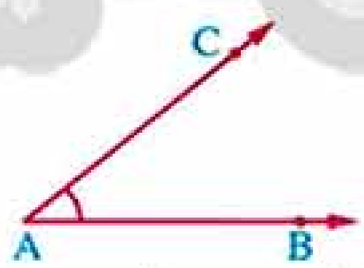
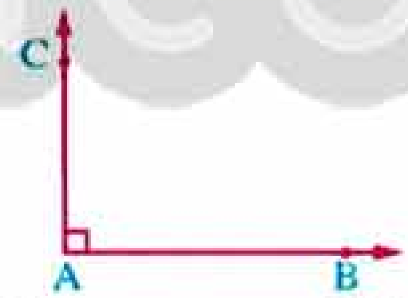
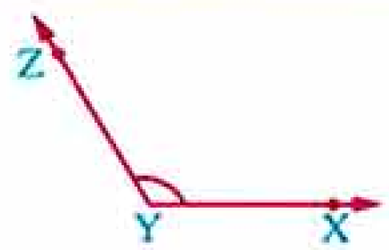
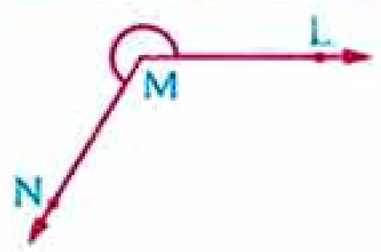


- A degree is divided into parts smaller than it , and they are minute (`) and second (̏) where :

– The degree equals 60 minutes (1° = 60`) – The minute equals 60 seconds (1` = 60 ̏)

The types of angles

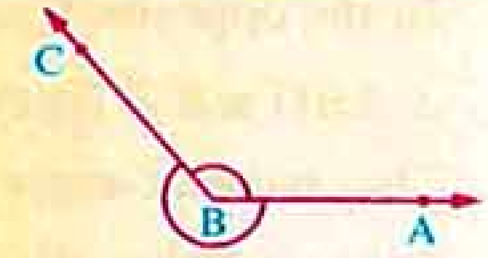
The angles are classified according to their measures as follows :

1 Zero angle  Its measure = 0° Its sides are coincident.	**2 Acute angle**  Its measure is more than 0° and less than 90°	**3 Right angle**  Its measure = 90°
4 Obtuse angle  Its measure is more than 90° and less than 180°	**5 Straight angle**  Its measure = 180° Its sides are forming one straight line.	**6 Reflex angle**  Its measure is more than 180° and less than 360°

Remark

In the opposite figure :

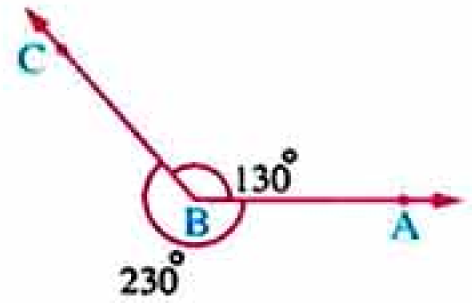
m (∠ ABC) + m (reflex ∠ ABC) = 360°



For example :

If m (∠ ABC) = 130°

, then m (reflex ∠ ABC) = 360° − 130° = 230°



Example 1 **Mention the type of each of the angles whose measures are as follows :**

1 32°	2 90°	3 110°	4 180°
5 250°	6 179° 60′	7 $180\frac{1}{4}°$	8 $159\frac{3}{8}°$

Solution

1 acute.	2 right.	3 obtuse.	4 straight.
5 reflex.	6 straight.	7 reflex.	8 obtuse.

TRY 1 by yourself

Complete the following two tables :

1

m (∠ ABC)	45°	180°	200°	150°	90°	94° 10′	89° 61′
Its type	....	....	....	....	....	....	....

2

m (∠ ABC)	135°	58°	80°	100°	110°	$52°\frac{1}{2}$	89° 60′
m (reflex ∠ ABC)	....	....	....	....	....	....	....

Some relations between the angles

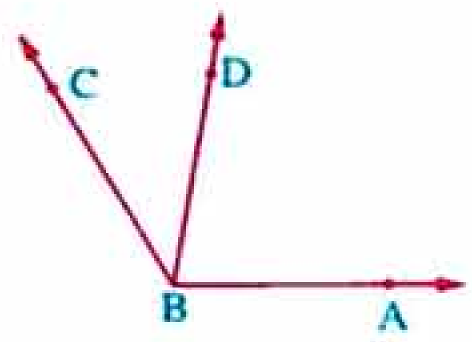
Adjacent angles

Two angles are said to be adjacent if they have a common vertex and a common side and the other two sides are on opposite sides of this common side.

UNIT 4

For example :

In the opposite figure :

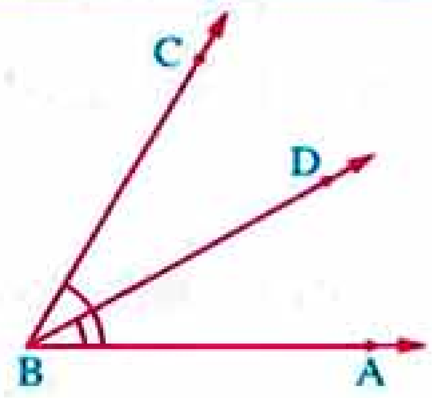


∠ ABD and ∠ DBC are two adjacent angles , for :

- They have a common vertex B and a common side $\overrightarrow{BD}$
- The two other sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on two opposite sides of $\overrightarrow{BD}$

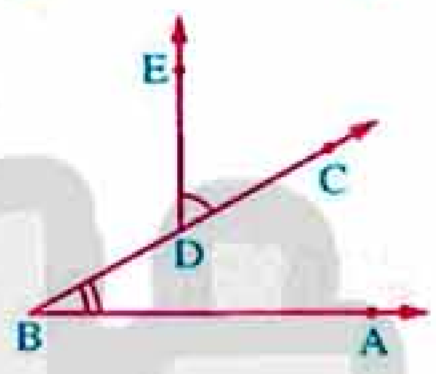
Remarks

1. **In the opposite figure :**



∠ ABD and ∠ ABC are not two adjacent angles because the two sides $\overrightarrow{BD}$ and $\overrightarrow{BC}$ are on the same side of the common side $\overrightarrow{BA}$

2. **In the opposite figure :**



∠ ABC and ∠ CDE are not two adjacent angles because they have not a common vertex and also they have not a common side.

Complementary angles

Two angles are said to be complementary if the sum of their measures is 90°

For example :

The two angles whose measures are 55° and 35° are called two complementary angles because 55° + 35° = 90°

Remarks

1. The two complementary angles are either acute angles or one of them is zero angle and the other is a right angle.
2. The complements of the same angle (or the equal angles in measure) are equal in measure.

   i.e. If ∠ A complements ∠ B , ∠ C complements ∠ B , then m (∠ A) = m (∠ C)

Supplementary angles

Two angles are said to be supplementary if the sum of their measures is 180°

For example :

The two angles whose measures are 143° and 37° are called two supplementary angles because 143° + 37° = 180°

Remarks

1. The two supplementary angles are either one of them is obtuse and the other is acute or each of them is a right angle or one of them is zero angle and the other is a straight angle.
2. The supplements of the same angle (or the equal angles in measure) are equal in measure.
   i.e. If $\angle A$ supplements $\angle B$ and $\angle C$ supplements $\angle B$, then $m(\angle A) = m(\angle C)$

TRY 2 by yourself

Complete the following :

1. The angle whose measure is $75°$ complements an angle of measure ....° and supplements an angle of measure ....°
2. The angle whose measure is ....° complements an angle of measure $67°$ and supplements an angle of measure ....°
3. The angle whose measure is ....° complements an angle of measure .... ° and supplements an angle of measure $154°$

The two adjacent supplementary angles

Two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are supplementary.

i.e. In the opposite figure :

If $\overleftrightarrow{AB} \cap \overrightarrow{CD} = \{C\}$

Therefore , $m(\angle ACD) + m(\angle DCB) = 180°$ "Straight angle"

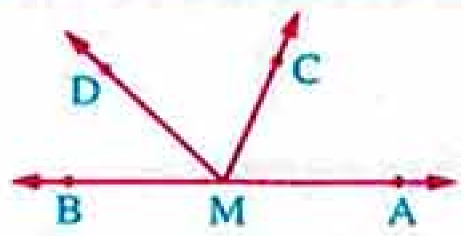
And if $m(\angle ACD) = 135°$

Then $m(\angle DCB) = 180° - 135° = 45°$

Remark

If $M \in \overleftrightarrow{AB}$, and $\overrightarrow{MC}$ and $\overrightarrow{MD}$ are drawn on one side of $\overleftrightarrow{AB}$,

then $m(\angle AMC) + m(\angle CMD) + m(\angle DMB) = 180°$



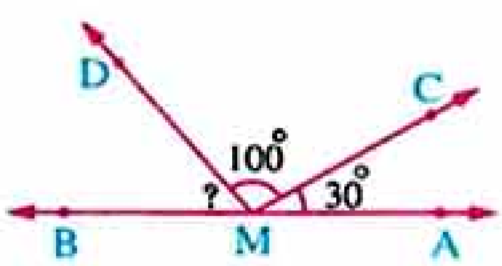
For example :

In the opposite figure :

If $M \in \overleftrightarrow{AB}$, $m(\angle AMC) = 30°$

, $m(\angle CMD) = 100°$

, then : $m(\angle DMB) = 180° - (30° + 100°) = 180° - 130° = 50°$



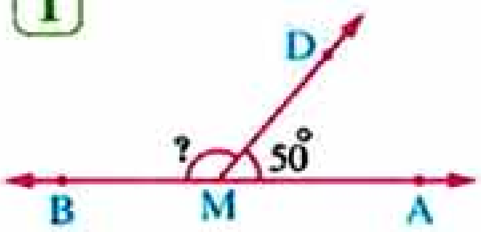
UNIT 4

TRY 3 by yourself

In each of the following figures :

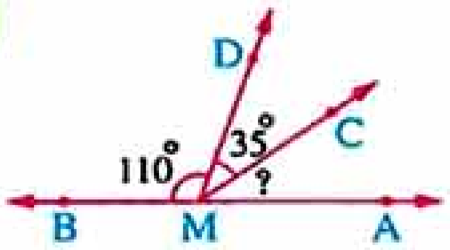
If $M \in \overleftrightarrow{AB}$, then find the measure of the angle marked by (?) :

1



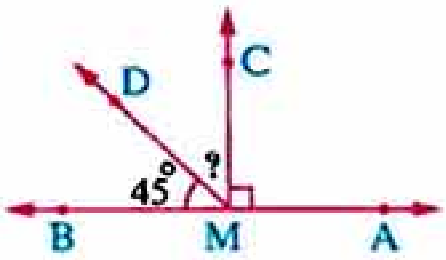
m (∠ DMB) = ...°

2



m (∠ AMC) = ...°

3



m (∠ DMC) = ...°

The two outer sides of two adjacent angles

If two adjacent angles are supplementary , then their outer sides are on the same straight line.

For example :

- **In the following figure :**

$\overrightarrow{MA}$ and $\overrightarrow{MC}$ are on the same straight line because :

m (∠ AMB) + m (∠ BMC) = 50° + 130°
= 180°

- **In the following figure :**

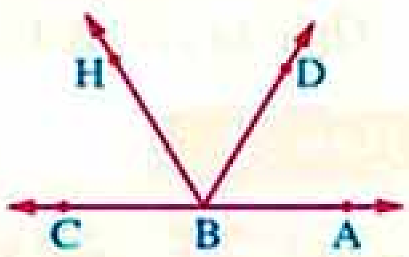
$\overrightarrow{YX}$ and $\overrightarrow{YL}$ are not on the same straight line because :

m (∠ XYZ) + m (∠ ZYL) = 38° + 141°
= 179° ≠ 180°

Remark

In the opposite figure :

If m (∠ ABD) + m (∠ DBH) + m (∠ HBC) = 180° ,
then $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on the same straight line.

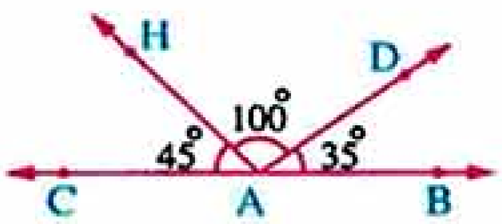


For example :

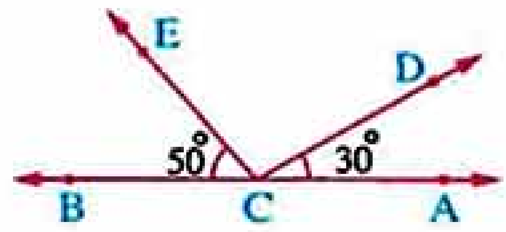
In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are on the same straight line.

because : m (∠ BAD) + m (∠ DAH) + m (∠ HAC)
= 35° + 100° + 45° = 180°



Example 2 **In the opposite figure :**



If m (∠ ACD) = 30° , m (∠ ECB) = 50°

and m (∠ DCE) = 2 m (∠ ECB)

State with giving the reason if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line or not.

Solution

$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.

The reason :

m (∠ DCE) = 2 × 50° = 100° because m (∠ DCE) = 2 m (∠ ECB)

i.e. m (∠ ACD) + m (∠ DCE) + m (∠ ECB) = 30° + 100° + 50° = 180°

Remark

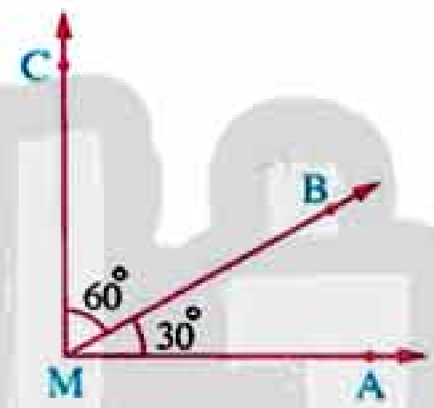
If the two adjacent angles are complementary angles , then their outer sides are perpendicular.

For example :

In the opposite figure :

$\overrightarrow{MA} \perp \overrightarrow{MC}$

Because : m (∠ AMB) + m (∠ BMC) = 30° + 60° = 90°

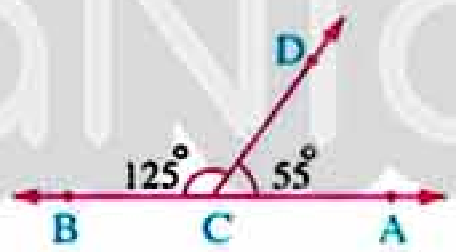


TRY 4 *by yourself*

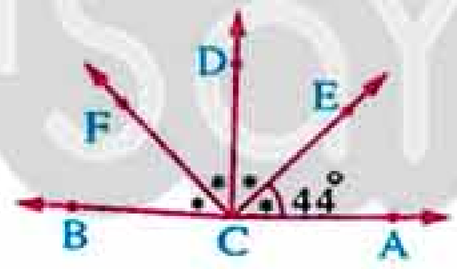
In each of the following figures ,

state if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line or not , and why ?

1



2



Answers *of try by yourself*

1 1 acute , straight , reflex , obtuse , right , obtuse , obtuse

2 225° , 302° , 280° , 260° , 250° , $307\frac{1}{2}$° , 270°

2 1 15° , 105° 2 23° , 157° 3 26° , 64°

3 1 130° 2 35° 3 45°

4 1 Yes , the reason : m (∠ ACD) + m (∠ BCD) = 180°

2 No , the reason : m (∠ ACE) + m (∠ ECD) + m (∠ DCF) + m (∠ FCB) = 176° ≠ 180°

LESSON 2

The Relations between the Angles (Follow)

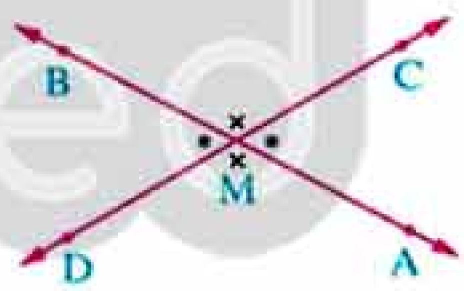
Vertically opposite angles (V.O.A.)

If two straight lines intersect , then the measures of each two vertically opposite angles are equal.

In the opposite figure :

If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at M

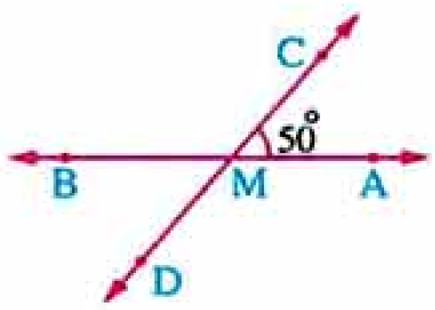
Then :



- ∠ AMC and ∠ BMD are vertically opposite angles , then m (∠ AMC) = m (∠ BMD)
- Also , ∠ CMB and ∠ AMD are vertically opposite angles , then m (∠ CMB) = m (∠ AMD)

For example :

In the opposite figure :



If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

, m (∠ AMC) = 50°

, then m (∠ DMB) = m (∠ AMC) = 50° (vertically opposite angles)

, m (∠ CMB) = 180° – m (∠ AMC) = 180° – 50° = 130°

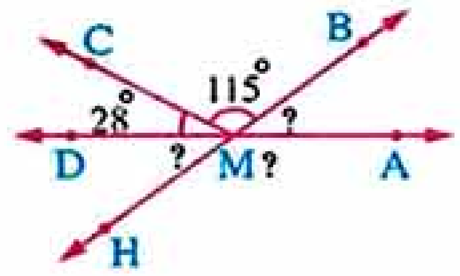
, then m (∠ AMD) = m (∠ CMB) = 130° (vertically opposite angles)

Example 1 **In the opposite figure :**

$\overleftrightarrow{AD} \cap \overleftrightarrow{BH} = \{M\}$,

m (∠ CMD) = 28° and m (∠ BMC) = 115°

Find the measures of the angles marked by (?)



Solution

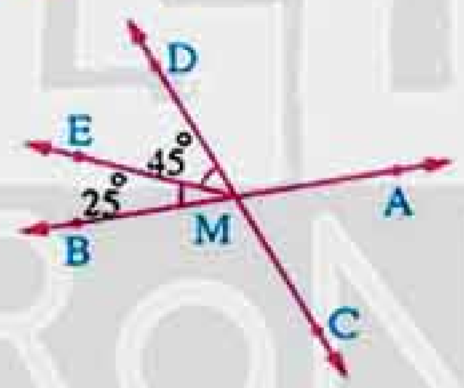
- m (∠ AMB) = 180° – (115° + 28°) = 180° – 143° = 37°
  Because : m (∠ AMB) + m (∠ BMD) = 180°
- m (∠ DMH) = 37°
  Because : m (∠ DMH) = m (∠ AMB) (vertically opposite angles)
- m (∠ AMH) = 143°
  Because : m (∠ AMH) = m (∠ BMD) (vertically opposite angles)

TRY 1 *by yourself*

In each of the following figures :

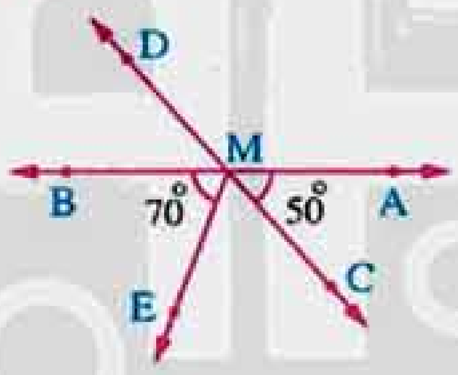
If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, find the measure of the required angle under each figure :

1



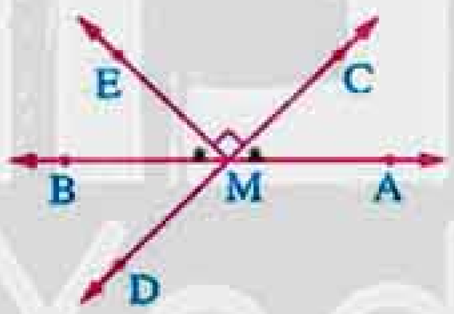
m (∠ AMC) = ...°

2



m (∠ DME) = ...°

3



m (∠ BMD) = ...°

Accumulative angles at a point

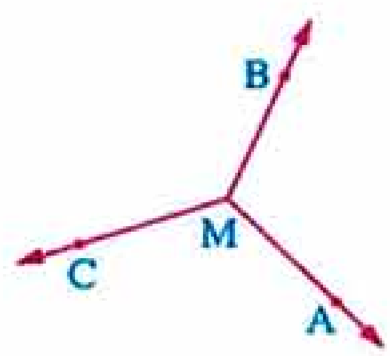
The sum of the measures of the accumulative angles at a point is 360°

In the opposite figure :

If $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ are rays having the same starting point M

The angles ∠ AMB , ∠ BMC and ∠ CMA are called accumulative angles at the point M and

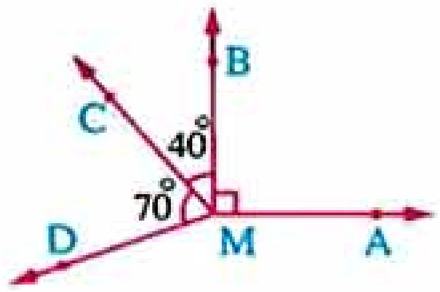
m (∠ AMB) + m (∠ BMC) + m (∠ CMA) = 360°



UNIT 4

For example :

In the opposite figure :



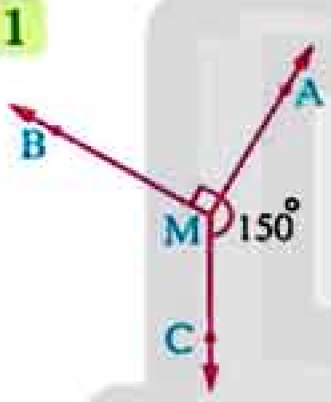
If $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$ and $\overrightarrow{MD}$ are rays having the same starting point M

, then m (∠ AMB) + m (∠ BMC) + m (∠ CMD) + m (∠ DMA) = 360°

So , m (∠ DMA) = 360° – (90° + 40° + 70°) = 160°

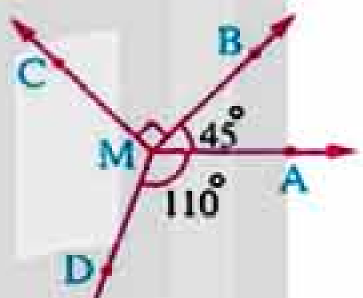
Example 2 **In each of the following figures , find the measure of the required angle under each figure :**

1



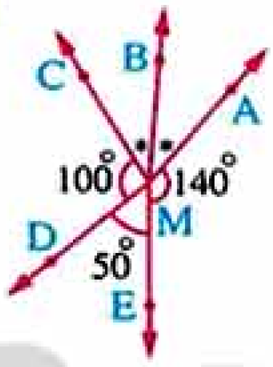
m (∠ BMC) = ... °

2



m (∠ CMD) = ... °

3



m (∠ AMB) = ... °

Solution

1 m (∠ BMC) = 360° – (150° + 90°) = 120°

2 m (∠ CMD) = 360° – (110° + 90° + 45°) = 115°

3 m (∠ AMC) = 360° – (140° + 50° + 100°) = 70°

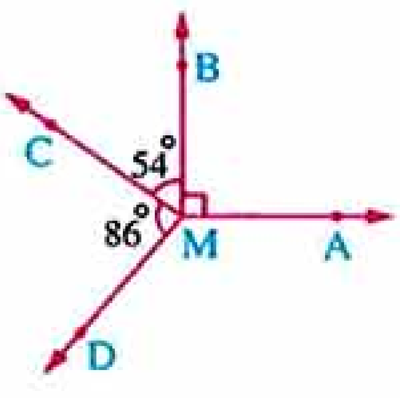
m (∠ AMB) = m (∠ BMC) = $\frac{70^\circ}{2}$ = 35°

TRY 2 by yourself

In each of the following figures :

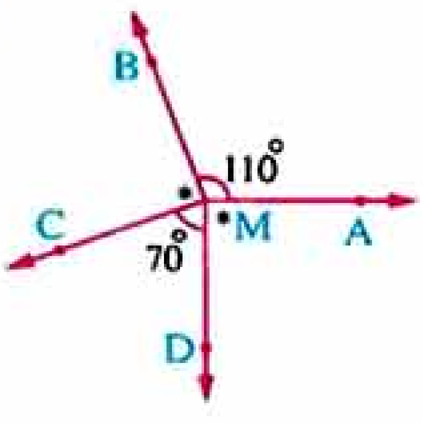
Find the measure of the required angle under each figure :

1



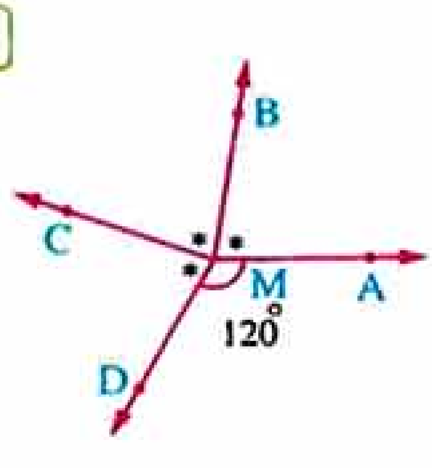
m (∠ AMD) = ... °

2



m (∠ AMD) = ... °

3



m (∠ BMD) = ... °

The angle bisector

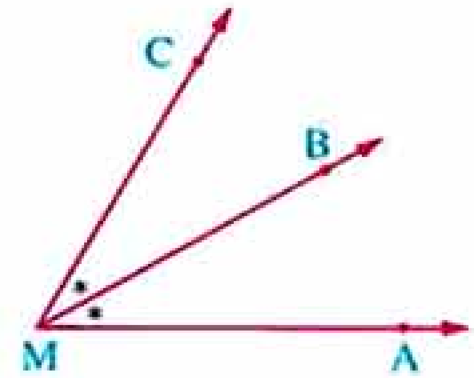
It is the ray that divides the angle into two halves (two equal angles in measure)

In the opposite figure :

$\overrightarrow{MB}$ bisects $\angle AMC$

i.e. $m(\angle AMB) = m(\angle BMC) = \frac{1}{2}\, m(\angle AMC)$

or $m(\angle AMC) = 2\, m(\angle AMB) = 2\, m(\angle BMC)$



For example :

If $m(\angle AMB) = 30°$

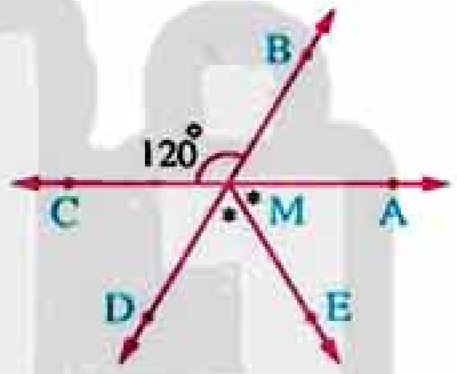
, then $m(\angle AMC) = 60°$

Example 3 **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, $m(\angle BMC) = 120°$

and $\overrightarrow{ME}$ bisects $\angle AMD$

Find : $m(\angle EMC)$



Solution

$m(\angle AMD) = 120°$

Because : $m(\angle AMD) = m(\angle BMC)$ (vertically opposite angles)

, $m(\angle EMD) = 60°$

Because : $\overrightarrow{ME}$ bisects $\angle AMD$

, $m(\angle CMD) = 180° - 120° = 60°$

, then $m(\angle EMC) = 60° + 60° = 120°$

UNIT 4

TRY 3 by yourself

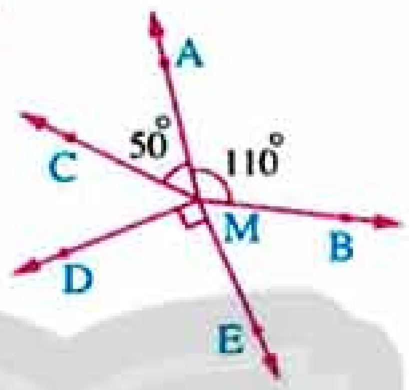
In each of the following figures :

If $\overrightarrow{MC}$ bisects $\angle$ AMD , find the measure of the required angle under each figure :

1

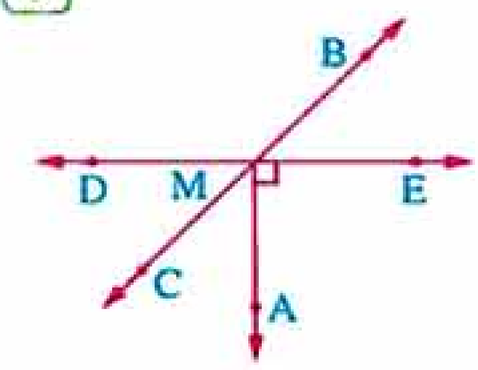
If M $\in \overleftrightarrow{AB}$, then m ($\angle$ DMC) = ...°

2



m ($\angle$ EMB) = ...°

3



m ($\angle$ BME) = ...°

Answers of try by yourself

1	1 70°	2 120°	3 45°
2	1 130°	2 90°	3 160°
3	1 67.5°	2 60°	3 45°

LESSON

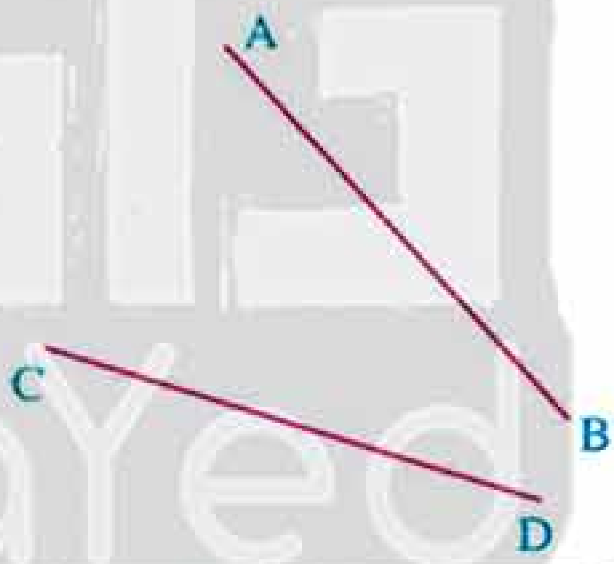
3 Congruence

Two geometric figures are congruent if they are fit exactly on top of each other.
We use the symbol ≡ to represent the congruence , and the following examples of congruence of some geometric figures :

First : Congruence of two line segments

In the opposite figure :

The two line segments $\overline{AB}$ and $\overline{CD}$ are congruent and by measuring we find that they are equal in length and the length of each one is 4 cm.



Generally

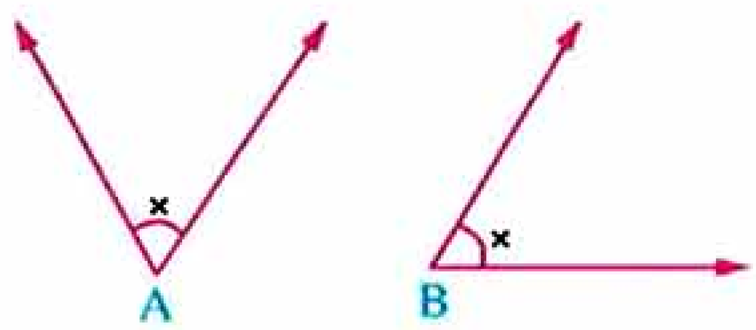
Two line segments are congruent if they are equal in length.

If the length of $\overline{XY}$ = the length of $\overline{ZL}$, then $\overline{XY} \equiv \overline{ZL}$

Second : Congruence of two angles

In the opposite figure :

The two angles ∠ A and ∠ B are congruent and by measuring we find that they are equal in measure and the measure of each angle is 60°



Generally

Two angles are congruent if they are equal in measure.

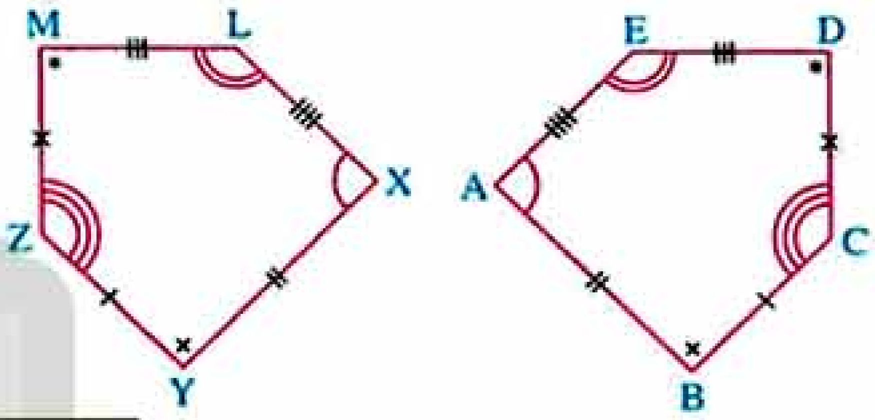
If m (∠ C) = m (∠ D) , then ∠ C ≡ ∠ D

UNIT 4

Third : Congruence of two polygons

Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.

For example :



The two opposite polygons are congruent because :

each two corresponding sides are equal in length.

i.e. AB = XY , BC = YZ , CD = ZM ,
DE = ML and EA = LX

and each two corresponding angles are equal in measure.

i.e. m (∠ A) = m (∠ X) , m (∠ B) = m (∠ Y) , m (∠ C) = m (∠ Z) ,
m (∠ D) = m (∠ M) and m (∠ E) = m (∠ L)
and we write the polygon ABCDE ≡ the polygon XYZML

Remark

It is better to write the name of two congruent polygons in the same order of their corresponding vertices.

For example :

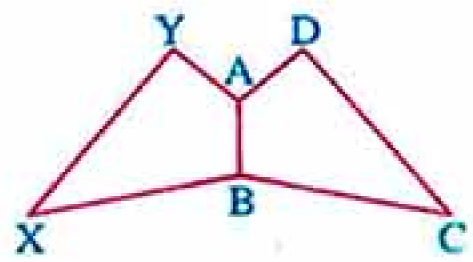
- The vertex A $\xleftrightarrow{\text{Corresponds to}}$ the vertex X
- The vertex B $\xleftrightarrow{\text{Corresponds to}}$ the vertex Y
- The vertex C $\xleftrightarrow{\text{Corresponds to}}$ the vertex Z
- The vertex D $\xleftrightarrow{\text{Corresponds to}}$ the vertex M
- The vertex E $\xleftrightarrow{\text{Corresponds to}}$ the vertex L

Remark

If the two polygons are congruent , then each side and each angle in one of them is congruent to its corresponding element in the other polygon.

For example :

If the figure ABCD ≡ the figure ABXY , then :



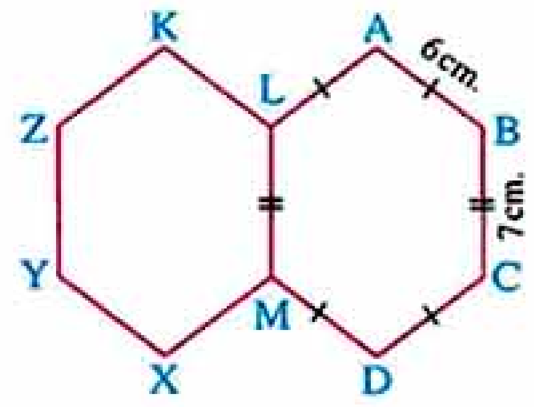
1. BC = BX , AD = AY , CD = XY
2. m (∠ D) = m (∠ Y) , m (∠ C) = m (∠ X) ,
   m (∠ DAB) = m (∠ YAB)
   and m (∠ ABC) = m (∠ ABX)

Notice that :

$\overleftrightarrow{AB}$ is the axis of symmetry of the polygon CDAYXB and divides it into two congruent polygons.

Example

In the opposite figure :



If the polygon ABCDML ≡ the polygon KZYXML , AB = CD = AL = DM = 6 cm. and BC = LM = 7 cm.

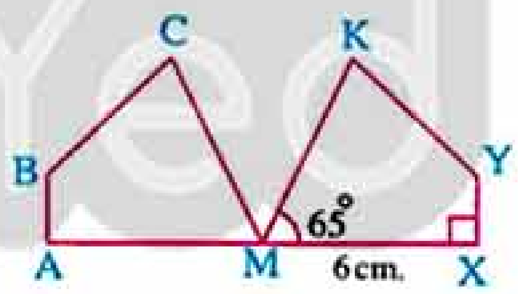
1. Write what you deduce from congruence of the two polygons.
2. Find the perimeter of the polygon MXYZKL

Solution

1. We deduce from the congruence of the two polygons ABCDML and KZYXML that :
   - The corresponding sides are equal in length.
     i.e. KL = AL = 6 cm. , KZ = AB = 6 cm. , ZY = BC = 7 cm. , YX = CD = 6 cm. and XM = DM = 6 cm.
   - The corresponding angles are equal in measure.
     i.e. m (∠ K) = m (∠ A) , m (∠ Z) = m (∠ B) , m (∠ Y) = m (∠ C) , m (∠ X) = m (∠ D) , m (∠ XML) = m (∠ DML) and m (∠ MLK) = m (∠ MLA)
2. The perimeter of the polygon MXYZKL
   = the perimeter of the polygon MDCBAL
   = MD + DC + CB + BA + AL + LM = 6 + 6 + 7 + 6 + 6 + 7 = 38 cm.

TRY *by yourself*

In the opposite figure :



If M ∈ $\overleftrightarrow{AX}$, m (∠ XMK) = 65° , $\overleftrightarrow{XY} \perp \overleftrightarrow{XM}$

the figure XYKM ≡ the figure ABCM and XM = 6 cm.

Complete the following :

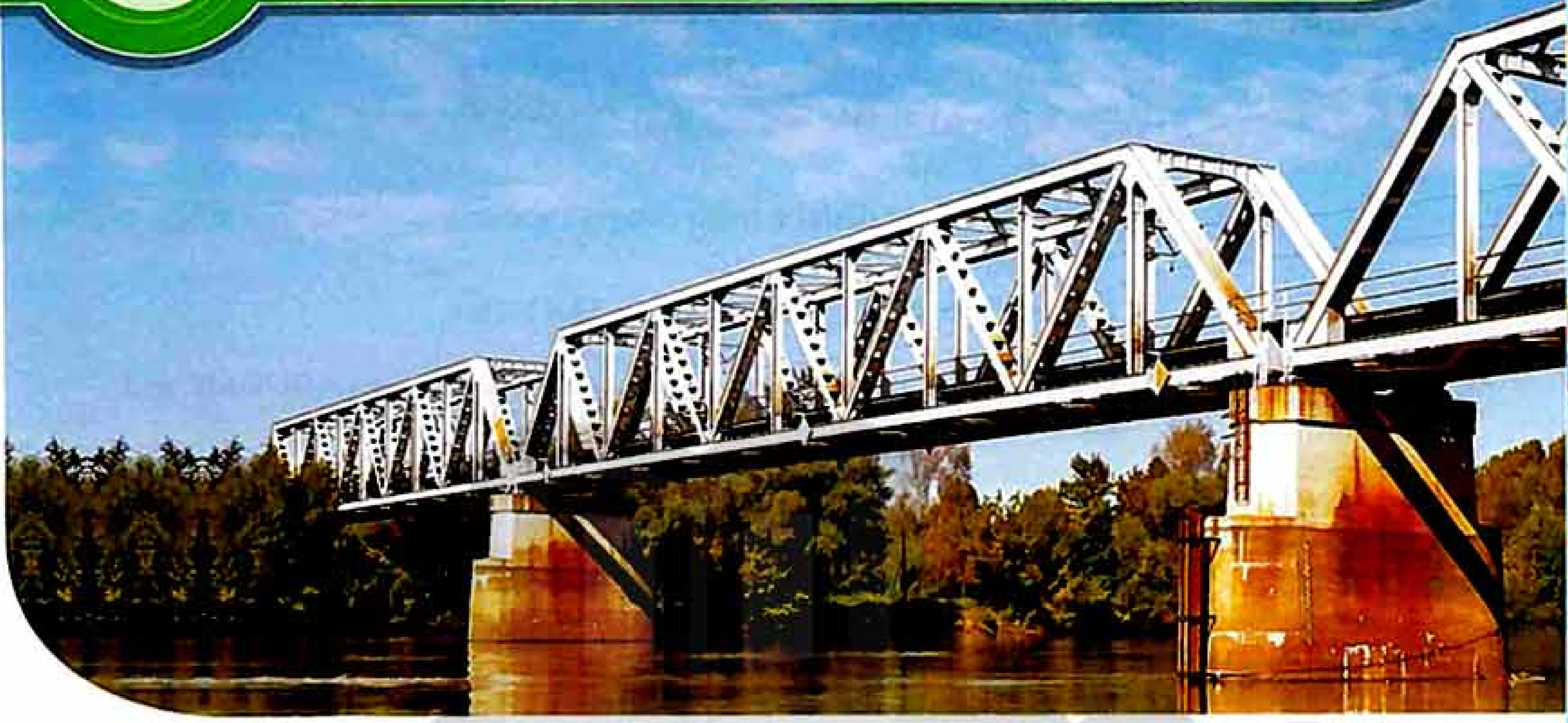
1. $\overline{XY}$ ≡ ....
2. YK = ...
3. AM = ... cm.
4. AX = ... cm.
5. m (∠ Y) = m (∠ ....)
6. m (∠ X) = m (∠ ....)
7. m (∠ CMA) = ...°
8. m (∠ A) = ...°
9. m (∠ KMC) = ...°
10. m (∠ CMX) = ...°

Answers of try by yourself

1. $\overline{AB}$ 2. BC 3. 6 4. 12 5. B
6. A 7. 65° 8. 90° 9. 50° 10. 115°

LESSON

4 Congruent Triangles



We know that any triangle has three sides and three angles , these three sides and three angles are known as the six elements of the triangle.

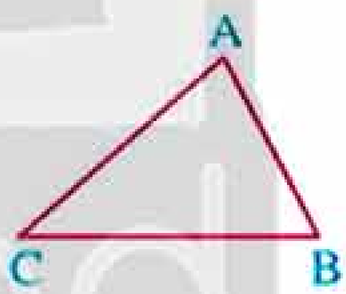
For example :

The six elements of the triangle ABC are

three sides : $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

and three angles :

$\angle A$, $\angle B$ and $\angle C$



> The two triangles are congruent if each element of the 6 elements of one of them is congruent to the corresponding element in the other triangle.

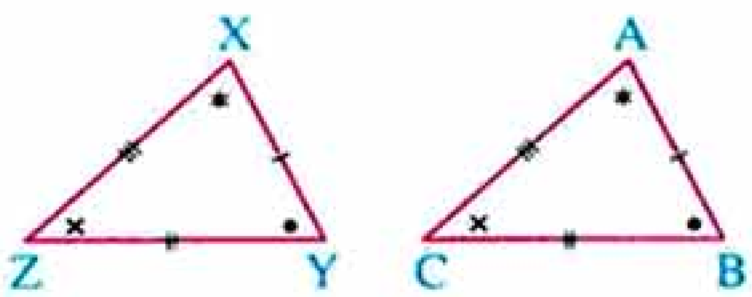
For example :

If ABC and XYZ are two triangles in which :

1. AB = XY , AC = XZ and BC = YZ

2. m ($\angle$ A) = m ($\angle$ X) , m ($\angle$ B) = m ($\angle$ Y) and m ($\angle$ C) = m ($\angle$ Z)

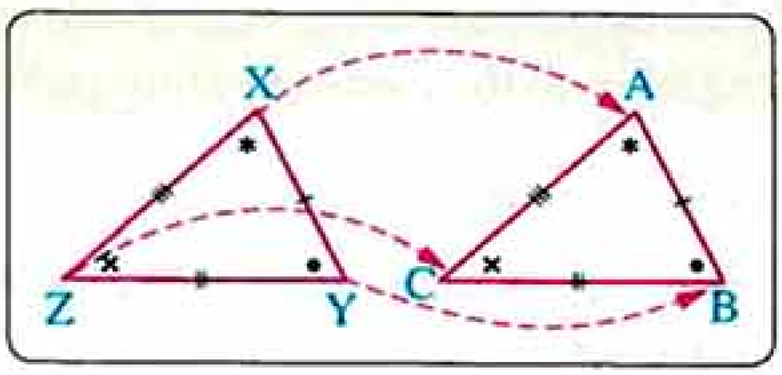
, then $\Delta ABC \equiv \Delta XYZ$



Remarks

1 In the two previous triangles , we notice that :

- The vertex X $\xleftrightarrow{\text{Corresponds to}}$ the vertex A
- The vertex Y $\xleftrightarrow{\text{Corresponds to}}$ the vertex B
- The vertex Z $\xleftrightarrow{\text{Corresponds to}}$ the vertex C



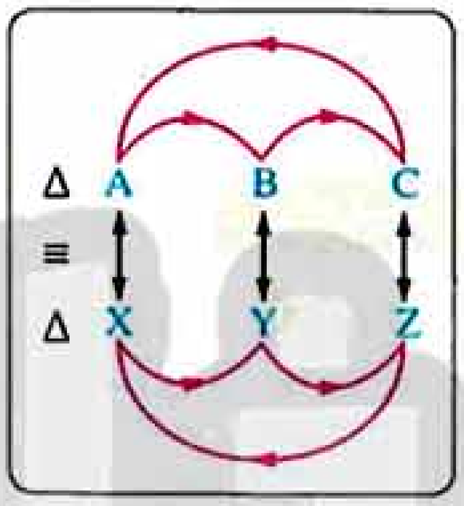
and when we write two congruent triangles , it is better to write them in the same order of their corresponding vertices.

$\Delta ABC \equiv \Delta XYZ$ or $\Delta ACB \equiv \Delta XZY$ or ...

2 If two triangles are congruent , then each element of the six elements of one of the two triangles is congruent to the corresponding element of the other triangle.

i.e. If $\Delta ABC \equiv \Delta XYZ$, then we deduce that :

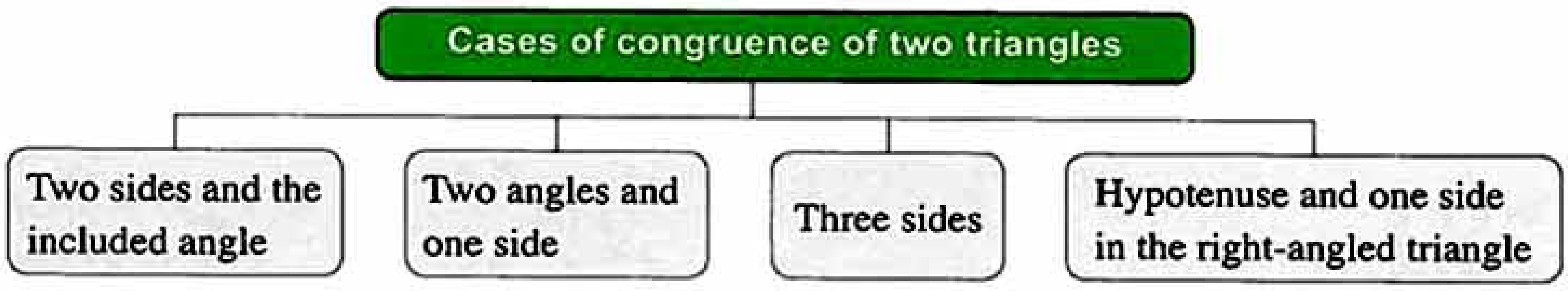
- First : $\overline{AB} \equiv \overline{XY}$, $\overline{BC} \equiv \overline{YZ}$ and $\overline{CA} \equiv \overline{ZX}$
- Second : $\angle A \equiv \angle X$, $\angle B \equiv \angle Y$ and $\angle C \equiv \angle Z$



Cases of congruence of two triangles

From the previous , we knew that the two triangles be congruent when each element of the six elements of one is congruent to the corresponding element of the other triangle , and in the following we will study how to prove that two triangles are congruent by proving that three elements only are congruent to the corresponding elements of the other , in this case , the three other elements are congruent of the two triangles.

In the following , the different cases of congruence of two triangles :



UNIT 4

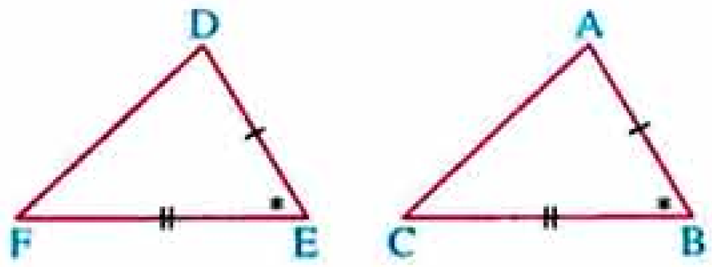
The first case (Two sides and the included angle S.A.S.)

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

For example :

If ABC and DEF are two triangles in which :

$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \overline{BC} \equiv \overline{EF} \\ \angle B \equiv \angle E \end{cases}$$



, then Δ ABC $\equiv \Delta$ DEF and we deduce that :

$$\begin{cases} \overline{AC} \equiv \overline{DF} \\ \angle A \equiv \angle D \\ \angle C \equiv \angle F \end{cases}$$

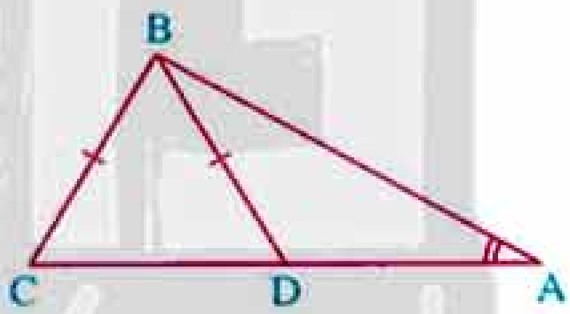
Remark

In the case of congruence of two triangles by two sides and the included angle , the included angle should be between the two sides.

For example :

Although ABC and ABD are two triangles in which :

$$\begin{cases} BC = BD \\ \overline{AB} \text{ is a common side} \\ \angle A \text{ is a common angle} \end{cases}$$



but it is clear that Δ ABC is not congruent to Δ ABD because $\angle$ A is not included between the two sides in each of the two triangles.

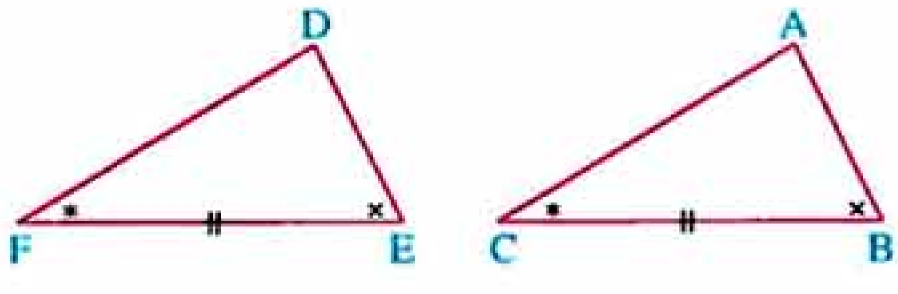
The second case (Two angles and one side A.S.A.)

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

For example :

If ABC and DEF are two triangles in which :

$$\begin{cases} \overline{BC} \equiv \overline{EF} \\ \angle B \equiv \angle E \\ \angle C \equiv \angle F \end{cases}$$



, then Δ ABC $\equiv \Delta$ DEF and we deduce that :

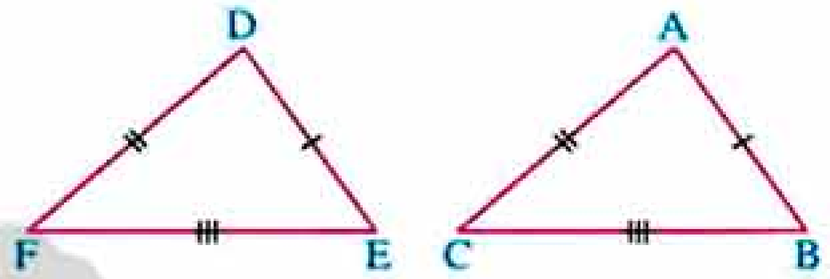
$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \overline{AC} \equiv \overline{DF} \\ \angle A \equiv \angle D \end{cases}$$

The third case (Three sides S.S.S.)

Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.

For example :

If ABC and DEF are two triangles in which :



$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \overline{BC} \equiv \overline{EF} \\ \overline{AC} \equiv \overline{DF} \end{cases}$$

, then $\Delta ABC \equiv \Delta DEF$ and we deduce that :

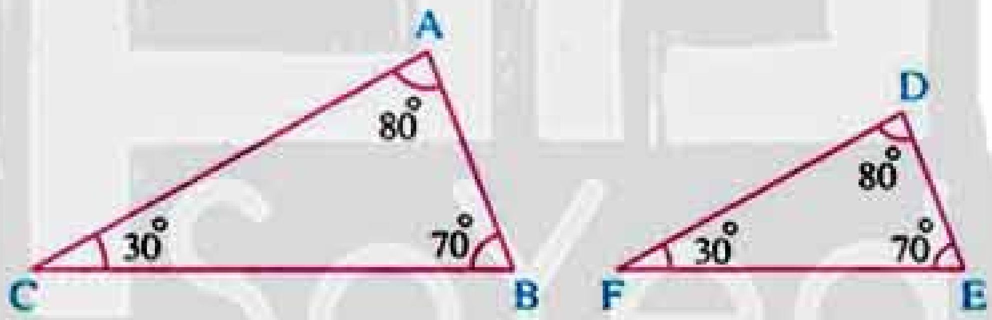
$$\begin{cases} \angle A \equiv \angle D \\ \angle B \equiv \angle E \\ \angle C \equiv \angle F \end{cases}$$

Remark

If each angle of one triangle is congruent to the corresponding angle of the other triangle , it is not necessary for the two triangles to be congruent.

For example :

Although the measures of the corresponding angles of the two triangles ABC and DEF are equal , but it is clear that the two triangles are not congruent.

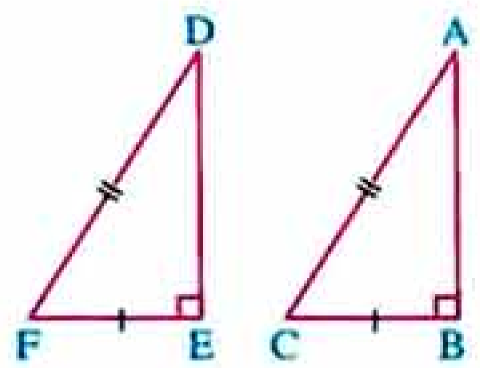


The fourth case (Hypotenuse and one side in the right-angled triangle R.H.S.)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

For example :

If ABC and DEF are two triangles in which :



$$\begin{cases} \overline{AC} \equiv \overline{DF} \\ \overline{BC} \equiv \overline{EF} \\ m(\angle B) = m(\angle E) = 90° \end{cases}$$

, then $\Delta ABC \equiv \Delta DEF$ and we deduce that :

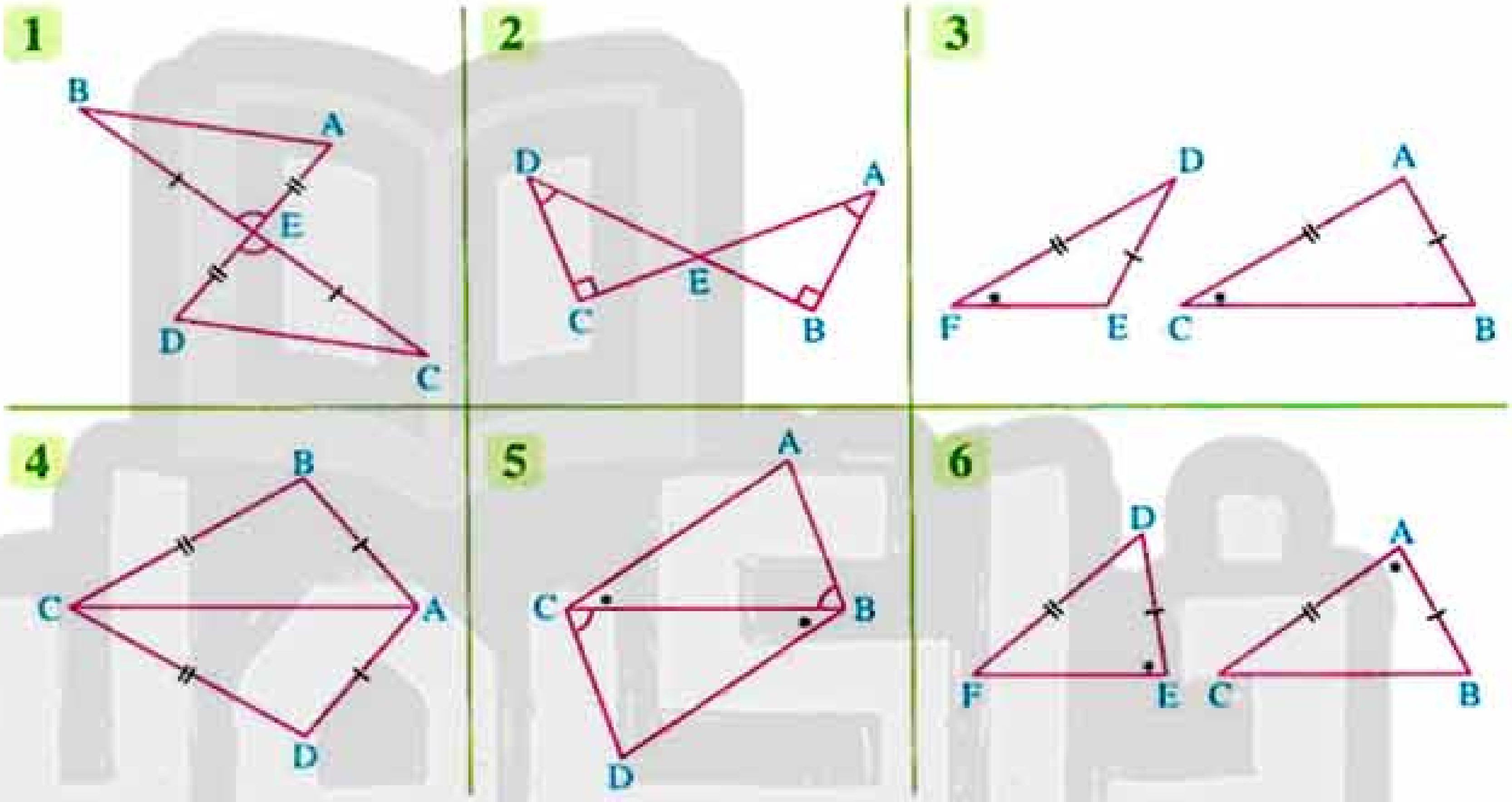
$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \angle A \equiv \angle D \\ \angle C \equiv \angle F \end{cases}$$

UNIT 4

Remark

The two right-angled triangles are congruent if the two sides of the right angle in one of them are congruent to the corresponding elements in the other triangle. (This case is an application of the first case of congruence of two triangles)

Example 1 In each of the following figures , show if the two triangles are congruent or not , give reason (Given that the similar signs denote the congruency of the elements marked by these signs)



Solution

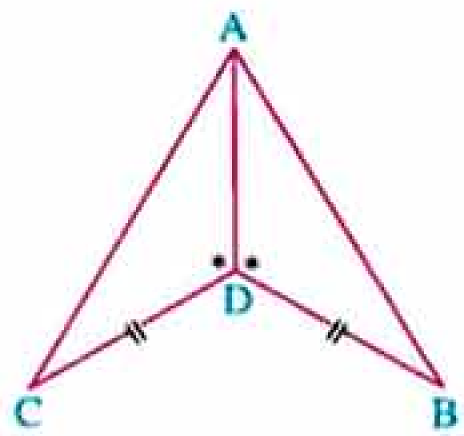
1. The two triangles are congruent (two sides and the included angle "S.A.S.")
2. The given data is not enough to prove the congruence of the two triangles.
3. The two triangles are not congruent because the given angle is not included between the two sides.
4. The two triangles are congruent (three sides "S.S.S.")
5. The two triangles are congruent (two angles and a side "A.S.A.")
6. The two triangles are not congruent because the two congruent angles are not corresponing.

Example 2 **In the opposite figure :**

BD = CD and m (∠ ADB) = m (∠ ADC)

Is Δ ABD ≡ Δ ACD ?

, then explain why $\overrightarrow{AD}$ bisects ∠ A



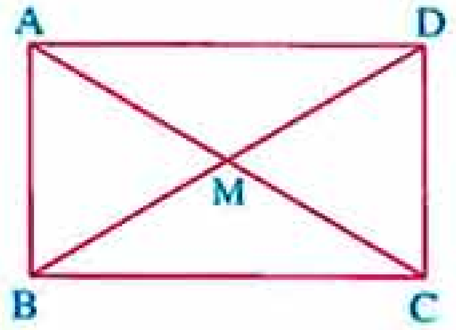
Solution Yes , $\Delta ABD \equiv \Delta ACD$ "two sides and included angle" we deduce from the congruence that : $m(\angle BAD) = m(\angle CAD)$

i.e. $\overrightarrow{AD}$ bisects $\angle A$

Example 3 **In the opposite figure :**

ABCD is a rectangle whose diagonals intersect at M

Is $\Delta ABC \equiv \Delta DCB$? Why ?



Solution Yes , $\Delta ABC \equiv \Delta DCB$, because : $m(\angle ABC) = m(\angle DCB) = 90^\circ$

, AC = BD (two diagonals of the rectangle)

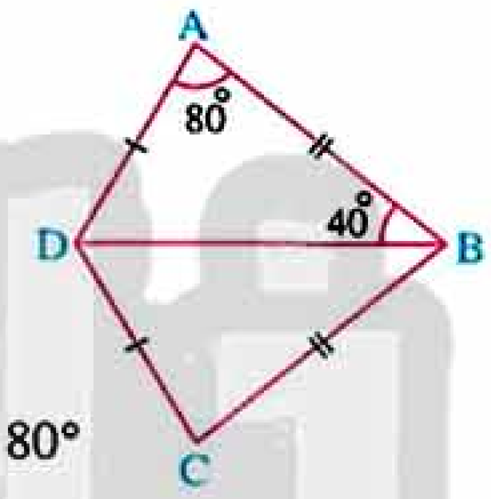
and $\overline{BC}$ is a common side.

Example 4 **In the opposite figure :**

BA = BC , DA = DC ,

$m(\angle ABD) = 40^\circ$ and $m(\angle BAD) = 80^\circ$

Find : $m(\angle ADC)$ showing the steps of the solution.



Solution In ΔABD , since $m(\angle ABD) = 40^\circ$, $m(\angle BAD) = 80^\circ$

Then $m(\angle ADB) = 180^\circ - (40^\circ + 80^\circ) = 60^\circ$

Since $\Delta ABD \equiv \Delta CBD$ (S.S.S.)

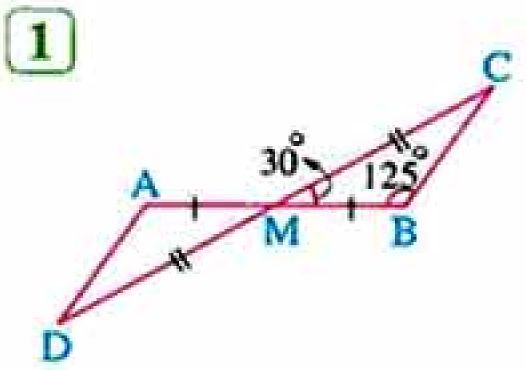
Then $m(\angle ADB) = m(\angle CDB) = 60^\circ$

Then $m(\angle ADC) = 60^\circ + 60^\circ = 120^\circ$

TRY by yourself

In each of the following figures, find the required under each figure :

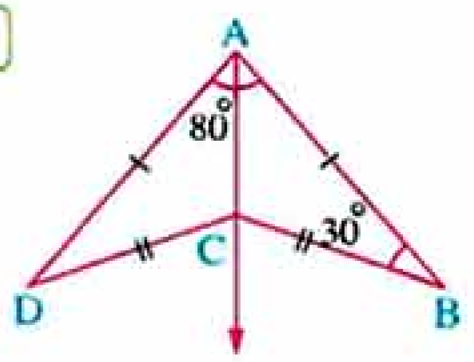
1



$\overline{AB} \cap \overline{CD} = \{M\}$

$m(\angle D) = \dots\dots\dots ^\circ$

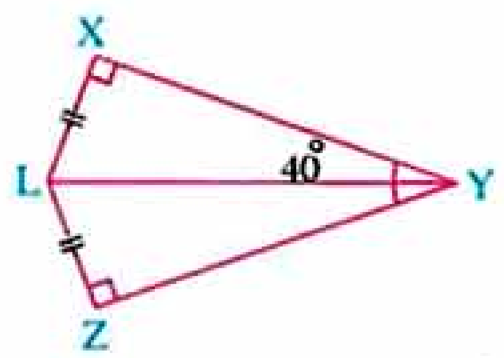
2



$m(\angle D) = \dots\dots\dots ^\circ$

, $m(\angle BAC) = \dots\dots\dots ^\circ$

3



$m(\angle XLY) = \dots\dots\dots ^\circ$

Answers of try by yourself

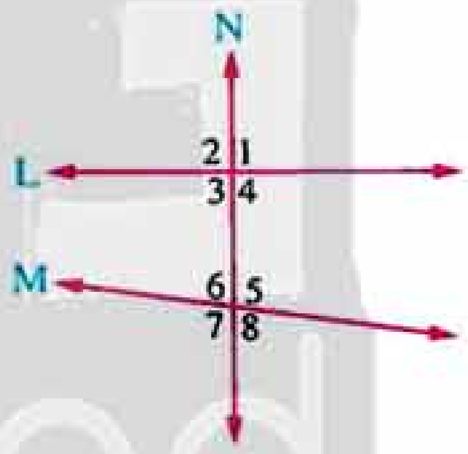
1 25° 2 30° , 40° 3 70°

LESSON 5

Parallelism

Angles formed from two straight lines and a transversal

In the opposite figure :



The straight line N intersects the two straight lines L and M

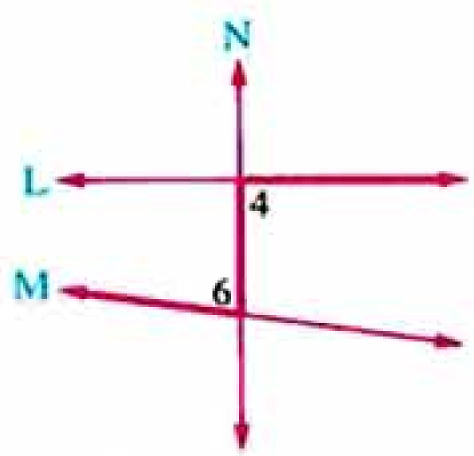
The straight line N is called "a transversal".

In this case , we get eight angles (at each point of intersection four angles are formed) and these eight angles could be classified according to their position relative to the transversal as follows :

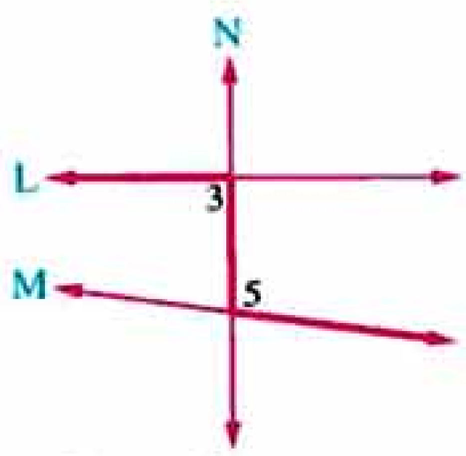
- Alternate angles.
- Corresponding angles.
- Interior angles on the same side of the transversal.

In the following , we will represent each pair of the previous pairs of angles :

1 Pairs of alternate angles

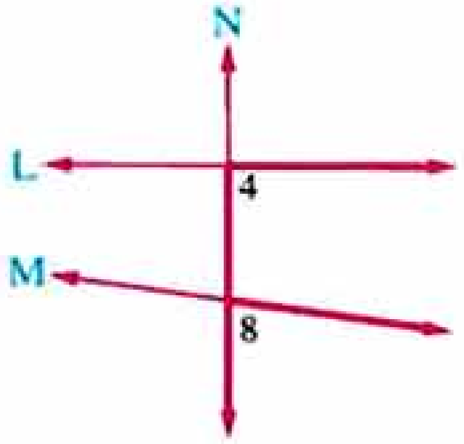


$\angle 4$ and $\angle 6$ are alternate angles.

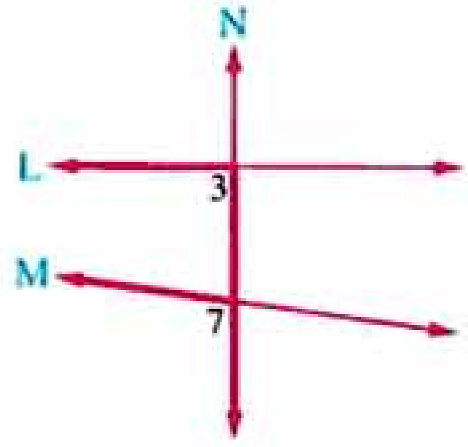


$\angle 3$ and $\angle 5$ are alternate angles.

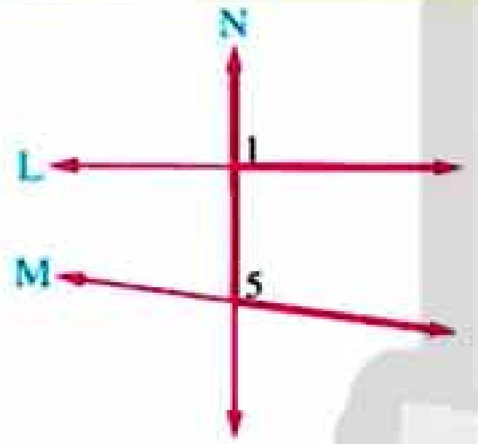
2 Pairs of corresponding angles



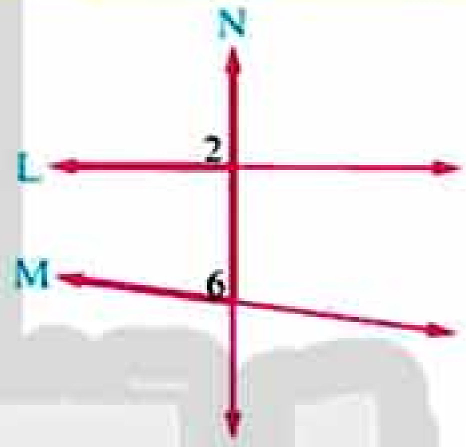
$\angle 4$ and $\angle 8$ are corresponding angles.



$\angle 3$ and $\angle 7$ are corresponding angles.

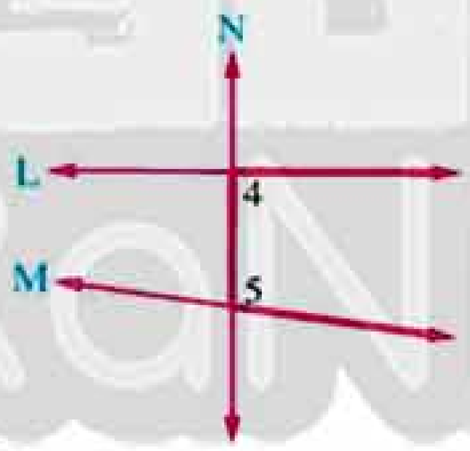


$\angle 1$ and $\angle 5$ are corresponding angles.

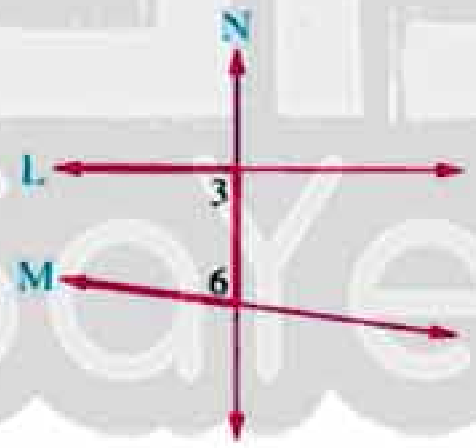


$\angle 2$ and $\angle 6$ are corresponding angles.

3 Pairs of interior angles on the same side of the transversal



$\angle 4$ and $\angle 5$ are interior angles on the same side of the transversal.



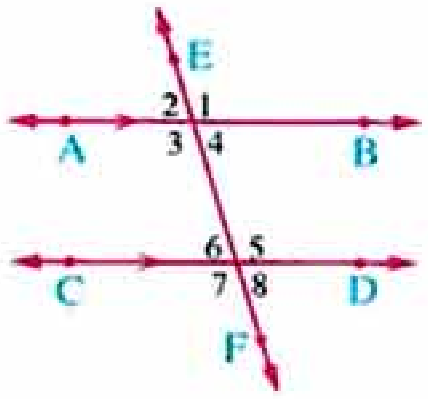
$\angle 3$ and $\angle 6$ are interior angles on the same side of the transversal.

Relation between pairs of angles formed from two parallel straight lines and a transversal to them

If two parallel straight lines are intersected by a transversal , then any two result angles from the intersection either congruent or supplementary.

For example :

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ is a transversal to them



UNIT 4

by measuring , you find that :

1 m (∠ 3) = m (∠ 5) , m (∠ 4) = m (∠ 6)

Generally

If a straight line intersects two parallel straight lines , then each two alternate angles are equal in measure.

2 m (∠ 1) = m (∠ 5) , m (∠ 2) = m (∠ 6) , m (∠ 3) = m (∠ 7) , m (∠ 4) = m (∠ 8)

Generally

If a straight line intersects two parallel straight lines , then each two corresponding angles are equal in measure.

3 m (∠ 3) + m (∠ 6) = 180° , m (∠ 4) + m (∠ 5) = 180°

Generally

If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are supplementary.

Example 1 **In each of the following figures , find the measure of the angle which is marked by " ? " giving reason.**

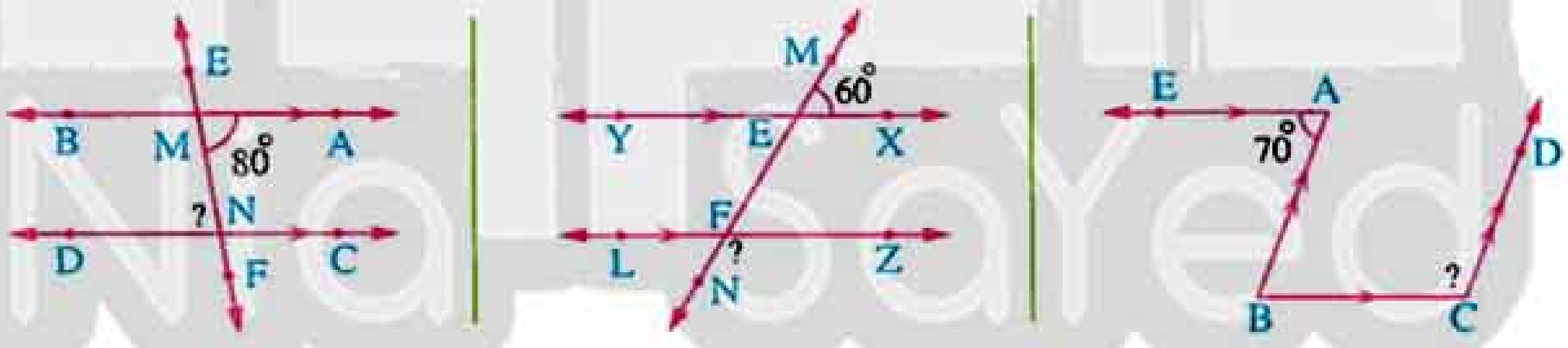


Fig. (1) **Fig. (2)** **Fig. (3)**

Solution

Fig. (1) : m (∠ MND) = 80°

because : m (∠ MND) = m (∠ AMN) (alternate angles)

Fig. (2) : m (∠ ZFN) = 120°

because : m (∠ ZFE) = m (∠ XEM) = 60° (corresponding angles)

Then : m (∠ ZFN) = 180° – 60° = 120°

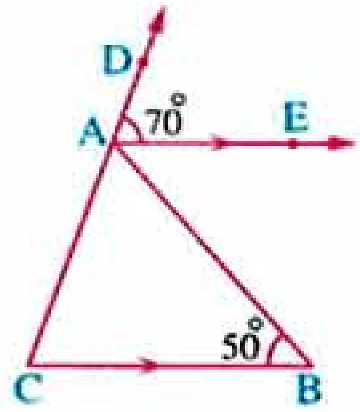
Fig. (3) : m (∠ BCD) = 110°

because : m (∠ B) = m (∠ A) = 70° (alternate angles)

, since ∠ B , ∠ BCD are two interior angles in the same side of the transversal , then m (∠ BCD) = 180° – 70° = 110°

Example 2 In the opposite figure : $\overrightarrow{AE} \parallel \overrightarrow{BC}$, $D \in \overrightarrow{CA}$, m (∠ DAE) = 70° and m (∠ B) = 50° Find giving reason :

1. m (∠ EAB)
2. m (∠ C)
3. m (∠ EAC)

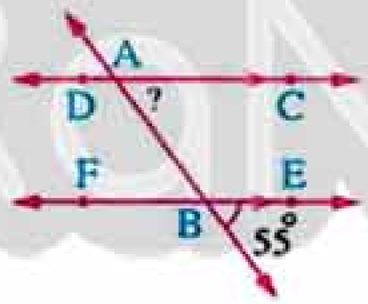


Solution

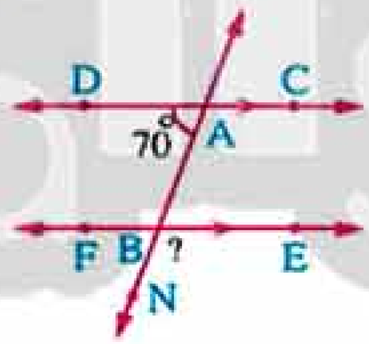
1. m (∠ EAB) = 50° because : m (∠ EAB) = m (∠ B) (alternate angles)
2. m (∠ C) = 70° because : m (∠ C) = m (∠ EAD) (corresponding angles)
3. m (∠ EAC) = 110° because : ∠ EAC , ∠ C are two interior angles in the same side of the transversal , then m (∠ EAC) = 180° – 70° = 110°
   or : because : m (∠ DAE) + m (∠ EAC) = 180°
   , then m (∠ EAC) = 180° – 70° = 110°

TRY 1 by yourself

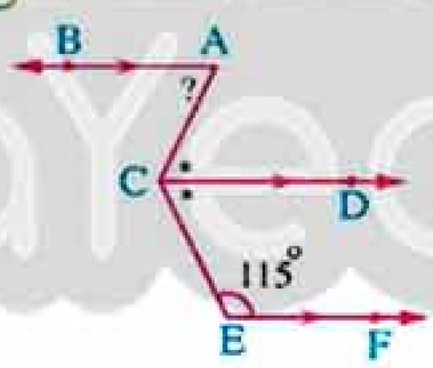
In each of the following figures , find the measure of the angle which is written under each figure :

1. 

m (∠ CAB) = ...°

2. 

m (∠ EBN) = ...°

3. 

m (∠ A) = ...°

How to prove that two straight lines are parallel ?

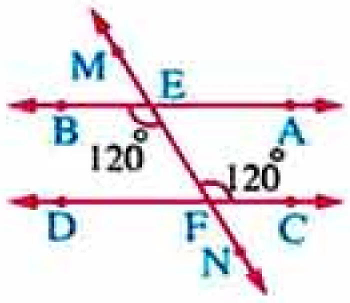
The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases is satisfied :

1. Two alternate angles have the same measure.
2. Two corresponding angles have the same measure.
3. Two interior angles in the same side of the transversal are supplementary.

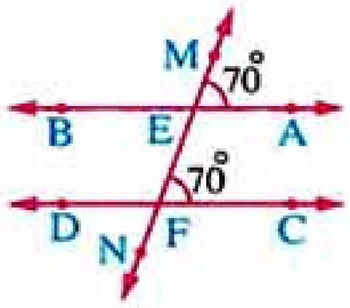
UNIT 4

Notice that :

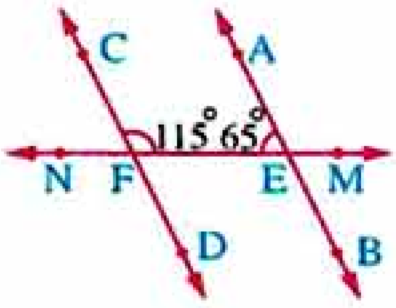
In each of the following figures where : $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two straight lines and $\overleftrightarrow{MN}$ is a transversal to them.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ because :
m (∠ BEF) = m (∠ EFC) = 120°
and they are two alternate angles.

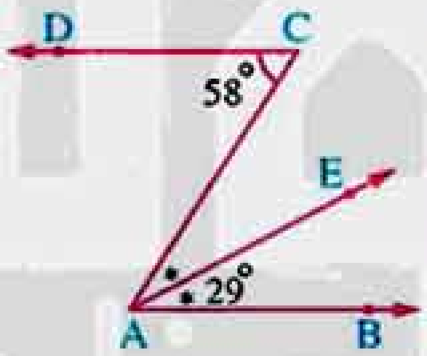


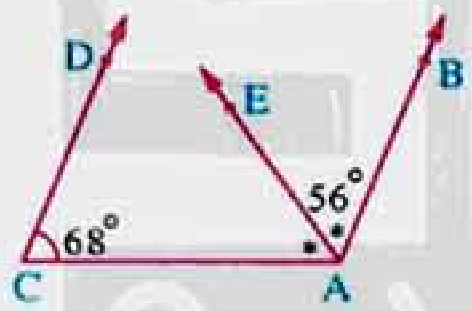
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ because :
m (∠ AEM) = m (∠ CFE) = 70°
and they are two corresponding angles.

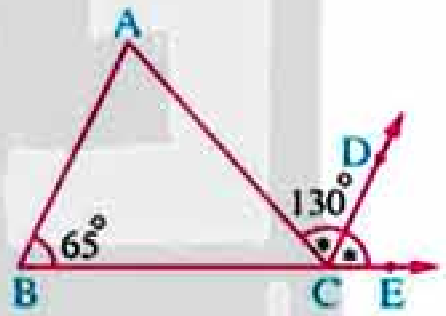


$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ because :
m (∠ AEF) + m (∠ CFE) = 65° + 115° = 180°
and they are interior angles in the same side of the transversal.

Example 3 In each of the following figures, show why $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$:

1 

2 

3 

Solution

1 m (∠ BAC) = 29° × 2 = 58°

i.e. m (∠ BAC) = m (∠ C) and they are two alternate angles ,
therefore : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

2 m (∠ CAB) = 56° × 2 = 112°

i.e. m (∠ CAB) + m (∠ C) = 112° + 68° = 180°
and they are interior angles in one side of the transversal ,
therefore : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

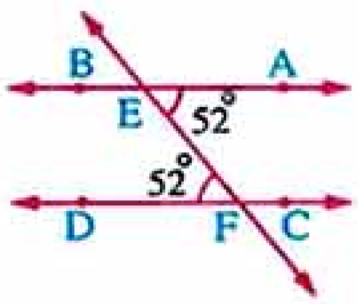
3 m (∠ ECD) = $\frac{130°}{2}$ = 65°

i.e. m (∠ ECD) = m (∠ B) and they are corresponding angles ,
therefore : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

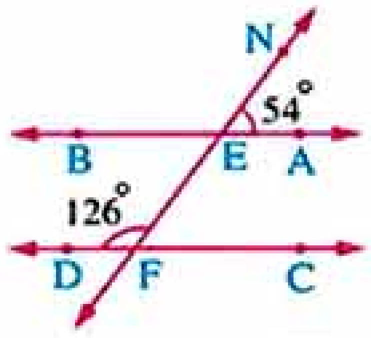
TRY 2 *by yourself*

In each of the following figures , why is $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$?

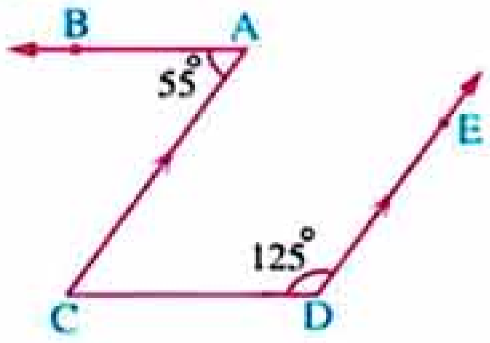
1



2



3



Geometric facts

1 The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other.
And vice versa , if two coplaner straight lines are perpendicular to a third one , then the two straight lines are parallel.

For example :

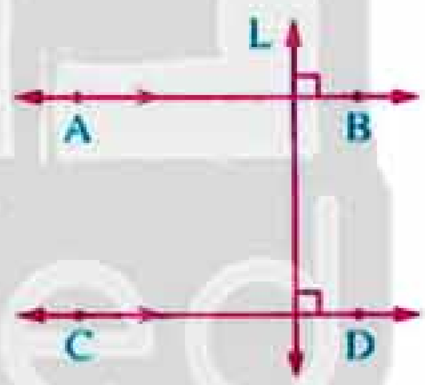
In the opposite figure :

If $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$, the straight line L is drawn perpendicular to $\overleftrightarrow{AB}$, then the straight line $L \perp \overleftrightarrow{CD}$

and if $\overleftrightarrow{AB} \perp$ the straight line L

, $\overleftrightarrow{CD} \perp$ the straight line L

, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



2 If two straight lines are parallel to a third straight line , then these two straight lines are parallel.

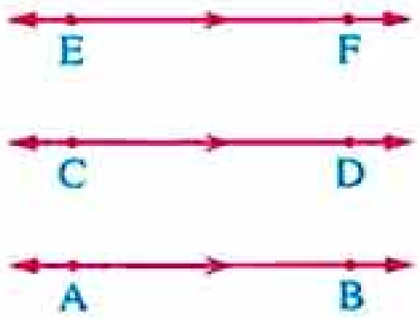
For example :

In the opposite figure :

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

, $\overleftrightarrow{EF}$ is drawn parallel to $\overleftrightarrow{CD}$

, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$



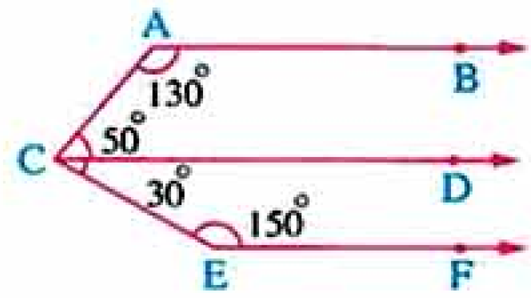
UNIT 4

Example 4 **In the opposite figure :**

If m (∠ A) = 130° , m (∠ ACD) = 50°

, m (∠ DCE) = 30° and m (∠ E) = 150°

Is $\overrightarrow{AB}$ // $\overrightarrow{EF}$? Why ?



Solution $\overrightarrow{AB}$ // $\overrightarrow{CD}$ because : m (∠ A) + m (∠ ACD) = 130° + 50° = 180°

"interior angles on the same side of the transversal"

, $\overrightarrow{EF}$ // $\overrightarrow{CD}$ because : m (∠ E) + m (∠ DCE) = 150° + 30° = 180°

"interior angles on the same side of the transversal"

Then , $\overrightarrow{AB}$ // $\overrightarrow{EF}$

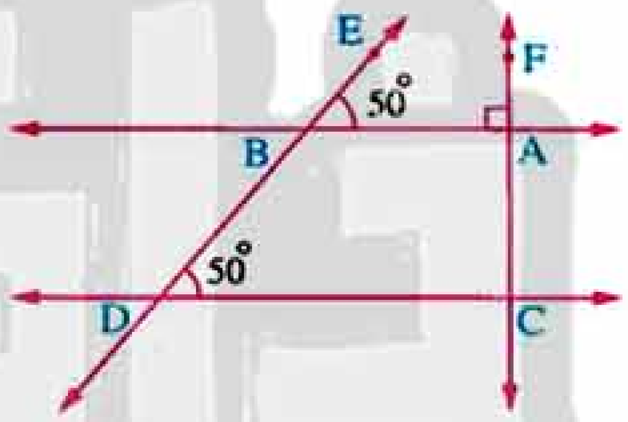
TRY 3 by yourself

In the opposite figure :

If m (∠ ABE) = m (∠ CDB) = 50°

and $\overleftrightarrow{FC} \perp \overleftrightarrow{AB}$

Is $\overleftrightarrow{FC} \perp \overleftrightarrow{CD}$? Why ?



3 If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.

For example :

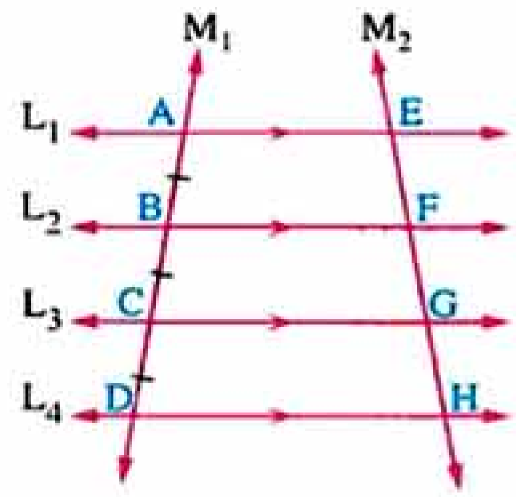
In the opposite figure :

If L_1 // L_2 // L_3 // L_4

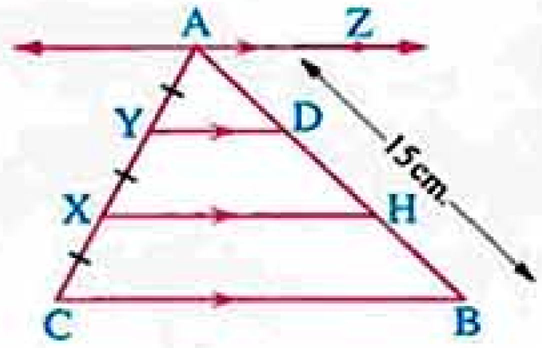
, M_1 and M_2 are two transversals

where AB = BC = CD

, then EF = FG = GH



Example 5 In the opposite figure :



$\overleftrightarrow{AZ} \,//\, \overline{YD} \,//\, \overline{XH} \,//\, \overline{CB}$

, AY = YX = XC and AB = 15 cm.

Find the length of $\overline{BD}$ showing the reason.

Solution

Since , $\overleftrightarrow{AZ} \,//\, \overline{YD} \,//\, \overline{XH} \,//\, \overline{CB}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are their transversals ,

AY = YX = XC

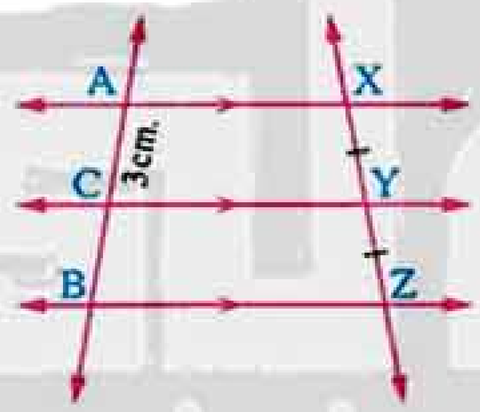
Then , AD = DH = HB = $\frac{15}{3}$ = 5 cm.

Then , BD = 5 + 5 = 10 cm.

TRY 4 by yourself

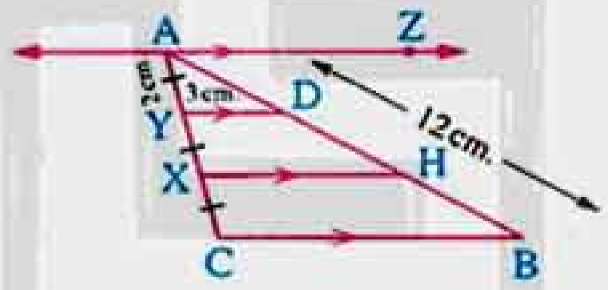
Complete under each figure of the following figures :

1



AB = ... cm.

2



BH = ... cm.

The perimeter of Δ ADY = ... cm.

Answers of try by yourself

1 [1] 55° [2] 110° [3] 65°

2 [1] The reason : m ($\angle$ AEF) = m ($\angle$ EFD) = 52° and they are alternate angles.

[2] The reason : m ($\angle$ BEF) = m ($\angle$ AEN) = 54° (vertically opposite angles) , then m ($\angle$ BEF) + m ($\angle$ EFD) = 54° + 126° = 180° and they are interior angles in the same side of the transversal.

[3] The reason : m ($\angle$ C) = 180° − 125° = 55° , then m ($\angle$ A) = m ($\angle$ C) = 55° and they are alternate angles.

3 Yes , the reason : m ($\angle$ ABE) = m ($\angle$ CDB) = 50° and they are corresponding angles , therefore $\overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$ and since $\overleftrightarrow{FC} \perp \overleftrightarrow{AB}$, therefore $\overleftrightarrow{FC} \perp \overleftrightarrow{CB}$

4 [1] 6 [2] 4 , 9

LESSON

6 Geometric Constructions

First : Constructing a perpendicular from a point outside a straight line:

If $\overleftrightarrow{AB}$ is a given straight line and $C \notin \overleftrightarrow{AB}$ as shown in fig. (1)

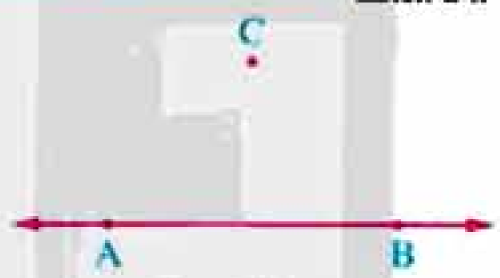


Fig. (1)

The required is constructing the perpendicular to $\overleftrightarrow{AB}$ from C

Procedure :

1. Using the compasses at C as a centre and with a suitable radius , draw an arc to intersect $\overleftrightarrow{AB}$ at the two points D and E as shown in fig. (2)

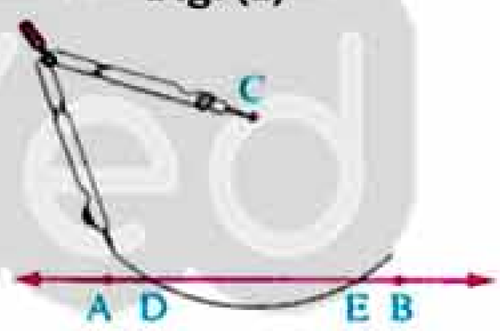


Fig. (2)

2. At D and E as centres and with a suitable radius (greater than $\frac{1}{2}$ DE) draw two arcs to intersect each other at L as shown in fig. (3)

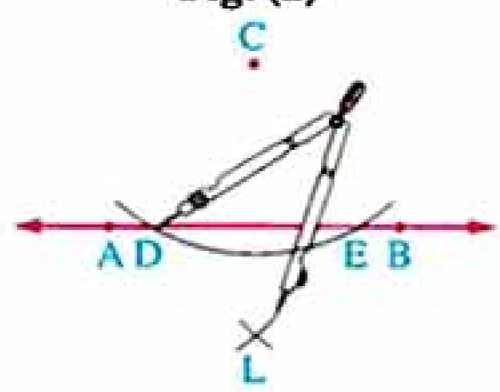


Fig. (3)

3. Draw $\overleftrightarrow{CL}$ to be the straight line passing through C perpendicular to $\overleftrightarrow{AB}$ as shown in fig. (4)

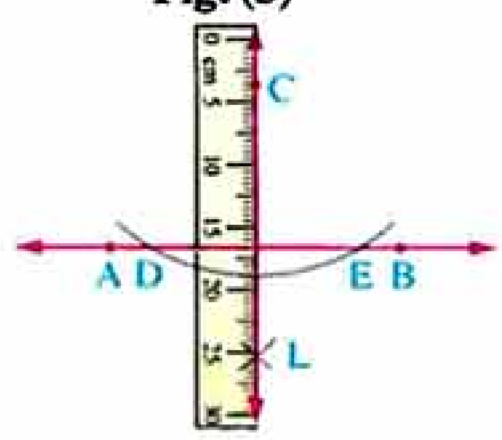


Fig. (4)

TRY 1 by yourself

Draw a perpendicular from a point outside a straight line.

Second : Drawing a perpendicular to a straight line that passes through a point which belongs to that straight line :

If $\overleftrightarrow{AB}$ is a given straight line.

$C \in \overleftrightarrow{AB}$ as shown in fig. (1)

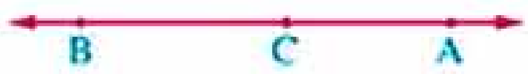


Fig. (1)

The required is drawing a perpendicular to $\overleftrightarrow{AB}$ from the point C

Procedure :

1. Place the sharp point of the compasses at C and adjust it with suitable length , then draw two arcs in two different sides of C to intersect $\overleftrightarrow{AB}$ at D and E as shown in fig. (2)

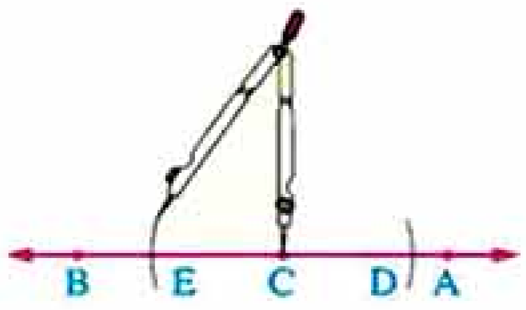


Fig. (2)

2. Place the sharp point of the compasses at each of the points D and E and adjust it with length greater than half the length of $\overline{DE}$, then draw two arcs to intersect at point X as shown in fig. (3)

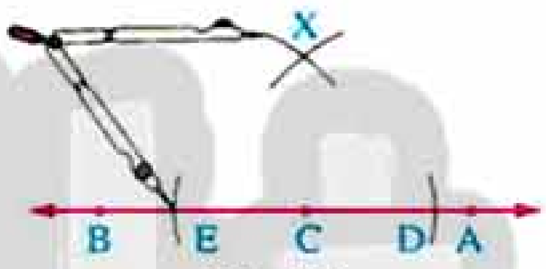


Fig. (3)

3. Draw $\overline{XC}$, then $\overline{XC}$ is perpendicular to $\overleftrightarrow{AB}$ as shown in fig. (4)


Fig. (4)

TRY 2 by yourself

Draw a perpendicular to a straight line from a point which belongs to that straight line.

The axis of symmetry of a line segment :

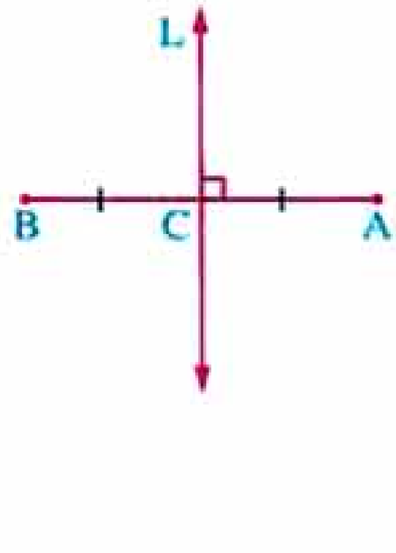
It is the straight line perpendicular to it from its midpoint.

In the opposite figure :

If C is the midpoint of $\overline{AB}$ and the straight line

$L \perp \overline{AB}$ from the point C

Then the straight line L is the axis of symmetry of the line segment $\overline{AB}$



UNIT 4

Third : Bisecting a given line segment "Constructing the symmetry axis of a given line segment" :

If $\overline{AB}$ is a given line segment as shown in fig. (1)

The required is constructing the symmetry axis of the line segment $\overline{AB}$ (The perpendicular to $\overline{AB}$ from its midpoint).

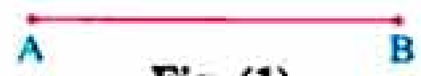


Fig. (1)

Procedure :

1. Using the compasses at A as a centre and with a radius greater than $\frac{1}{2}$ AB , draw two arcs in the opposite sides of $\overline{AB}$ as shown in fig. (2)

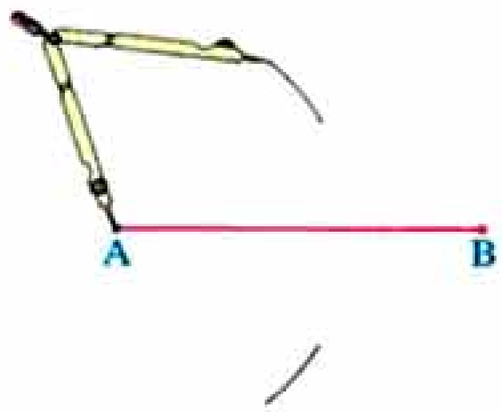


Fig. (2)

2. Using the compasses at B as a centre and with the same radius , draw two other arcs to intersect the previous two arcs at D and E as shown in fig. (3)

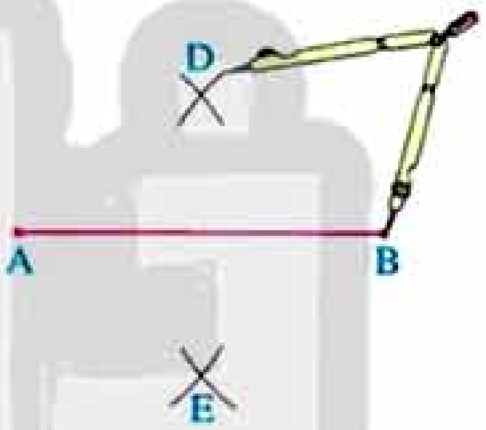


Fig. (3)

3. Draw $\overleftrightarrow{DE}$ to cut $\overline{AB}$ at a point as C which is the midpoint of $\overline{AB}$,
$\overleftrightarrow{DE} \perp \overline{AB}$
Then $\overleftrightarrow{DE}$ is the perpendicular to $\overline{AB}$ from its midpoint
i.e. $\overleftrightarrow{DE}$ is the axis of symmetry of $\overline{AB}$ as shown in fig. (4)

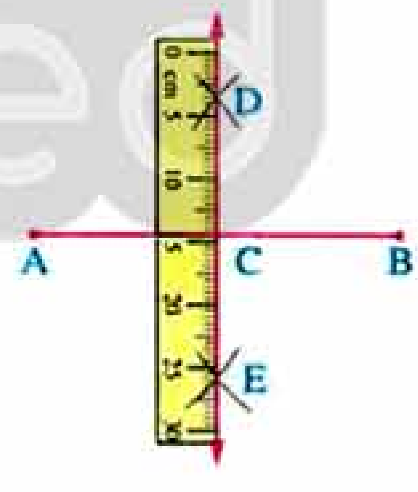


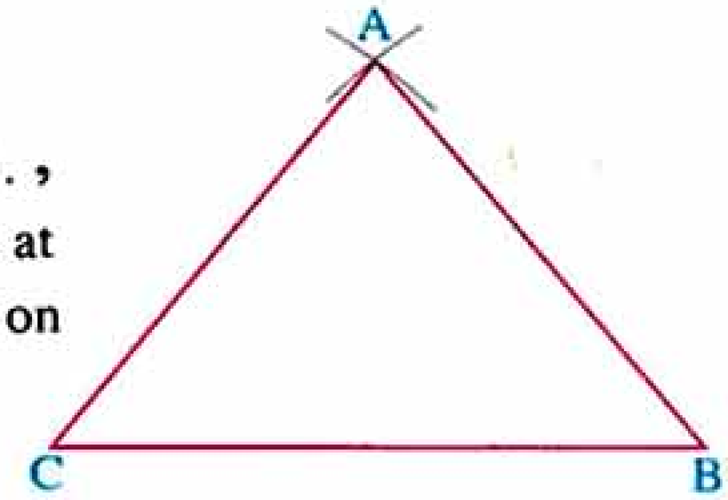
Fig. (4)

TRY 3 by yourself ***Draw a line segment of length 5 cm. , then draw its symmetry axis.***

Example 1 Using the geometric instruments , draw Δ ABC in which AB = AC = 4 cm. , BC = 5 cm. , then draw the axes of symmetry of its three sides.
Are the axes of symmetry of the three sides concurrent (i.e. intersecting at one point) ? (Don't remove the arcs)

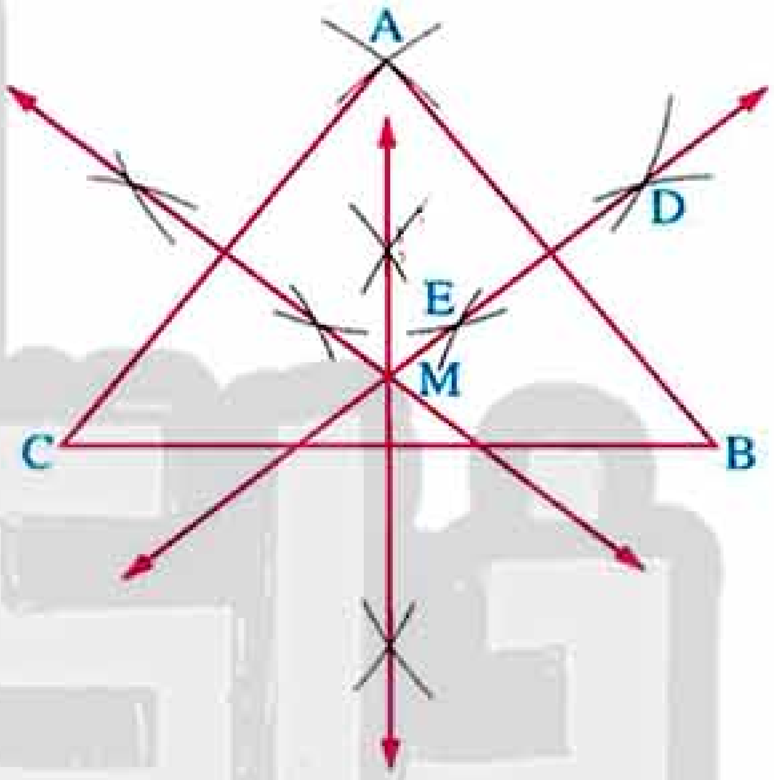
Solution

First : Drawing Δ ABC



1. Draw $\overline{BC}$ such that BC = 5 cm.
2. Using the compasses with length 4 cm. , place the sharp point of the compasses at each point of B and C , draw two arcs on one side of $\overline{BC}$ to intersect at A
3. Draw $\overline{BA}$ and $\overline{CA}$ to get Δ ABC

Second : Drawing the axes of symmetry of the sides of Δ ABC



1. Place the sharp point of the compasses at A with length greater than half the length of $\overline{AB}$

   i.e. more than 2 cm. , draw two arcs on two different sides of $\overline{AB}$
2. Place the sharp point of the compasses at B and with the same previous length , draw two other arcs to intersect the previous arcs at the two points D and E
3. Draw $\overleftrightarrow{DE}$, then $\overleftrightarrow{DE}$ is the axis of symmetry of the side $\overline{AB}$
4. Do the same previous steps to draw the axes of symmetry of $\overline{AC}$ and $\overline{BC}$
5. We notice that the three axes of symmetry are concurrent

   (**i.e.** they are intersecting at one point M)

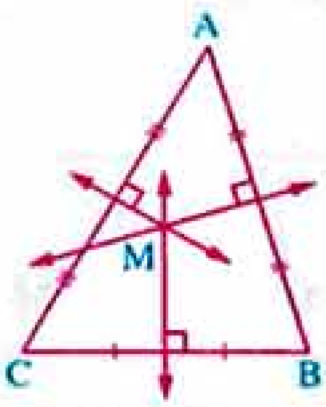
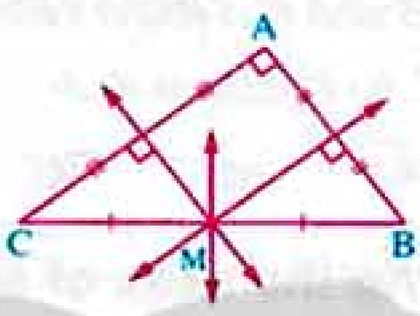
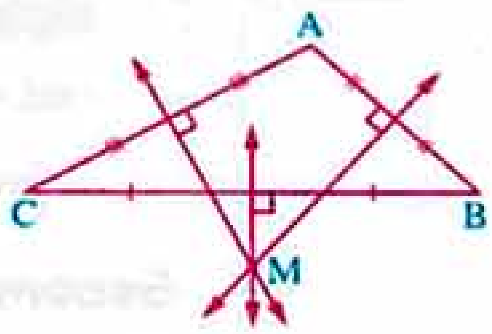
Notice that :

We can draw without writing the steps of the construction but don't remove the arcs.

UNIT 4

Remarks

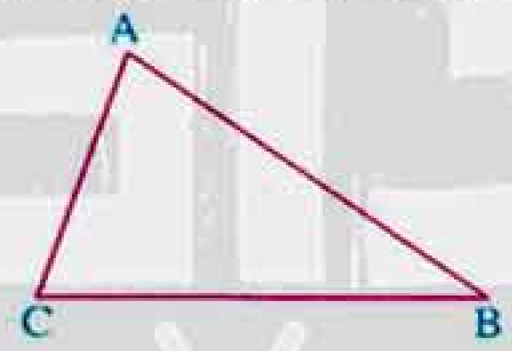
• The axes of symmetry of the sides of any triangle are intersecting at one point (say M). The position of M differs according to the type of the triangle as follows :

Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
		
M is inside the triangle.	M is the midpoint of the hypotenuse.	M is outside the triangle.

• The lengths of the line segments joining the point of intersection of the axes of symmetry and the vertices of the triangle are equal in all previous cases.
i.e. AM = BM = CM

TRY 4 *by yourself*

Draw the axis of symmetry of each side of D ABC and check that the three axes intersect at one point.



Fourth : Constructing the bisector of a given angle :

If ∠ ABC is a given angle as shown in fig. (1)

The required is constructing the bisector of ∠ ABC
"Using the compasses and the ruler"

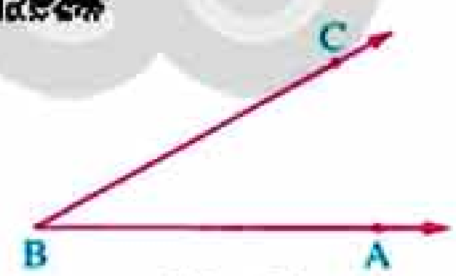


Fig. (1)

Procedure :

1. Using the compasses and with a suitable radius at the vertex of the angle B as a centre , draw an arc to intersect $\overrightarrow{BA}$ and $\overrightarrow{BC}$ (the two sides of ∠ ABC) at D and E respectively as shown in fig. (2)

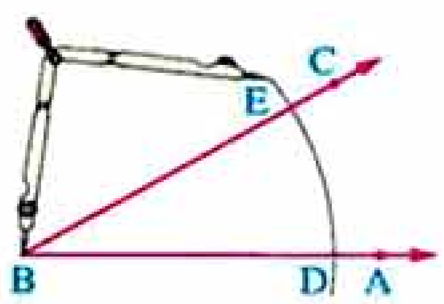


Fig. (2)

2 Taking D and E as centres and using the compasses with a suitable radius , draw two arcs to intersect at the point X as shown in fig. (3)

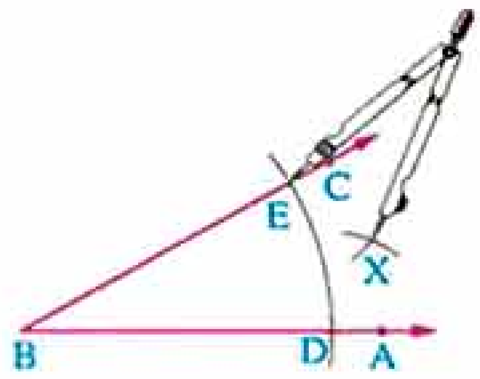


Fig. (3)

3 Draw $\overrightarrow{BX}$ to be the bisector of ∠ ABC as shown in fig. (4)

Notice that :

$\overleftrightarrow{BX}$ is the axis of symmetry of ∠ ABC

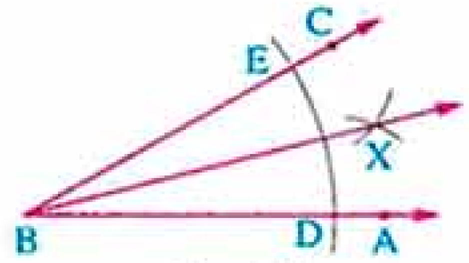


Fig. (4)

TRY 5 by yourself ***Draw an angle of measure 80° , then construct the bisector of this angle.***

Fifth : Constructing an angle to be congruent to a given angle (without using protractor) :

∠ ABC is a given angle as shown in fig. (1)

Fig. (1)

The required is drawing ∠ XYZ such that ∠ XYZ is congruent to ∠ ABC
i.e. m (∠ XYZ) = m (∠ ABC)

Procedure :

1 Draw $\overrightarrow{YL}$ to represent one of the sides of the required angle as shown in fig. (2)

Fig. (2)

2 Using the compasses with B as a centre and with a suitable radius , draw an arc to cut $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at D and E respectively as shown in fig. (3)

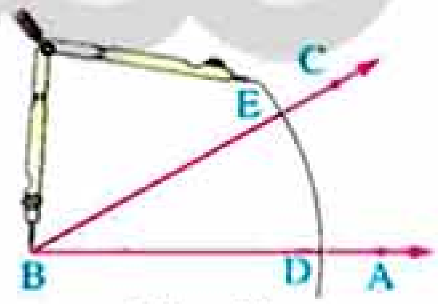


Fig. (3)

3 With Y as a centre and with the same radius , draw an arc to cut the ray $\overrightarrow{YL}$ at X as shown in fig. (4)

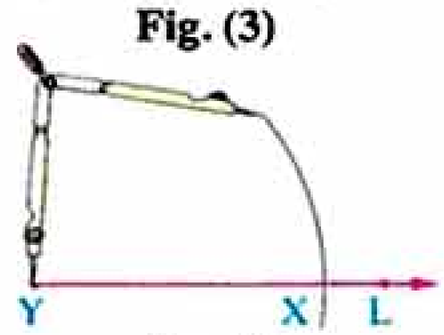


Fig. (4)

4 With X as a centre and with radius equal to the length of $\overline{DE}$, draw another arc to cut the previous arc at Z as shown in fig. (5)

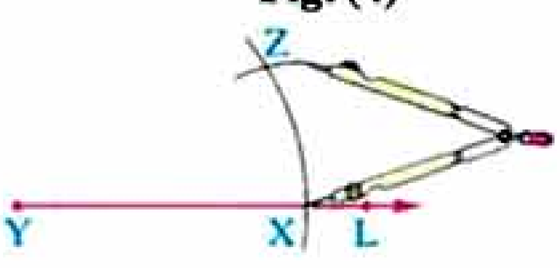


Fig. (5)

UNIT 4

5 Draw $\overrightarrow{YZ}$, then $\angle XYZ$ is the required angle as shown in fig. (6)

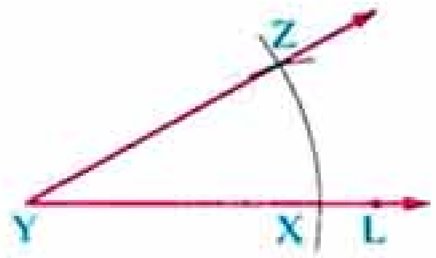


Fig. (6)

TRY 6 by yourself

Draw $\angle B$ of measure 50°, then without using the protractor draw $\angle C$ congruent to $\angle B$

Sixth : Drawing a straight line from a given point parallel to given straight line :

$\overleftrightarrow{AB}$ is a given straight line and $C \notin \overleftrightarrow{AB}$ as shown in fig. (1)

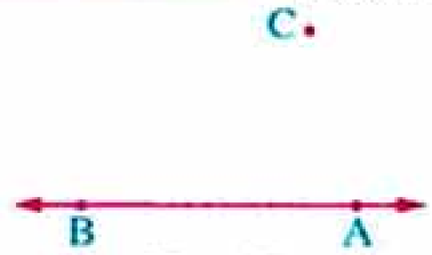


Fig. (1)

The required is drawing a straight line passing through the point C parallel to $\overleftrightarrow{AB}$

Procedure :

1 Draw the straight line $\overleftrightarrow{XY}$ passing through the point C and cutting $\overleftrightarrow{AB}$ at Y as shown in fig. (2)

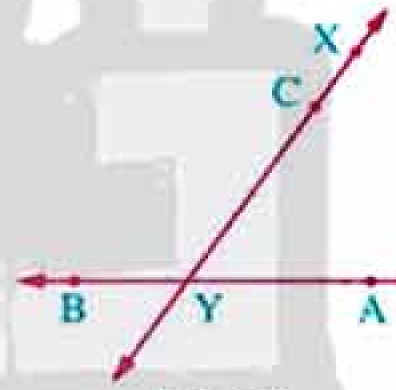


Fig. (2)

2 Draw at C the angle XCD corresponding to $\angle AYX$ such that $\angle XCD \equiv \angle XYA$ using the previous construction , then $\overleftrightarrow{CD}$ is the straight line which passes through the point C and parallel to $\overleftrightarrow{AB}$ as shown in fig. (3)

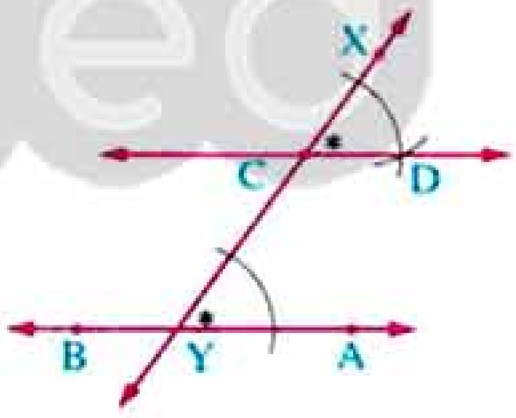
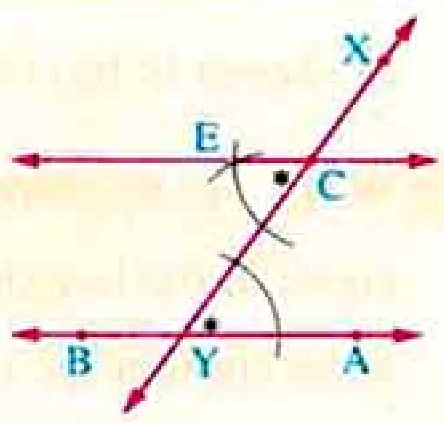


Fig. (3)

Remark

In the previous activity , we can replace the second step by drawing $\angle YCE$ at the point C in the alternate position with $\angle AYC$ such that $\angle YCE \equiv \angle AYC$, then $\overleftrightarrow{CE}$ will be the straight line which passes through the point C and parallel to $\overleftrightarrow{AB}$ as shown in the opposite figure.



Example 2 Draw ΔABC in which AB = 7 cm. , m (∠ A) = 50° , m (∠ B) = 70° , then bisect $\overline{AC}$ at D , then draw $\overleftrightarrow{DE}$ // $\overline{AB}$ to cut $\overline{BC}$ at E , then by measuring , find :

1. The length of each of $\overline{BE}$ and $\overline{CE}$ What do you notice ?
2. The length of $\overline{DE}$ What do you notice ?

Solution

- Using the ruler and the protractor , draw ΔABC
- Using the campasses , bisect $\overline{AC}$ at D
- Using the ruler and the compasses , draw ∠ CDE such that : ∠ CDE ≡ ∠ A

So $\overrightarrow{DE}$ // $\overline{AB}$, then by measuring , we find that :

1. BE ≃ 3.1 cm. , CE ≃ 3.1 cm.

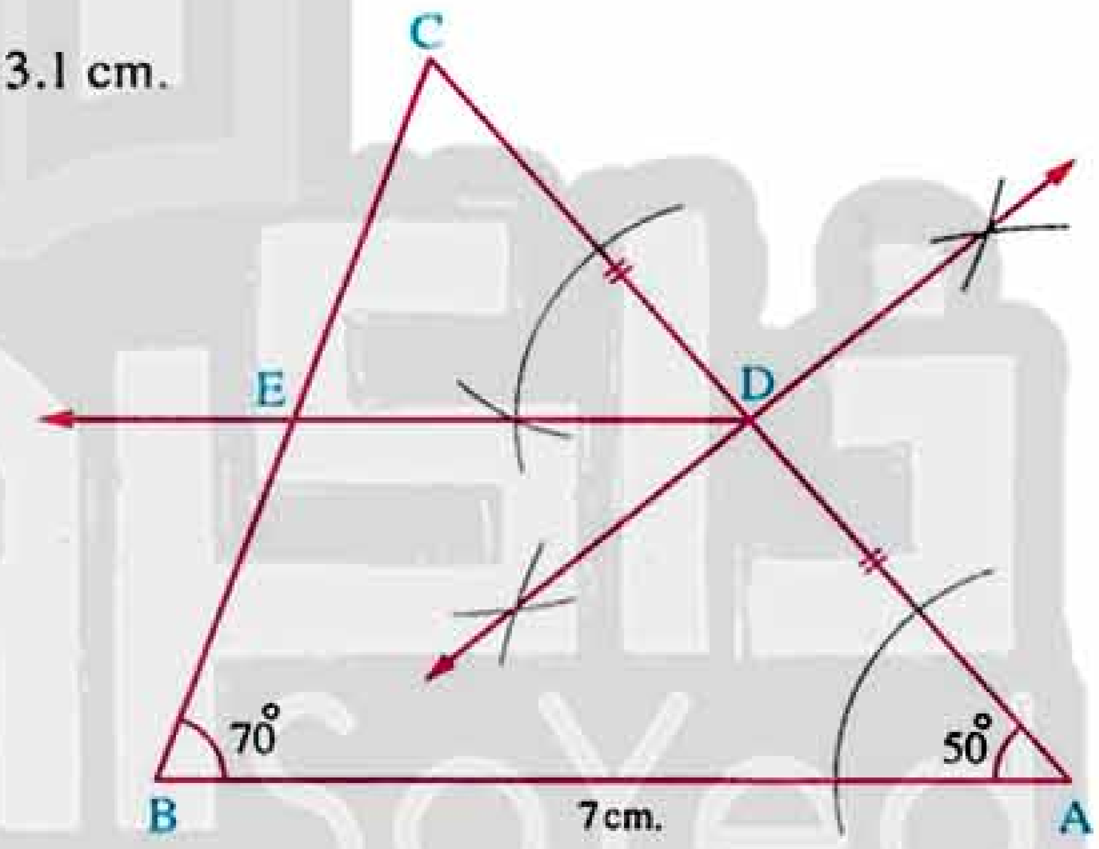
   We notice that E is the midpoint of $\overline{BC}$

   i.e. BE = CE

2. DE = 3.5 cm.

   We notice that :

   $DE = \frac{1}{2} AB$



TRY 7 by yourself

Using the geometric tools , draw the equilateral triangle ABC whose side length is 6 cm. , then bisect ∠ A by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D , then draw $\overrightarrow{DE}$ // $\overline{AB}$ to cut $\overline{AC}$ at E , then find by measuring the length of $\overline{DE}$, and the length of $\overline{AE}$ what do you notice ? (Don't remove the arcs)

Activities

using computer

Activity 1 : Using Excel 2007 program to find the product of two integers.

Activity 2 : Using Excel 2007 program to find the quotient of two integers.

Activity 3 : Using Excel 2007 program to verify that : $a^m \times a^n = a^{m+n}$

Activity 4 : Using Excel 2007 program to verify that : $a^m \div a^n = a^{m-n}$, $m \geq n$, $a >$ zero

Activity 5 : Using Excel 2007 program to verify that : $(a+b)^2 = a^2 + 2\,a\,b + b^2$

Activity 6 : Using Excel 2007 program to verify that : $(a-b)^2 = a^2 - 2\,a\,b + b^2$

Activity 7 : Using Excel 2007 program to verify that : $(a+b)(a-b) = a^2 - b^2$

Activity 1 Using Excel 2007 program to find the product of two integers

1. Using the task bar, click "Start" then from the menu "All Programs" select "Microsoft Office", then select "Microsoft Excel"

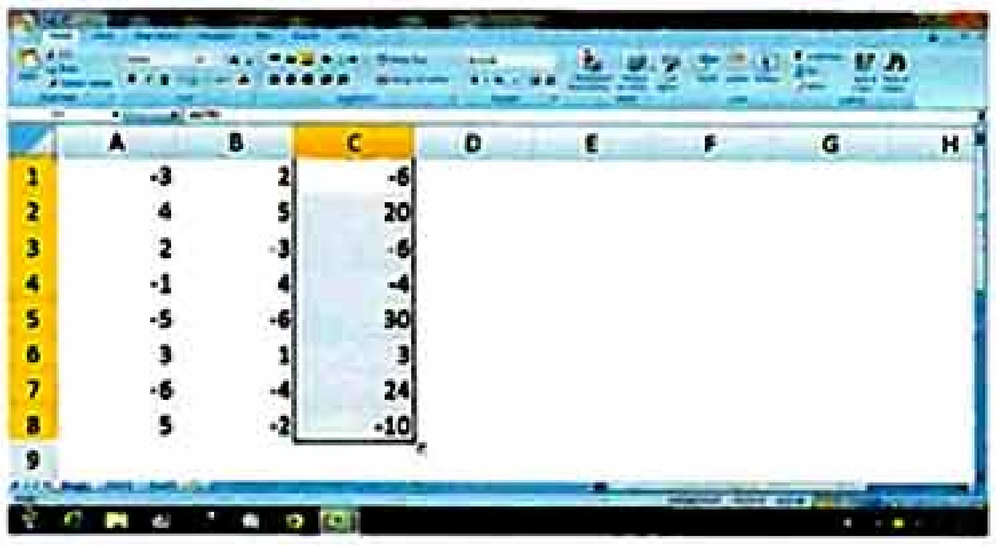
2. Fill in any two columns (Say A and B) with a set of integers as shown in the opposite screen.

3. Find the product of each number in column A by its corresponding in column B as follows :

- Stop at cell C1
- Write = A1 * B1 as shown in the opposite screen.

4. Click ENTER and you will obtain the product of the number in cell A1 by the number in cell B1 as shown in the opposite screen.

5. To find the product of the remaining numbers in the columns A and B, stop at cell C1 and move the mouse at the small square at the corner of cell C1 till the pointer changes to the form (+) , then perform (Auto fill) by copying the formula from cell C1 to cell C8, then you obtain the opposite screen.



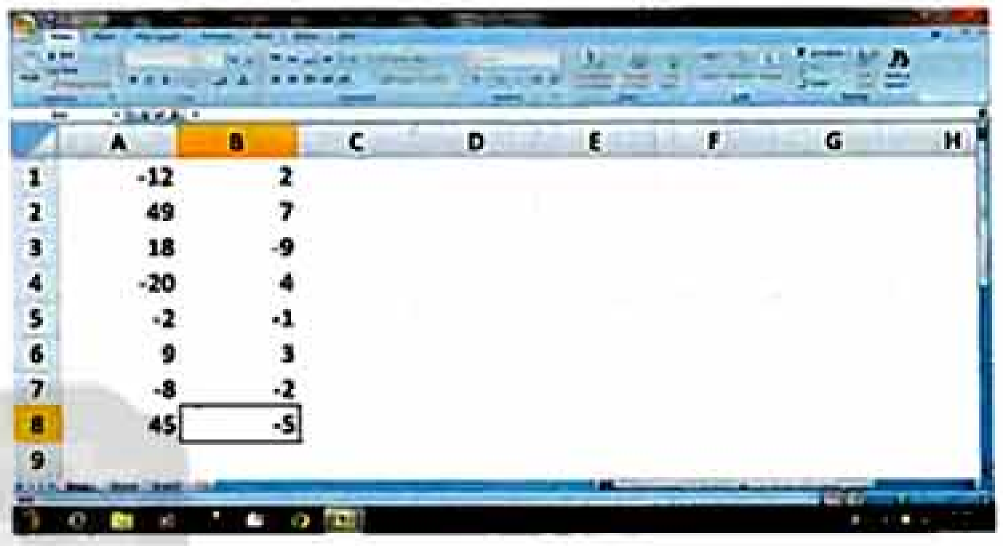
(Deduce the signs rule of multiplication)

Activities

Activity 2 Using Excel 2007 program to find the quotient of two integers

1. Select the program "Microsoft Excel" as in activity (1)

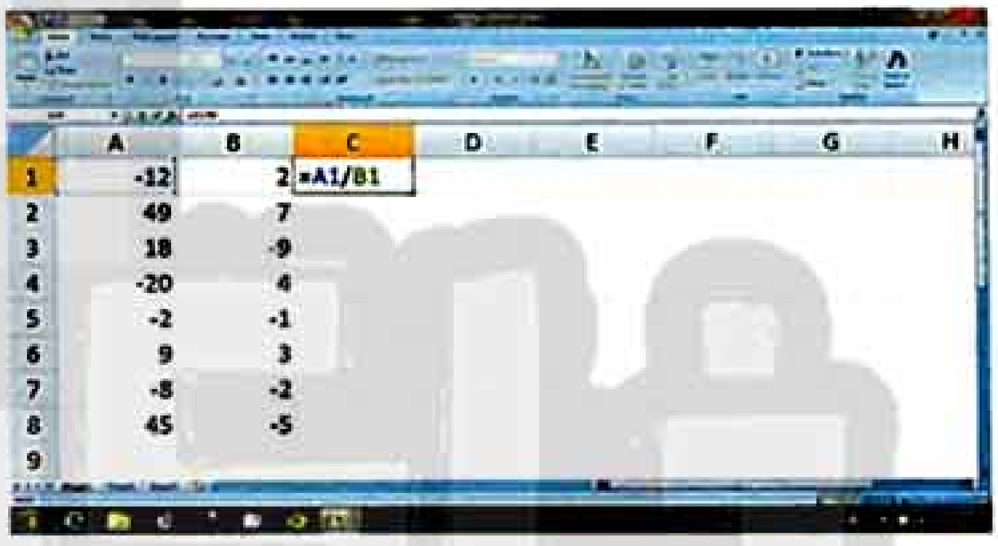
2. Fill in any two columns (say A and B) by a set of integers as shown in the opposite screen.



3. Find the quotient of each number in column A by its corresponding in column B as follows:

   • Stop at cell C1

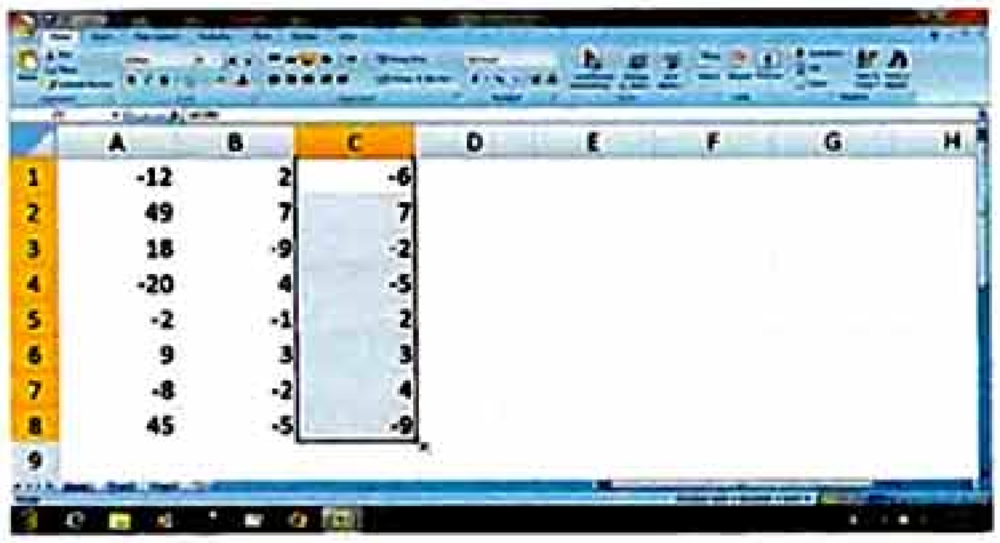
   • Write = A1 / B1 as shown in the opposite screen.



4. Click ENTER and you will obtain the quotient of the number in cell A1 by its corresponding in cell B1 as shown in the opposite screen.

5. To find the quotient of the remaining numbers in the columns A and B, stop at cell C1 and move the mouse at the small square at the corner of cell C1 till the pointer changes to the cell C1 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell C1 to cell C8, then you obtain the opposite screen.

   (Deduce the signs rule of division)



Activity 3 Using Excel 2007 program to verify that : $a^m \times a^n = a^{m+n}$

1 Select the program "Microsoft Excel" as in activity (1)

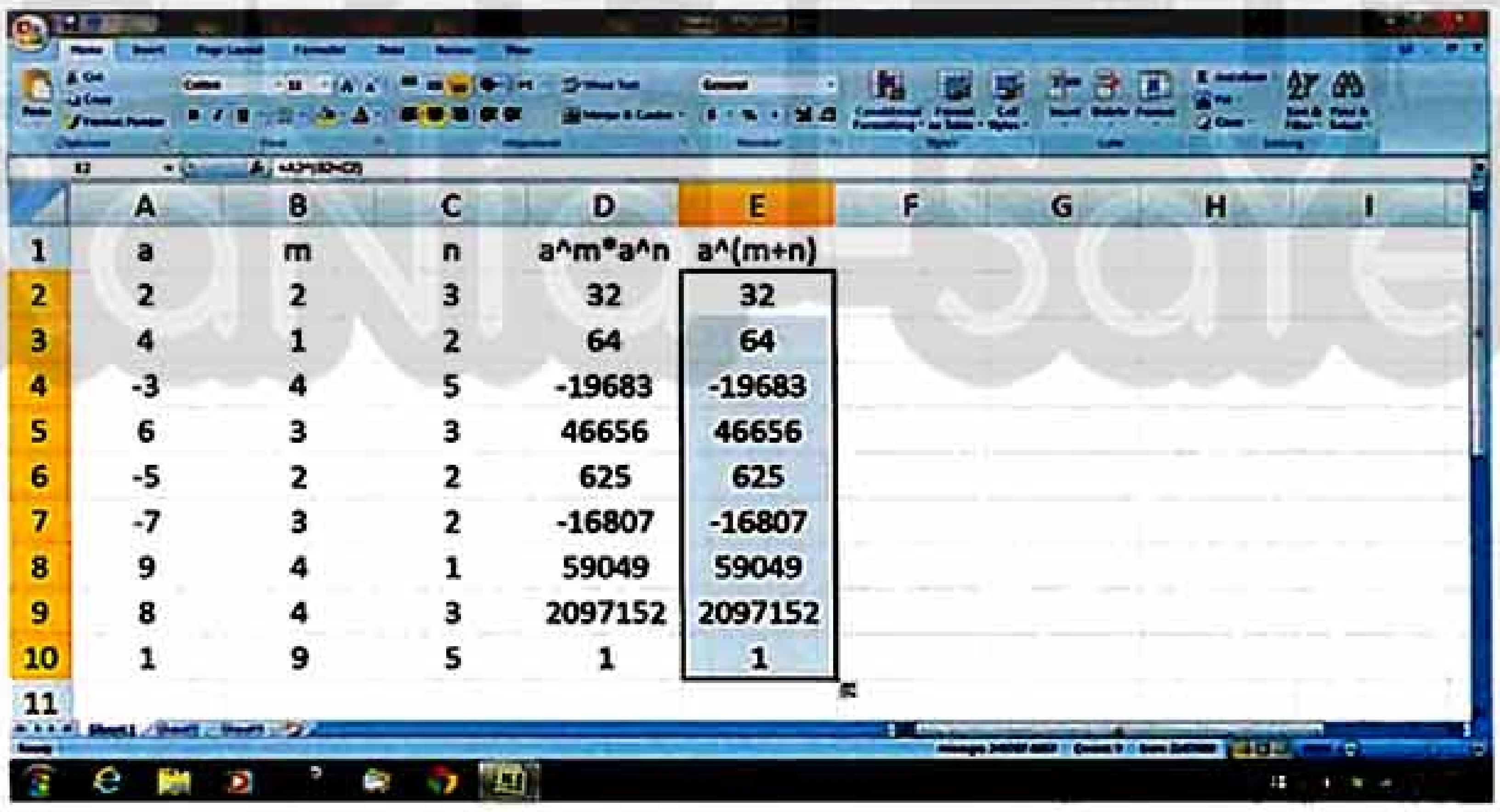
2 Fill in the column A with a set of positive or negative numbers that represent different values for the symbol a, and fill in the columns B and C with a set of positive numbers that represent different values for each m and n respectively.

3 Stop at cell D2 and write = A2^ B2 * A2^ C2, then click ENTER.

4 Stop at cell D2 and move the mouse at the small square at the corner of cell D2 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell D2 to cell D10

5 Stop at cell E2 and write = A2^ (B2 + C2), then click ENTER.

6 Stop at cell E2 and move the mouse at the small square at the corner of cell E2 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell E2 to cell E10, you obtain the following screen :



	A	B	C	D	E
1	a	m	n	a^m*a^n	a^(m+n)
2	2	2	3	32	32
3	4	1	2	64	64
4	-3	4	5	-19683	-19683
5	6	3	3	46656	46656
6	-5	2	2	625	625
7	-7	3	2	-16807	-16807
8	9	4	1	59049	59049
9	8	4	3	2097152	2097152
10	1	9	5	1	1
11					

- By comparing the results in the columns D and E, you can deduce that : $a^m \times a^n = a^{m+n}$
- Noticing that the numbers which filled in column A, we deduce that the previous rule is right when the bases are positive or negative.

Activities

Activity 4 Using Excel 2007 program to verify that : $a^m \div a^n = a^{m-n}$, $m \geq n$, $a >$ zero

1. Select the program "Microsoft Excel" as in activity (1)

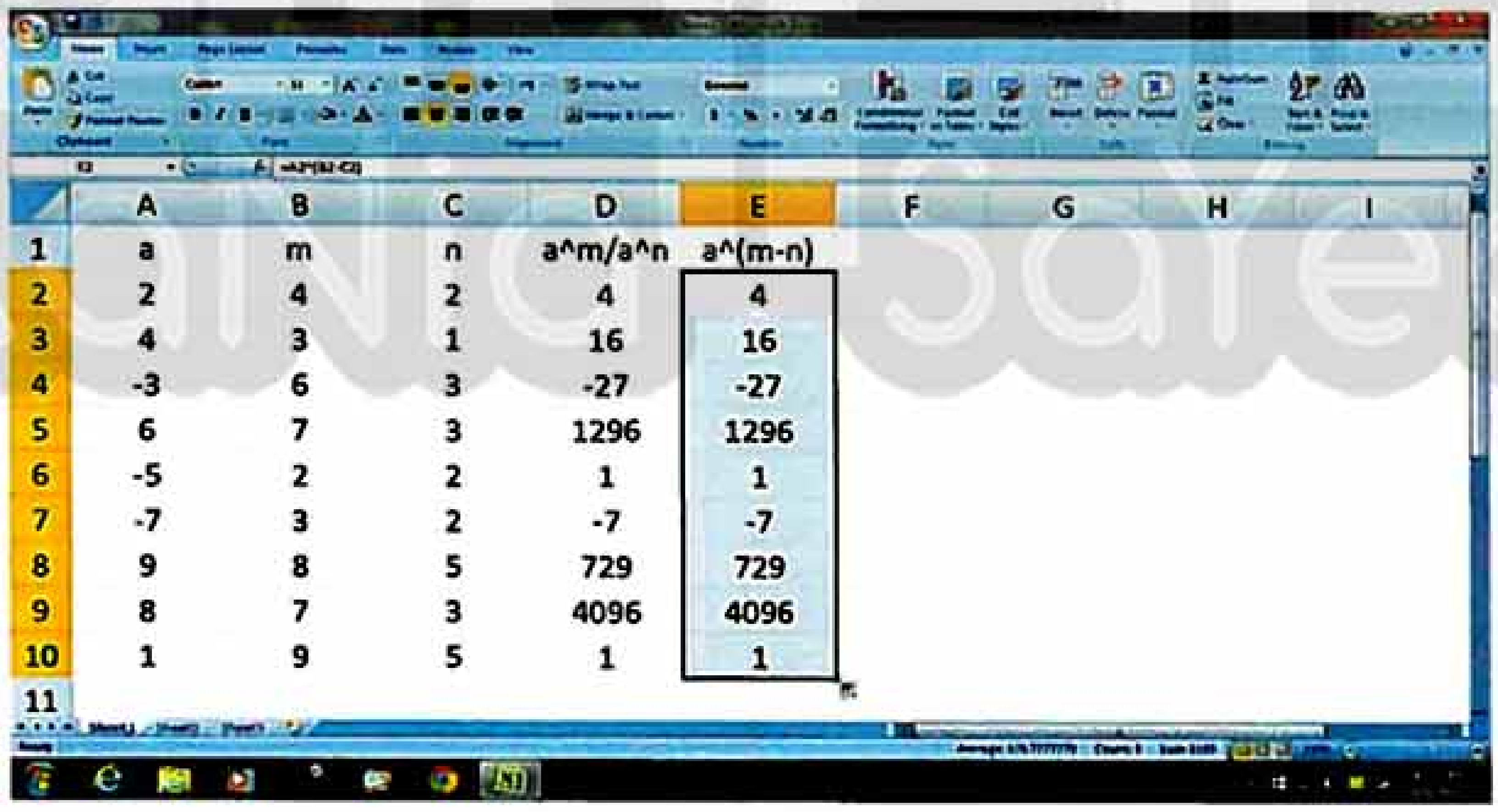
2. Fill the columns A with a set of positive or negative numbers that represent different values for the symbol a, and fill in the columns B and C with a set of positive numbers that represent different values for each m and n respectively.

3. Stop at cell D2 and write = A2^ B2 / A2^ C2, then click ENTER.

4. Stop at cell D2 and move the mouse at the small square at the corner of cell D2 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell D2 to cell D10

5. Stop at cell E2 and write = A2^ (B2 – C2), then click ENTER.

6. Stop at cell E2 and move the mouse at the small square at the corner of cell E2 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell E2 to cell E10, you obtain the following screen :



	A	B	C	D	E
1	a	m	n	a^m/a^n	a^(m-n)
2	2	4	2	4	4
3	4	3	1	16	16
4	-3	6	3	-27	-27
5	6	7	3	1296	1296
6	-5	2	2	1	1
7	-7	3	2	-7	-7
8	9	8	5	729	729
9	8	7	3	4096	4096
10	1	9	5	1	1

- By comparing the results in the columns D and E, you can deduce that : $a^m \div a^n = a^{m-n}$
- Noticing that the numbers which filled in column A, we deduce that the previous rule is right when the bases are positive or negative.

Activity 5 Using Excel 2007 program to verify that : $(a + b)^2 = a^2 + 2ab + b^2$

1 Select the program "Microsoft Excel" as in activity (1)

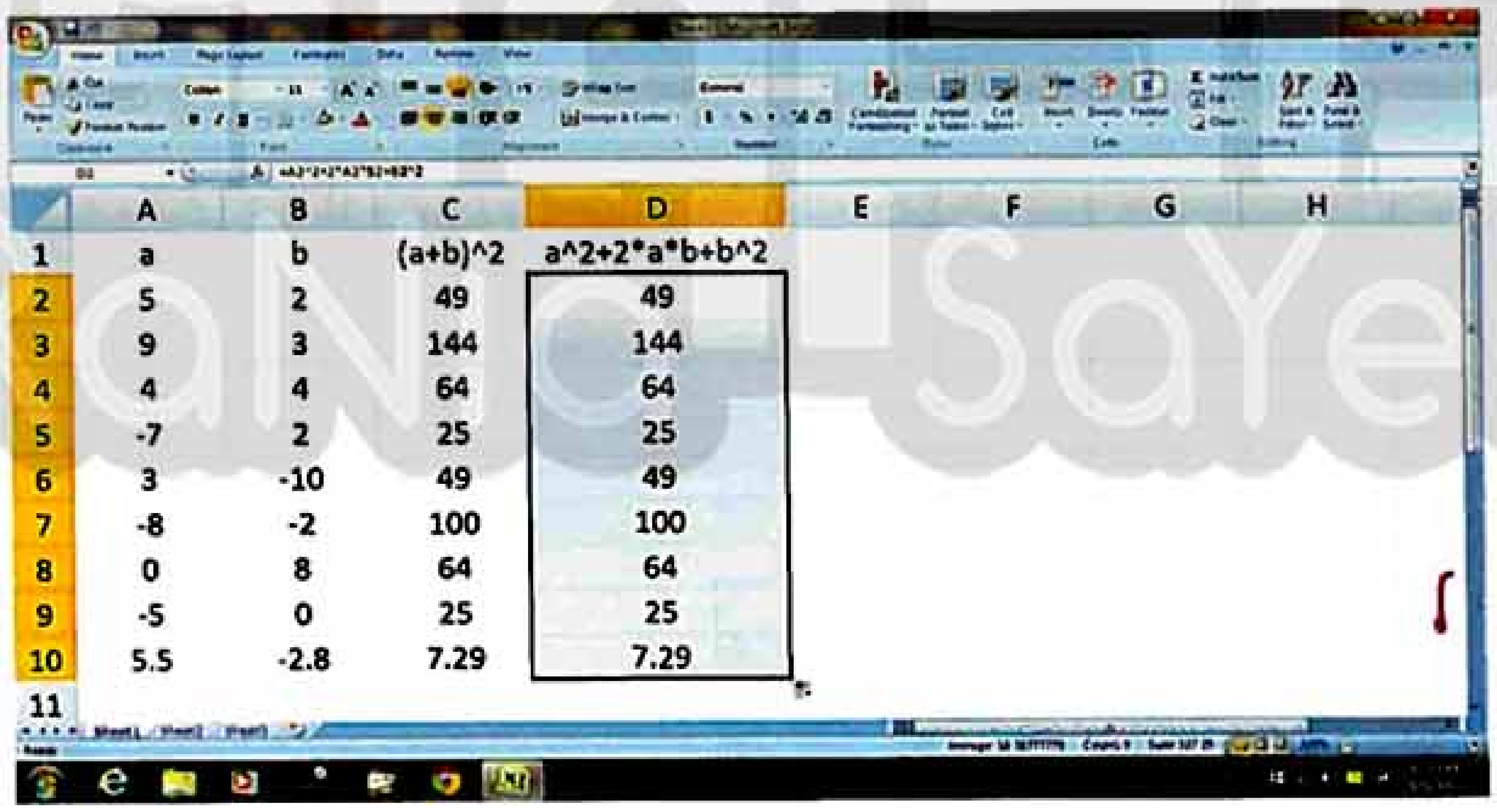
2 Fill in the columns A and B with a set of different numbers that represent different values for each a and b respectively.

3 Stop at cell C2 and write = (A2 + B2)^ 2 , then click ENTER.

4 Stop at cell C2 and move the mouse at the small square at the corner of cell C2 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell C2 to cell C10

5 Stop at cell D2 and write = A2 ^ 2 + 2 * A2 * B2 + B2 ^ 2, then click ENTER.

6 Stop at cell D2 and move the mouse at the small square at the corner of cell D2 till the pointer changes to the form (+), then perform (Auto fill) by copying the formula from cell D2 to cell D10, you obtain the following screen :



	A	B	C	D
1	a	b	(a+b)^2	a^2+2*a*b+b^2
2	5	2	49	49
3	9	3	144	144
4	4	4	64	64
5	-7	2	25	25
6	3	-10	49	49
7	-8	-2	100	100
8	0	8	64	64
9	-5	0	25	25
10	5.5	-2.8	7.29	7.29

- By comparing the results in the columns C and D, you can deduce that : $(a + b)^2 = a^2 + 2ab + b^2$
- Noticing that the numbers which filled in columns A and B, we deduce that the previous rule is right for all numbers.

Activities

Activity 6 Using Excel 2007 program to verify that : $(a-b)^2 = a^2 - 2ab + b^2$

By the same procedure of activity (5) but with writing = (A2 – B2)^2 in step (3) and writing = A2 ^ 2 – 2 * A2 * B2 + B2 ^ 2 in step (5) you obtain the following screen :

	A	B	C	D
1	a	b	(a-b)^2	a^2-2*a*b+b^2
2	5	2	9	9
3	9	3	36	36
4	4	4	0	0
5	-7	2	81	81
6	3	-10	169	169
7	-8	-2	36	36
8	0	8	64	64
9	-5	0	25	25
10	5.5	-2.8	68.89	68.89

- By comparing the results in the columns C and D, you can deduce that : $(a-b)^2 = a^2 - 2ab + b^2$
- Noticing that the rule is right for all numbers.

Activity 7 Using Excel 2007 program to verify that : $(a+b)(a-b) = a^2 - b^2$

By the same procedure of activity (5) but with writing = (A2 + B2) * (A2 – B2) in step (3) and writing = A2 ^ 2 – B2 ^ 2 in step (5) you obtain the following screen :

	A	B	C	D
1	a	b	(a+b)(a-b)	a^2-b^2
2	5	2	21	21
3	9	3	72	72
4	4	4	0	0
5	-7	2	45	45
6	3	-10	-91	-91
7	-8	-2	60	60
8	0	8	-64	-64
9	-5	0	25	25
10	5.5	-2.8	22.41	22.41

- By comparing the results in the columns C and D, you can deduce that : $(a+b)(a-b) = a^2 - b^2$
- Noticing that the rule is right for all numbers.

EL-MOASSER

In Mathematics

Exercises

For 1st Prep.
First Term

By
A group of supervisors

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CONTENTS

First Algebra and Statistics

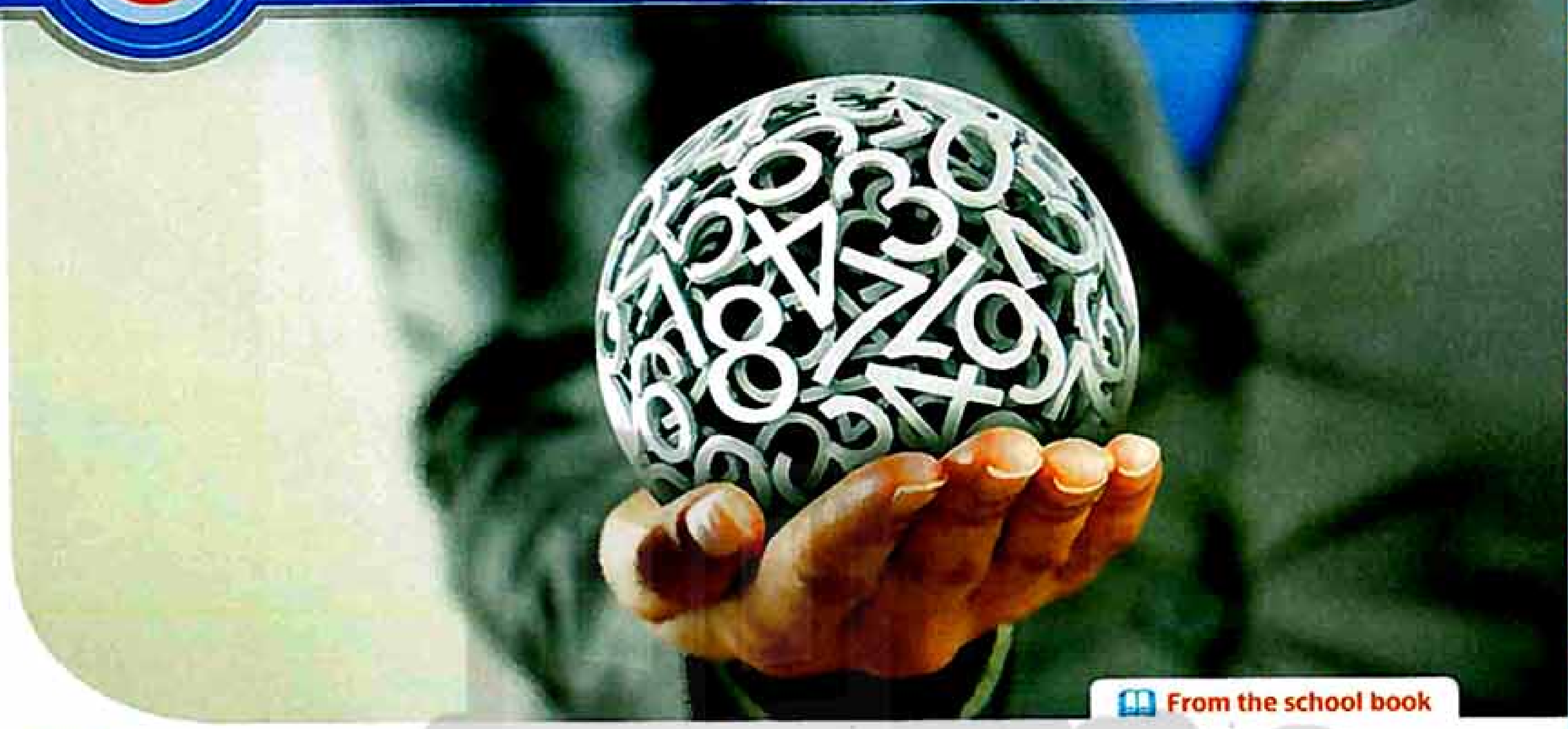
UNIT 1

Rational Numbers

Exercises of the unit :

1. Set of rational numbers.
2. Comparing and ordering rational numbers.
3. Adding and subtracting rational numbers.
4. Multiplying and dividing rational numbers.
5. Applications on rational numbers.
- Summary of unit one.
- Unit exams.

EXERCISE 1 Set of Rational Numbers



From the school book

1 **Which of the following numbers is rational and which is not rational ?**

$\frac{2}{3}$, zero , 6.5 , -1.8 , $12\frac{5}{6}$, $\frac{2-2}{3}$, $\frac{4}{5-5}$, 3^2 , $(-4)^{zero}$

2 **Which of the following numbers is an integer ?**

$\frac{15}{5}$, $\frac{4}{8}$, $-\frac{35}{7}$, $-\frac{14}{14}$, $-\frac{24}{5}$, $\frac{0}{5}$, $3\frac{1}{4}$

3 **Complete each of the following :**

1. $\frac{3}{4} = \frac{9}{\cdots\cdots} = \frac{\cdots\cdots}{8}$
2. $\frac{4}{5} = \frac{\cdots\cdots}{10} = \frac{16}{\cdots\cdots}$

4 **Put each of the following numbers in the simplest form :**

1. $\frac{15}{25}$
2. $-\frac{24}{56}$
3. $\frac{45}{20}$
4. $-\frac{132}{88}$

5 **Which of the following rational numbers can be written as a terminating decimal ?**

1. 📖 $\frac{7}{15}$
2. 📖 $\frac{7}{20}$
3. $\frac{5}{8}$
4. $-\frac{8}{9}$
5. $\frac{5}{11}$
6. $-\frac{13}{22}$
7. $\frac{17}{6}$
8. $2\frac{2}{5}$
9. $-1\frac{2}{3}$
10. $\left|-1\frac{2}{9}\right|$

6 **Write each rational number in the form $\frac{a}{b}$:**

1. -5
2. 📖 zero
3. 📖 0.75
4. 📖 -0.01
5. 5.4
6. 📖 30%
7. 4.5%
8. 📖 $8\frac{2}{3}$

Exercise 1

7 **Write the following rational numbers as a decimal and a percentage :**

1. $\frac{1}{6}$
2. $2\frac{1}{2}$
3. $-\frac{3}{20}$
4. $\frac{5}{9}$
5. $7\frac{3}{16}$
6. $\frac{16}{3}$

8 Why does the definition of a rational number $\frac{a}{b}$ state that $b \neq 0$?

9 **If $a = 2$, $b = 6$,**

show which of the following numbers is rational and which is not rational :

1. $\frac{b}{a}$
2. $-\frac{2}{a}$
3. $\frac{\text{zero}}{a+b}$
4. $\frac{2b}{a-2}$

10 **Complete the following :**

1. If $\frac{5}{a}$ is a rational number , then $a \neq$ ....
2. The number $\frac{3}{x-2}$ is a rational number if $x \neq$ ....
3. The number $\frac{2}{3x}$ is a rational number if $x \neq$ ....
4. The rational number $\frac{4-x}{x-3} = 0$ if $x =$ ....
5. The rational number $\frac{x-5}{x} = 0$ if $x =$ ....
6. $\frac{1}{4} =$ .... %
7. $\frac{21}{1000} =$ .... %
8. $|-0.4| =$ .... %

11 **Choose the correct answer from the given ones :**

1. If $-\frac{4}{5} = \frac{20}{x}$, then $x =$ ....

(a) 25 (b) – 25 (c) 5 (d) 100

2. The number $\frac{a-6}{a-4}$ is not a rational number if $a =$ ....

(a) 6 (b) 4 (c) 1 (d) zero

3. The rational number $\frac{a}{b}$ is an integer if ....

(a) $a < b$ (b) $a > b$

(c) b is a divisor of a (d) a is a divisor of b

UNIT 1

4 $0.\dot{5}\dot{7} =$ ....

(a) $\frac{57}{100}$ (b) $\frac{75}{99}$ (c) $\frac{575}{1000}$ (d) $\frac{19}{33}$

5 $\left|-\frac{8}{25}\right| =$ ....

(a) $-\frac{8}{25}$ (b) $-0.3\dot{2}$ (c) $0.\dot{3}\dot{2}$ (d) 32%

6 12% = ...

(a) $0.\dot{3}$ (b) 1.2 (c) $\frac{3}{25}$ (d) 0.012

7 The rational number $\frac{x}{-3}$ is negative if x ....

(a) > zero (b) < zero (c) ≥ zero (d) = zero

8 If $\frac{a}{b}$ is a rational number and ab = zero, then ....

(a) $a = 0, b \neq 0$ (b) $a \neq 0, b \neq 0$ (c) $a = 0, b = 0$ (d) $a \neq 0, b = 0$

9 The number $\frac{5x}{|x| - 2} \notin \mathbb{Q}$ if $x =$ ....

(a) zero (b) -1 (c) ± 2 (d) 5

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12 Write the rational number $\frac{a}{b}$ that equals $\frac{3}{5}$ and the sum of its two terms is 24

13 **If $x \in \mathbb{N}$, find the values of x which make each of the following an integer :**

1 $\frac{75}{x}$ 2 $\frac{15}{x+1}$

EXERCISE 2

Comparing and Ordering Rational Numbers

From the school book

1 Complete by rational numbers on the number line :

1. −2 $\square$ $-\frac{4}{3}$ −1 $\square$ $-\frac{1}{3}$ zero $\square$ $\frac{2}{3}$ 1 $\square$ $\frac{5}{3}$ 2

2. −1 $-\frac{5}{6}$ $\square$ $\square$ $\square$ $-\frac{1}{6}$ zero $\square$ $\square$ $\frac{3}{6}$ $\square$ $\square$ 1

2 Represent each of the following rational numbers on the number line :

1. $\frac{1}{3}$
2. $-\frac{1}{2}$
3. $\frac{5}{3}$
4. $-\frac{7}{4}$
5. $1\frac{1}{5}$
6. $-3\frac{1}{2}$
7. 0.4
8. $\left|-\frac{3}{5}\right|$

3 Write the correct sign "< , = or >" :

1. $-\frac{1}{2}$ $\square$ zero
2. $-\frac{3}{4}$ $\square$ $\frac{1}{4}$
3. $-4\frac{1}{2}$ $\square$ -5
4. $4\frac{1}{2}$ $\square$ 5
5. $\left|-\frac{3}{2}\right|$ $\square$ $\frac{1}{2}$
6. $\left|\frac{15}{2}\right|$ $\square$ $7\frac{1}{2}$

4 Put the suitable sign " > , < or = " in the space in each of the following :

1. $\frac{1}{4}$ .... $\frac{1}{6}$
2. $-\frac{5}{7}$ .... $-\frac{3}{2}$
3. $\frac{9}{5}$ .... $1\frac{2}{3}$
4. $-3\frac{1}{2}$ .... $-\frac{20}{6}$
5. 0.5 .... $\frac{2}{8}$
6. 1.6 .... $\left|-\frac{8}{5}\right|$

UNIT 1

5 **Arrange the following rational numbers descendingly :**

$\frac{3}{10}$, $\frac{7}{30}$, $-\frac{1}{3}$, $-\frac{1}{5}$ and $\frac{4}{15}$

6 **Arrange the following rational numbers in an ascending order :**

$\frac{3}{4}$, $-\frac{5}{8}$, $-\frac{7}{12}$ and $\frac{2}{3}$

7 **Write a rational number in each of the following :**

1. $\frac{2}{5} < \square < \frac{3}{5}$
2. $-\frac{2}{3} < \square < -\frac{1}{3}$
3. $\frac{1}{8} < \square < \frac{1}{4}$
4. $-\frac{2}{7} < \square < -\frac{3}{14}$

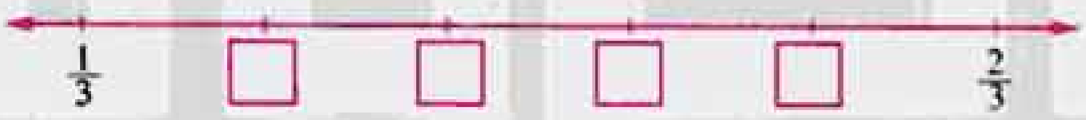
8 **Write two rational numbers lying between :**

1. $\frac{1}{2}$ and $\frac{4}{5}$
2. $-\frac{3}{4}$ and $-\frac{2}{3}$
3. 0.3 and $\frac{3}{5}$

9 **Write four rational numbers between each of the following pairs of numbers :**

1. $\frac{1}{2}$ and $\frac{11}{12}$
2. $-\frac{4}{9}$ and $-\frac{5}{6}$
3. zero and 3

10 **Complete by rational numbers on the number line :**



11 Identify and write four rational numbers between $\frac{3}{2}$ and $\frac{3}{4}$, such that one of them is an integer.

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12 Find the integer lying between $\frac{11}{3}$, $\frac{11}{2}$, and between $\frac{9}{4}$, $\frac{25}{6}$ at the same time. « 4 »

13 **In the opposite number line :**

If OA = OB , find the value of x

« – 10 »

EXERCISE 3 Adding and Subtracting Rational Numbers

📖 From the school book

1 Complete the following :

1. The additive identity element in $\mathbb{Q}$ is ....
2. The additive inverse of the number $\frac{3}{7}$ is ....
3. The additive inverse of the number $-\frac{4}{9}$ is ....
4. $\frac{-6}{-11}$ is the additive inverse of the number ....
5. The additive inverse of the number $\left(\frac{2}{3}\right)^{\text{zero}}$ is ....
6. The additive inverse of the number $\left(-\frac{2}{7}\right)^{\text{zero}}$ is ....
7. The additive inverse of the number $\left|-\frac{4}{5}\right|$ is ....
8. The additive inverse of the number zero is ....

2 Find the result of each of the following in the simplest form :

1. $\frac{3}{7}+\frac{2}{7}$
2. $-\frac{2}{9}+\frac{2}{9}$
3. $\frac{7}{8}-\frac{3}{8}$
4. 📖 $-\frac{3}{5}-\frac{9}{5}$
5. $\frac{5}{6}+\left(-\frac{4}{6}\right)$
6. $\frac{5}{9}+\left|-\frac{4}{9}\right|$

3 Calculate the value of each of the following in its simplest form :

1. 📖 $\frac{1}{4}+\frac{25}{8}$
2. $\frac{1}{5}-\frac{2}{3}$
3. 📖 $-\frac{9}{12}+\frac{3}{16}$
4. 📖 $-\frac{3}{10}+\left(-\frac{2}{5}\right)$
5. $-\frac{15}{18}+\frac{12}{16}$
6. $-\frac{2}{5}-\frac{3}{15}$
7. $\frac{3}{7}-\left(-\frac{2}{5}\right)$
8. $-\frac{5}{6}-\left(-\frac{3}{4}\right)$
9. 📖 $\frac{19}{10}+\left(-\frac{39}{100}\right)$

UNIT 1

4 Find the value of each of the following in its simplest form :

1 $3\frac{2}{7}+2\frac{3}{7}$
2 $9\frac{1}{5}-7\frac{3}{5}$
3 $-10\frac{7}{8}-\left(-4\frac{5}{8}\right)$
4 $\frac{1}{4}+2\frac{3}{8}$
5 $6\frac{2}{3}-3\frac{1}{6}$
6 $-15\frac{1}{2}+2\frac{3}{8}$
7 $-2\frac{1}{2}-12\frac{1}{16}$
8 $2\frac{3}{8}-\frac{1}{4}$
9 $-2+13\frac{3}{7}$

5 Calculate each of the following in its simplest form :

1 $\frac{2}{5}+0.2$
2 $\left|-5\frac{1}{2}\right|-\frac{1}{4}$
3 $50\%+\frac{1}{4}$
4 $25\%+\left(-\frac{1}{4}\right)$
5 $\frac{2}{3}-0.\dot{3}$

6 Choose the correct answer from the given ones :

1 $\frac{3}{4}+50\ \%=$ ....
(a) 75 % (b) 150 % (c) $\frac{5}{4}$ (d) $\frac{3}{2}$

2 Subtracting $\frac{1}{5}$ from $\frac{6}{5}$ gives ....
(a) 1 (b) – 1 (c) $-\frac{3}{5}$ (d) $\frac{7}{5}$

3 Subtracting $\frac{1}{3}$ from $-\frac{4}{3}$ gives ....
(a) – 1 (b) 1 (c) $-\frac{5}{3}$ (d) $\frac{5}{3}$

4 Subtracting $\frac{1}{7}$ from zero gives ....
(a) zero (b) $\frac{1}{7}$ (c) $-\frac{1}{7}$ (d) $\frac{6}{7}$

5 Subtracting $-\frac{3}{2}$ from zero gives ....
(a) zero (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1

6 .... $-\frac{1}{2}=-1$
(a) $1\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-1\frac{1}{2}$

7 $\frac{3}{5}+$ .... = zero
(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) 1 (d) zero

8 If $A+\frac{6}{7}=$ zero , then A = ...
(a) zero (b) 1 (c) $\frac{6}{7}$ (d) $-\frac{6}{7}$

9 If $\left(A+\frac{1}{4}\right)$ is the additive inverse of the number $\frac{3}{4}$, then A = ...
(a) $-\frac{3}{4}$ (b) $-\frac{1}{4}$ (c) – 1 (d) 1

10 $-[12+(-9)]=$ ....
(a) 3 (b) – 3 (c) 21 (d) – 21

Exercise 3

11 $-[(-3)+(-7)] = \cdots\cdots$

(a) 4 (b) -4 (c) 10 (d) -10

12 If $x = 2$, $y = 3$ and $z = 4$, then $\frac{x}{y} - \frac{z}{x} = \cdots\cdots$

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

13 If $\frac{5}{7} + \frac{x}{2} = \frac{25}{35}$, then $2x = \cdots\cdots$

(a) 2 (b) $\frac{5}{7}$ (c) zero (d) $\frac{11}{2}$

7 **Use the number line to find the result of each of the following :**

1 $\frac{1}{5} + \frac{2}{5}$ 2 $\frac{5}{8} - \frac{3}{8}$ 3 $-\frac{1}{3} + \frac{5}{3}$ 4 $-\frac{3}{4} + \left(-\frac{1}{4}\right)$

8 **Put (✓) for the correct statement and (✗) for the incorrect one :**

1 $\frac{9}{16} - \left(-\frac{3}{4}\right) = \frac{9}{16} + \left(-\frac{3}{4}\right)$ ( )

2 $-3\frac{1}{6} - \left(-7\frac{1}{12}\right) = -3\frac{1}{6} + 7\frac{1}{12}$ ( )

3 $0 - \left(-\frac{13}{5}\right) = \frac{13}{5}$ ( )

4 $-\frac{3}{4} - \frac{2}{5} = -\frac{3}{4} + \frac{2}{5}$ ( )

9 **Write the property of addition used in each of the following :**

1 $\frac{7}{2} + \frac{9}{16} = \frac{9}{16} + \frac{7}{2}$

2 $\left[\frac{2}{3} + \left(-\frac{1}{3}\right)\right] + \left(-\frac{1}{6}\right) = \frac{2}{3} + \left[\left(-\frac{1}{3}\right) + \left(-\frac{1}{6}\right)\right]$

3 $\frac{3}{4} + \left(-\frac{3}{4}\right) = \text{zero}$

4 $\text{zero} + \left(-\frac{3}{4}\right) = -\frac{3}{4}$

10 **Find the sum of each of the following :**

1 $\frac{4}{7} + \text{zero}$

2 $\text{zero} + \left(-\frac{7}{10}\right)$

3 $\text{zero} - \left(-\frac{17}{4}\right)$

4 $\left[\frac{1}{4} + \left(-\frac{1}{4}\right)\right] + \frac{3}{4}$

5 $\frac{5}{6} + \left(-\frac{3}{6} + \frac{3}{6}\right)$

6 $\left[\frac{2}{9} + \left(-\frac{4}{9}\right)\right] + \left(-\frac{3}{9}\right)$

11 **Using the addition properties in $\mathbb{Q}$, find the result of each of the following in the simplest form :**

1 $\frac{1}{4} + \frac{1}{2} + \frac{3}{4}$

2 $\frac{2}{7} + \frac{3}{4} + \frac{5}{7} + \frac{1}{4}$

3 $\frac{5}{4} + \left(-\frac{13}{5}\right) + \left(-\frac{25}{4}\right) + \frac{28}{5}$

4 $\frac{5}{8} + \left(-\frac{3}{4}\right) + \frac{3}{8} + \frac{3}{4}$

UNIT 1

5 $\frac{2}{13} + \frac{1}{5} + \frac{11}{13} + \left(-\frac{6}{5}\right)$

6 $-\frac{3}{7} + \frac{1}{2} + \left(-\frac{1}{14}\right)$

7 $\frac{12}{18} + \frac{5}{9} + \frac{1}{3} + \left(-\frac{15}{27}\right)$

8 $\frac{2}{3} + \frac{4}{5} + \frac{3}{4}$

9 $7\frac{1}{4} + \left(-11\frac{1}{4}\right)$

10 $-13\frac{1}{8} + 7\frac{3}{8}$

12 **If $x = \frac{5}{6}$, $y = -\frac{1}{3}$ and $z = \frac{1}{2}$, find the value of each of the following :**

1 $x + z$ « $\frac{4}{3}$ »

2 $x + y$ « $\frac{1}{2}$ »

3 $x - y$ « $\frac{7}{6}$ »

4 $(y + z) - x$ « $-\frac{2}{3}$ »

13 If $a = \frac{1}{2}$, $b = -\frac{3}{2}$, find the value of $(a - b)^3$ « 8 »

14 **Complete the following :**

1 $14\frac{1}{2} + \left(-11\frac{1}{2}\right) = \ldots\ldots + \left[11\frac{1}{2} + \left(-11\frac{1}{2}\right)\right]$

2 $\frac{3}{32} + \left(-\frac{17}{32}\right) = \left[\frac{3}{32} + \left(-\frac{3}{32}\right)\right] + \ldots\ldots$

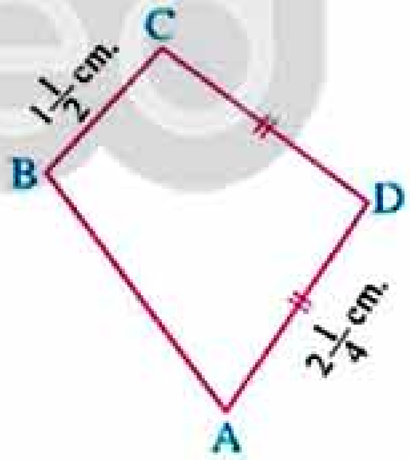
15 **Complete in the same pattern :**

1 $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, ... , ...

2 6 , $5\frac{1}{4}$, $4\frac{1}{2}$, ... , ... , ... , ... , $\frac{3}{4}$

Geometric Application

16 If the perimeter of the opposite figure equals $8\frac{2}{3}$ cm.
, calculate the length of $\overline{AB}$



« $2\frac{2}{3}$ cm. »

For excellent pupils

17 **In each of the following , find the value of x :**

1 $\left|x + \frac{1}{5}\right| = \frac{2}{5}$ « $\frac{1}{5}$ or $-\frac{3}{5}$ »

2 $\left|\frac{3}{4} - x\right| = \frac{1}{4}$ « $\frac{1}{2}$ or 1 »

18 **Find the result of the following :**

$\left(51\frac{1}{2} - 1\frac{1}{2}\right) + \left(52\frac{1}{2} - 2\frac{1}{2}\right) + \cdots + \left(99\frac{1}{2} - 49\frac{1}{2}\right) + \left(100\frac{1}{2} - 50\frac{1}{2}\right)$ « 2500 »

EXERCISE 4

Multiplying and Dividing Rational Numbers

From the school book

1 Complete the following :

1. The multiplicative identity of the rational numbers is ....
2. The multiplicative inverse of the number $\frac{3}{7}$ is ....
3. The multiplicative inverse of the number $-\frac{4}{9}$ is ....
4. The multiplicative inverse of the number -6 is ....
5. The multiplicative inverse of the number $3\frac{1}{2}$ is ....
6. The multiplicative inverse of the number 0.5 is ....
7. The multiplicative inverse of the number 1 is ....
8. The multiplicative inverse of the number -1 is ....
9. The multiplicative inverse of the number $\left(-\frac{3}{5}\right)^{\text{zero}}$ is ....
10. The multiplicative inverse of the number $\left|-\frac{3}{5}\right|$ is ....
11. The rational number $\frac{a-1}{5}$ has a multiplicative inverse if $a \neq$ ....
12. The rational number which has no multiplicative inverse is ....

2 Complete the following :

1. $\frac{2}{3} \times \left(-\frac{4}{5}\right) = -\frac{4}{5} \times$ ....
2. $7 \times \frac{\text{..........}}{7} = 1$
3. $\frac{2}{3} \times \frac{3}{2} =$ ....
4. $-\frac{4}{5} \times$ .... $= -\frac{4}{5}$
5. $-\frac{4}{11} \times$ .... $= 1$
6. $2\frac{3}{5} \times$ .... $= 1$

7 .... $\times 0.8 = 1$

8 $4 \times$ .... $= -5$

9 $\frac{2}{3}\left(2+\frac{1}{2}\right)=\frac{2}{3}\times 2+$ ....

10 $\frac{3}{9}=\frac{2}{3}\times\frac{\text{.........}}{8}$

11 If $\frac{x}{y}=\frac{2}{3}$, then $\frac{3x}{2y}=$ ....

12 If $\frac{a}{b}=70$, then $\frac{a}{2b}=$ ....

3 Put (✓) for the correct statement and (✗) for the incorrect one :

1 Every rational number has a multiplicative inverse. ( )

2 The multiplicative inverse of a rational number is an integer. ( )

3 The multiplicative inverse of the number $\frac{0}{7}$ is $\frac{7}{0}$ ( )

4 $2\frac{1}{5}$ is the multiplicative inverse for the rational number $5\frac{1}{4}$ ( )

5 $\left(\frac{2}{7}+\frac{3}{5}\right)$ is the multiplicative inverse for the rational number $\frac{35}{31}$ ( )

6 $\frac{3}{4}\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{1}{8}$ ( )

4 State the property of the multiplication of rational numbers used in each of the following statements :

1 $-\frac{1}{2}\times\frac{2}{3}=\frac{2}{3}\times\left(-\frac{1}{2}\right)$

2 $-\frac{3}{7}\times\left(-\frac{7}{3}\right)=1$

3 $-\frac{7}{20}\times\left(\frac{5}{2}\times 4\right)=\left(\frac{5}{2}\times 4\right)\times -\frac{7}{20}$

4 $\frac{5}{4}\times 1=\frac{5}{4}$

5 $0.8\times 0=0$

5 Find the result of each of the following in the simplest form :

1 $\frac{3}{5}\times\frac{2}{7}$

2 $-\frac{1}{2}\times\frac{2}{3}$

3 $-\frac{3}{8}\times\left(-\frac{5}{3}\right)$

4 $\frac{2}{6}\times -\frac{3}{4}$

5 $-\frac{2}{3}\times\frac{5}{8}$

6 $\frac{4}{5}\times\left(-\frac{3}{7}\right)$

7 $\left|-\frac{3}{7}\right|\times\left(-\frac{4}{3}\right)$

8 $\frac{1}{2}\times\left|-12\right|$

9 $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}$

6 Find the result of each of the following in the simplest form :

1 $\frac{4}{5}\div\frac{3}{7}$

2 $-\frac{1}{6}\div\frac{5}{2}$

3 $-\frac{4}{11}\div\left(-\frac{4}{11}\right)$

4 $\frac{5}{27}\div\frac{1}{9}$

5 $\frac{5}{6}\div\left(-\frac{15}{2}\right)$

6 $-\frac{5}{16}\div\left(-\frac{11}{8}\right)$

7 $-\frac{5}{8}\div\frac{5}{8}$

8 zero $\div\frac{3}{5}$

9 $\frac{3}{4}\div(-9)$

Exercise 4

7 **Find the result of each of the following in the simplest form :**

1. $3\frac{1}{2} \times (-4)$
2. $1\frac{1}{2} \times \left(-\frac{3}{2}\right)$
3. $-4\frac{2}{7} \times \left(-5\frac{1}{6}\right)$
4. $3\frac{1}{8} \times \left(-4\frac{1}{5}\right)$
5. $-0.5 \times \frac{2}{5}$
6. $2\frac{1}{2} \times 0.8$
7. $\left|-1\frac{1}{2}\right| \times \left|-\frac{5}{3}\right|$
8. $\left|-0.\dot{6}\right| \times 1\frac{1}{3}$

8 **Find the result of each of the following in the simplest form :**

1. $-2\frac{1}{5} \div \frac{11}{5}$
2. $-7\frac{5}{6} \div \frac{47}{100}$
3. $-4\frac{2}{7} \div 1\frac{1}{14}$
4. $-1 \div 2\frac{1}{4}$
5. $-4\frac{1}{3} \div \left(-3\frac{1}{4}\right)$
6. $0.5 \div 5\frac{1}{2}$
7. $-2\frac{3}{4} \div \left(-3\frac{1}{8}\right)$
8. $6\frac{1}{4} \div (-15)$
9. $2\frac{3}{5} \div \left(-1\frac{11}{15}\right)$

9 **Using the distribution property, find the value of each of the following in the simplest form :**

1. $\frac{5}{12} \times 3 + \frac{5}{12} \times 9$
2. $\frac{4}{9} \times 11 + \frac{4}{9} \times 16$
3. $4 \times \frac{8}{17} + 9 \times \frac{8}{17} + 4 \times \frac{8}{17}$
4. $\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11)$
5. $\frac{4}{5} \times 13 - \frac{4}{5} \times 22 + \frac{4}{5} \times 9$
6. $\frac{7}{12} \times 5 + 9 \times \frac{7}{12} - 2 \times \frac{7}{12}$
7. $\frac{7}{13} \times 6 + \frac{7}{13} \times 8 - \frac{7}{13}$
8. $\frac{27}{11} \times \frac{9}{4} - \frac{27}{11} \times \frac{1}{4} + \frac{27}{11} \times 9$
9. $-\frac{3}{7} \times 8 + 5 \times \left(-\frac{3}{7}\right) + \left(-\frac{3}{7}\right)$
10. $\frac{5}{2} \times \frac{13}{11} + \frac{5}{2} \times \left(-\frac{2}{11}\right) + \frac{5}{2}$
11. $\frac{22}{25} \times \frac{7}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$
12. $35 \times \frac{3}{4} + 35 \times \frac{1}{2} - 35 \times \frac{1}{4}$
13. $\frac{7}{15} \times \frac{4}{25} + \frac{16}{25} \times \frac{2}{3} + \frac{7}{15} \times \frac{1}{5} + \frac{16}{25} \times \left(-\frac{1}{5}\right)$

10 **Find the result of each of the following in the simplest form :**

1. $\left(\frac{3}{8} + \frac{5}{8}\right) \div \frac{5}{8}$
2. $\frac{3}{4} \times \left(\frac{1}{2} - \frac{1}{3}\right)$
3. $\left(-\frac{18}{5} \div \frac{9}{35}\right) \times \left(-\frac{3}{7}\right)$
4. $\left[-\frac{12}{25} \times \left(-\frac{5}{7}\right)\right] \div \left(-\frac{9}{14}\right)$
5. $\left(-1\frac{2}{3} \times 4\frac{2}{3}\right) \div 6\frac{1}{9}$
6. $\left(5\frac{1}{16} \div 6\frac{3}{4}\right) \times \left(-7\frac{5}{9}\right)$
7. $-\frac{5}{2} \div \left(\frac{3}{4} + \frac{1}{2} - \frac{1}{3}\right)$
8. $\left(2\frac{2}{5} \div \frac{3}{4}\right)\left(-\frac{4}{3} \div 2\right)$

UNIT 1

11 **Find the value of (n) in each of the following :**

1. $-\frac{7}{3} \times \left(-\frac{3}{7}\right) = n$
2. $n \times \frac{17}{3} = 1$
3. $-\frac{7}{3} \times n = 0$
4. $\frac{5}{7} \times n = \frac{5}{7}$
5. $n \times \left[\frac{1}{2} + \left(-\frac{3}{5}\right)\right] = n \times \frac{1}{2} + 5 \times \left(-\frac{3}{5}\right)$

12 If $a = 2$, $b = \frac{1}{2}$ and $c = \frac{3}{2}$, find in the simplest form the value of : $(a - b) \div c$ « 1 »

13 **If $x = -\frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, find the numerical value of each of the following :**

1. $x y z$
2. $x y + y z$

« $\frac{3}{4}$, $-\frac{5}{2}$ »

14 **If $a = 1\frac{3}{4}$, $b = \frac{12}{7}$ and $c = \frac{2}{3}$, then find the numerical value of each of the following :**

1. $a b c + 3$
2. $a b - c$

« 5 , $\frac{7}{3}$ »

15 If $a = \frac{3}{4}$ and $b = -\frac{5}{2}$, find in the simplest form the numerical value of : $\frac{a - b}{a + b}$ « $-\frac{13}{7}$ »

16 If $a = \frac{1}{3}$, $b = \frac{1}{2}$ and $c = -2$, find in the simplest form the value of : $(b - a)(b - c)$ « $\frac{5}{12}$ »

17 **If $x = \frac{3}{2}$, $y = -\frac{1}{4}$ and $z = -2$, find in the simplest form the numerical value of each of the following :**

1. $\frac{1}{x y z}$ « $\frac{4}{3}$ »
2. $x - (z \div y)$ « $-\frac{13}{2}$ »
3. $\frac{x}{y} - \frac{z}{y}$ « -14 »
4. $(x + z) \div (y - z)$ « $-\frac{2}{7}$ »
5. $\frac{x + y}{z}$ « $-\frac{5}{8}$ »

Life Applications

18 The weights of things on the surface of the moon = $\frac{1}{6}$ their weights on the surface of the Earth.

If the weight of a man on the Earth = $76\frac{4}{5}$ kg. , find his weight on the moon.

« $12\frac{4}{5}$ kg. »

19 If water flows through a pipe at a rate of $2\frac{1}{2}$ litres per minute, how long will it take to fill three containers 20 litres each ?

« 24 minutes »

20 How many pieces of wire the length of each is $3\frac{3}{4}$ metres can be cut from a wire of length 60 metres ?
Will any piece of wire be left over ?
If so, how long will it be ?

« 16 pieces »

For excellent pupils

21 Find the rational number which if we subtract $\left(\frac{2}{5} - \frac{1}{7}\right) \div \left(\frac{4}{35} + \frac{1}{7}\right)$ from it, the result will be 2

« 3 »

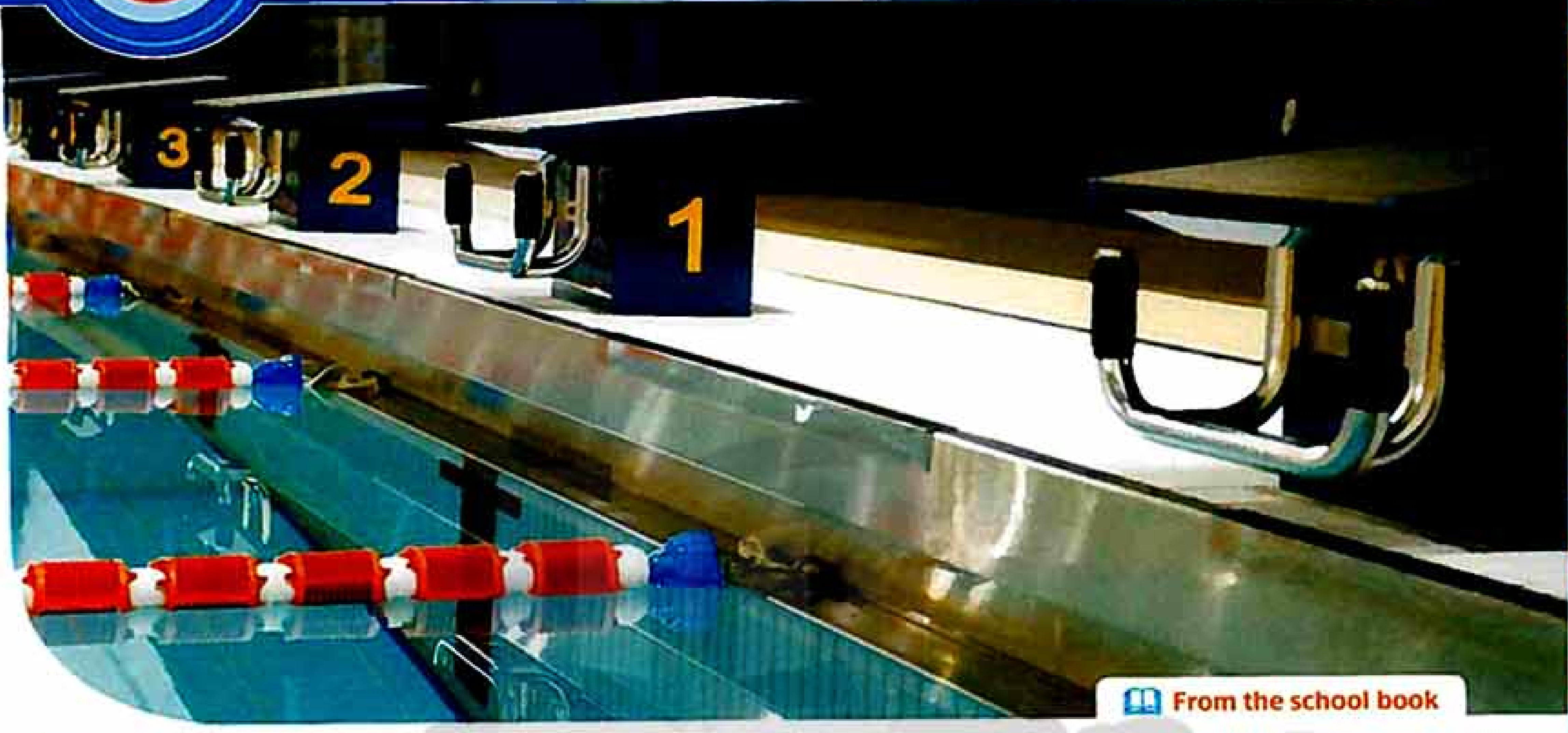
22 **Find the product of :**

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \cdots \times \frac{99}{100}$

What is the product when the last rational number is $\frac{n-1}{n}$?

EXERCISE 5

Applications on Rational Numbers



From the school book

1 **Find a rational number in the middle of the way (half-way) between :**

1. $\frac{3}{8}$, $\frac{5}{8}$
2. $\frac{2}{5}$, $\frac{4}{5}$
3. $-\frac{3}{4}$, $\frac{3}{4}$
4. $\frac{1}{2}$, $\frac{7}{8}$
5. $-\frac{1}{2}$, $-\frac{3}{4}$
6. 0.1 , $-\frac{2}{5}$
7. 📖 $-\frac{11}{9}$, $-\frac{13}{35}$
8. 📖 $-4\frac{3}{7}$, $8\frac{1}{3}$
9. zero , $\frac{2}{5}$

2 **Find a rational number lying at :**

1. One fourth of the way between $\frac{5}{7}$, $-\frac{3}{7}$ *from the side of the smaller number.*
2. One fourth of the way between $\frac{1}{3}$, 1 *from the side of the greater number.*
3. One third of the way between $-\frac{3}{5}$, $-\frac{4}{5}$ *from the side of the greater number.*
4. 📖 One third of the way between $\frac{4}{7}$, $1\frac{3}{4}$ *from the side of the smaller number.*
5. One fifth of the way between $-\frac{1}{2}$, $-\frac{2}{5}$ *from the side of the greater number.*
6. 📖 One fifth of the way between $-\frac{2}{3}$, $-\frac{3}{5}$ *from the side of the smaller number.*
7. One tenth of the way between $\frac{5}{6}$, $\frac{2}{3}$ *from the side of the smaller number.*
8. One eighth of the way between zero , $-1\frac{1}{2}$

Exercise 5

3 **Choose the correct answer from the given ones :**

1. If $a \times \frac{b}{2} = \frac{a}{2}$, $a \neq 0$, then $b = \dots\dots\dots$

   (a) $\frac{a}{2}$ (b) 0 (c) a (d) 1 (e) – a

2. If $\frac{x}{3} - 4 = 6$, then $\frac{x}{3} + \frac{2}{3} = \dots\dots\dots$

   (a) 1 (b) x (c) $\frac{32}{3}$ (d) 10 (e) $\frac{2x}{9}$

3. If $\frac{x}{y} = 1$, then $2x - 2y = \dots\dots\dots$

   (a) 4 (b) 2 (c) 1 (d) 0 (e) $\frac{1}{2}$

4. If $x + \frac{2}{x} = 5 + \frac{2}{5}$, then $x = \dots\dots\dots$

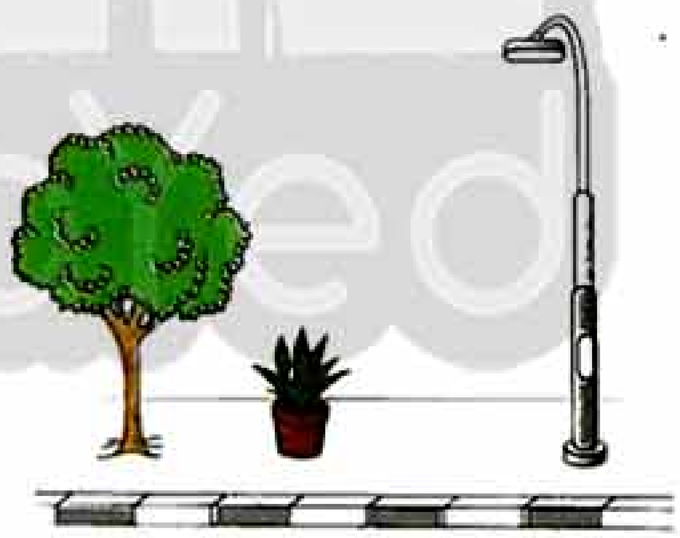
   (a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) 1 (d) $\frac{5}{2}$ (e) 5

5. If 5 a = 45 and b a = 1 , then b = ...

   (a) $\frac{1}{45}$ (b) $\frac{1}{9}$ (c) $\frac{1}{5}$ (d) 5 (e) 9

Life Application

4 In one of the projects of paving and afforesting roads , a tree was planted at a distance of 3.3 m. from the beginning of the road and a lamp post was fixed at a distance of $7\frac{1}{2}$ m. from the beginning of the road. If we want to put a flower bed at the third of the distance between them from the direction of the tree, at which distance should we put the flower bed from the beginning of the road ?



« 4.7 m. »

Summary of Unit 1

- The rational number is the number that can be expressed in the form $\frac{a}{b}$ where $a \in \mathbb{Z}$, $b \in \mathbb{Z}$, $b \neq 0$
- Each integer is a rational number, but not each rational number is an integer.
- If $\frac{a}{b}$ is a rational number, then $b \neq 0$
- If $\frac{a}{b}$ is a rational number equal to 0, then $a = 0$
- The value of the rational number $\frac{a}{b}$ does not change if its two terms are multiplied or divided by an integer not equal to 0
- To put a rational number $\frac{a}{b}$ in its simplest form, divide each of its terms by the highest common factor (H.C.F) between them.
- To write a rational number in the form of percentage we express it as $\frac{a}{100}$ which equals a%
- To write a rational number in the form of a terminated decimal, we make its denominator equal to 10, 100, 1000 or ...
- Between every two different rational numbers, there are an infinite number of rational numbers.
- If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then :

* $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
* $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$
* $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
* $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ (Where $\frac{c}{d} \neq 0$)

- The set of rational numbers is closed under addition , subtraction and multiplication operations , and not closed under division operation.
- Each of addition and multiplication operations in $\mathbb{Q}$ is commutative and associative, but each of subtraction and division in $\mathbb{Q}$ is not commutative and associative.

- Zero is the additive identity in $\mathbb{Q}$, and 1 is the multiplicative identity in $\mathbb{Q}$
- For each rational number $\frac{a}{b}$ there is an additive inverse to it that is $-\frac{a}{b}$, and for each rational number $\frac{a}{b}$ not equal to 0 , there is a multiplicative inverse to it that is $\frac{b}{a}$
- Multiplication in $\mathbb{Q}$ is distributed over addition and subtraction from right and from left.
- The number that lies at the middle of the way between two numbers

  = the smaller number + $\frac{1}{2}$ the distance between the two numbers

  or = the greater number − $\frac{1}{2}$ the distance between the two numbers.
- The number that lies at one third of the way between two numbers :

  * From the side of the smaller number

  = the smaller number + $\frac{1}{3}$ the distance between the two numbers.

  * From the side of the greater number

  = the greater number − $\frac{1}{3}$ the distance between the two numbers.

Exams on Unit One

Model 1

Answer the following questions :

1 Complete the following :

1. The remainder of subtracting $\frac{1}{5}$ from $-\frac{2}{5}$ equals ....
2. If $\frac{x-5}{x-7}=0$, then $x=$ ....
3. The additive inverse of the number $\left|-\frac{5}{6}\right|$ is ....
4. The number that lies in the middle of the way between $\frac{1}{2}$ and $\frac{3}{4}$ is ....
5. If $\frac{a}{b}=\frac{3}{5}$, then $\frac{5\,a}{3\,b}=$ ....

2 Choose the correct answer from the given ones :

1. The multiplicative inverse of the number $2\frac{1}{3}$ is ....

   (a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

2. $\frac{4}{7}$ .... $\frac{3}{5}$

   (a) $<$ (b) $>$ (c) $=$ (d) $\geq$

3. If $\frac{x}{x+5}$ is a rational number , then $x\neq$ ....

   (a) 0 (b) 1 (c) -5 (d) 5

4. If $\frac{3}{5}+x=\frac{3}{5}$, then $x=$ ....

   (a) 0 (b) 1 (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

5. If $3\,a=27$, $a\,b=1$, then $b=$ ....

   (a) 3 (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{9}$

6. $-1\div\frac{3}{5}=$ ....

   (a) $-\frac{3}{5}$ (b) $\frac{3}{5}$ (c) $-\frac{5}{3}$ (d) $\frac{5}{3}$

3 **[a] Use the distribution property to find the value of the following :**

$\frac{7}{12} \times \frac{23}{45} + \frac{17}{12} \times \frac{23}{45} - 2 \times \frac{23}{45}$

[b] Use the properties of addition of rational numbers to find the value of :

$\frac{5}{4} + (-\frac{13}{5}) + (-\frac{25}{4}) + \frac{28}{5}$

4 **[a]** Find four rational numbers between $\frac{1}{3}$ and $\frac{7}{9}$

[b] If $x = \frac{2}{3}$, $y = -\frac{1}{6}$ and $z = -3$, then find the value of each of the following :

1 $(x \div y) - (z \div y)$ 2 $\frac{x + y}{x z}$

5 **[a] Arrange the following numbers in a descending order :**

$\frac{3}{10}$, $\frac{7}{30}$, $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{4}{15}$

[b] Find the rational number that lies one third of the way between $\frac{4}{7}$ and $1\frac{3}{4}$ from the smaller number.

Model 2

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 The multiplicative inverse of the number $-\frac{3}{4}$ is ....

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$

2 If $a + \frac{3}{5} = 0$, then a = ...

(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{5}{3}$ (d) 0

3 If $\frac{x - 4}{5x} \in \mathbb{Q}$, then $x \neq$ ....

(a) – 5 (b) 4 (c) 5 (d) 0

4 If $a \times \frac{b}{3} = \frac{a}{3}$, then b = ...

(a) – a (b) 1 (c) $\frac{a}{3}$ (d) a

UNIT 1

5 If $\frac{a}{b} = 60$, then $\frac{a}{3b} = \dots\dots$

(a) 17 (b) 20 (c) 23 (d) 180

6 $0.\dot{1}\dot{2} = \dots\dots$

(a) $\frac{12}{100}$ (b) $\frac{21}{99}$ (c) $\frac{4}{33}$ (d) $\frac{3}{25}$

2 Complete the following :

1 The rational number which has no multiplicative inverse is ……….

2 If $\frac{x+7}{x+1} = 0$, then $x = \dots\dots$

3 The increase of $\frac{3}{4}$ than $\frac{2}{7}$ equals ……….

4 If $\frac{x}{y} = 1$, then $5x - 5y = \dots\dots$

5 The rational number that lies in the middle of the way between $\frac{2}{3}$ and $\frac{1}{4}$ is ……….

3 [a] Write four rational numbers between $\frac{5}{4}$ and $\frac{2}{3}$

[b] Use the distribution property to find the value of the following :

$\frac{3}{5} \times 7 + \frac{3}{5} \times 4 - \frac{3}{5}$

4 [a] Find a rational number lying at one fourth of the way between $\frac{3}{5}$ and $\frac{2}{3}$ from the side of the greater number.

[b] Write each of the following numbers in the form of $\frac{a}{b}$:

1 0.35 2 $0.\dot{1}\dot{5}$ 3 – 4

5 **[a] If $x = \frac{3}{4}$, $y = \frac{2}{3}$ and $z = \frac{4}{7}$, then find the numerical value of each of the following :**

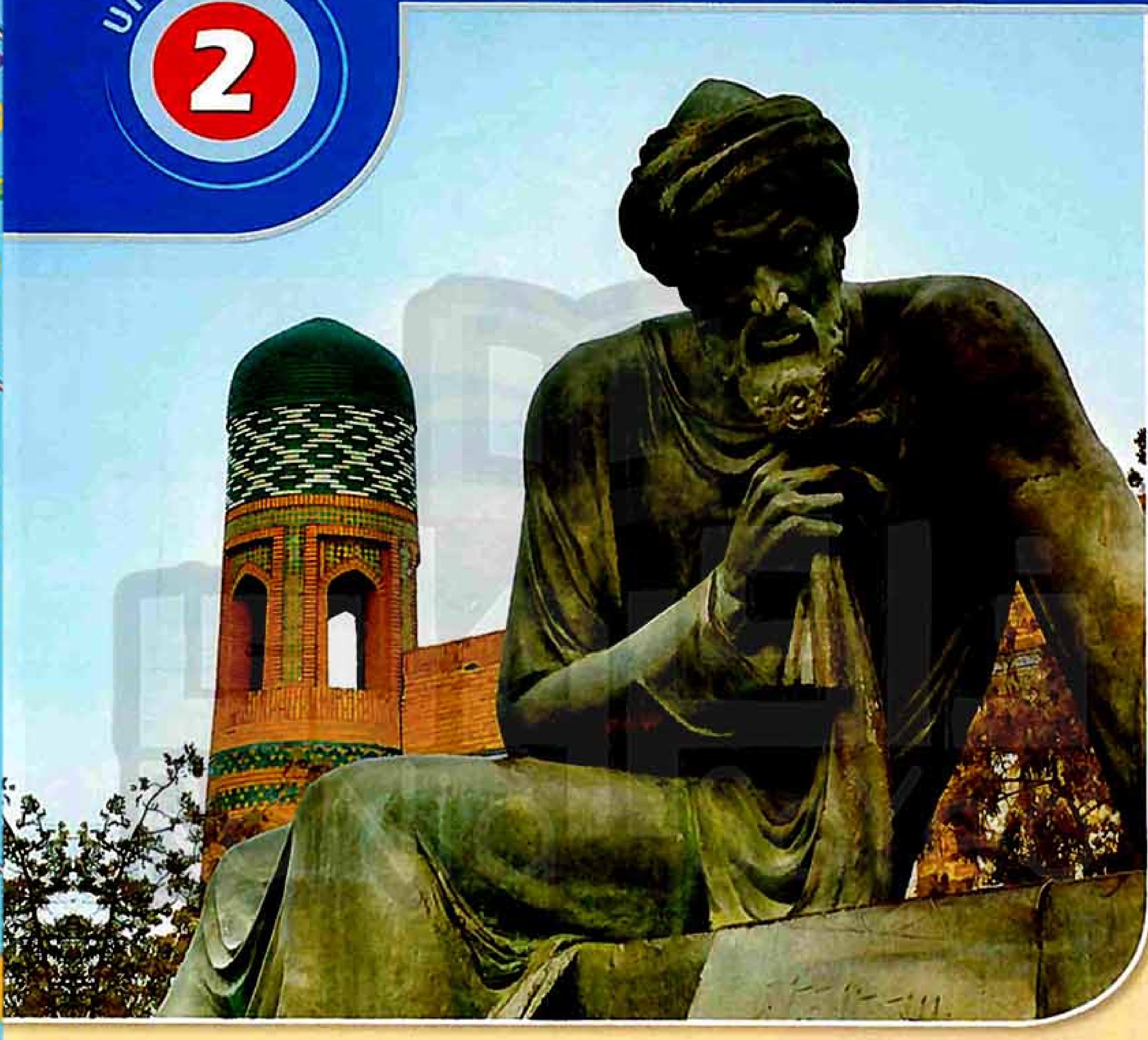
1 $\frac{1}{x y z}$ 2 $x y + y z$

[b] Represent each of the following rational numbers on the number line :

1 $\frac{3}{5}$ 2 $-3\frac{1}{2}$

Algebra

UNIT 2



Exercises of the unit :

6. Algebraic terms and algebraic expressions.
7. Like algebraic terms.
8. Adding and subtracting algebraic expressions.
9. Multiplying and dividing algebraic terms.
10. Multiplying a monomial by an algebraic expression.

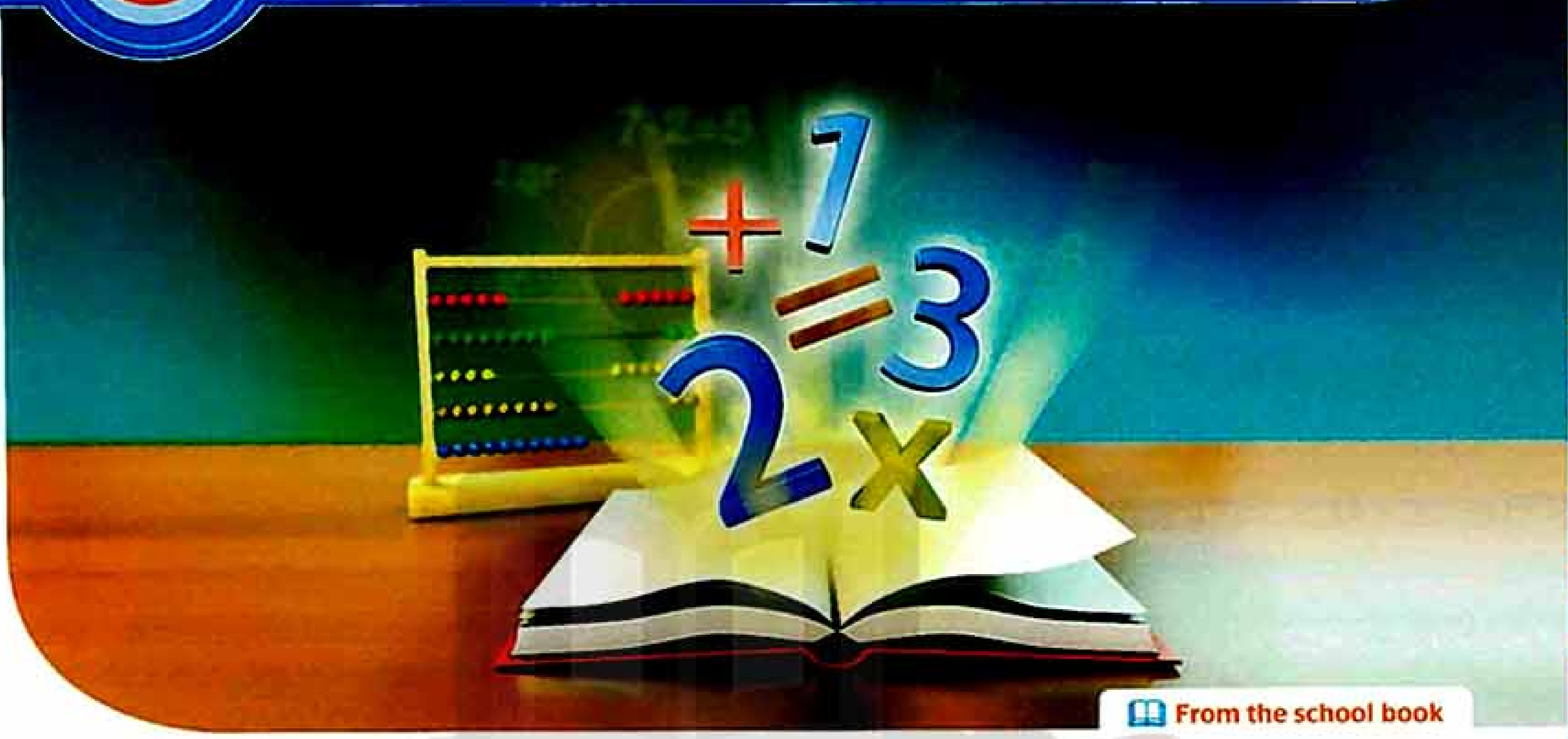
- Summary of the first part of unit two.
- Exams on the first part of unit two.

11. Multiplying a binomial by an algebraic expression.
12. Dividing an algebraic expression by a monomial.
13. Dividing an algebraic expression by another one.
14. Factorization by identifying the highest common factor (H.C.F.).

- Summary of the second part of unit two.
- Exams on the second part of unit two.

EXERCISE 6

Algebraic Terms and Algebraic Expressions



From the school book

1 Complete the following table :

Algebraic term	-7	$2ab^2$	3	$7ab^3c$	$-8x^2b$	xy^2
Coefficient	-7	2	....	....	....	....
Degree	zero	$1+2=3$	....	....	....	....

2 Complete the following table :

The algebraic expression	Number of terms	Name	Degree
$-3a^5b$	1	monomial	6
$3x^2+y$	2	binomial	2
$5x^3-7x+4$	....	trinomial	....
$2a^2b+3ab^2-a^2b^2$	....	....	....
$x^2y^2-3xy^4$	....	....	....
$a^2b-3ab^3+2a^3b^2+b^4$	....	....	....

3 Complete the following :

1. The degree of the term $3x^2y$ is .... and its coefficient is ....
2. The coefficient of the algebraic term $\frac{1}{2}x^3yz^2$ is .... and its degree is ....
3. The coefficient of the algebraic term x is .... and its degree is ....

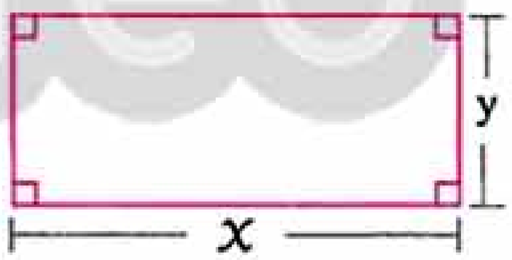
Exercise 6

4. The degree of the absolute term in an algebraic expression is ....
5. The coefficient of the algebraic term $(-2)^3$ is .... and its degree is ....
6. $5\,x^2 + 3$ is an algebraic expression of the .... degree.
7. The number of terms of the algebraic expression $5\,y^2 - 3\,xy + 2\,x^2$ is .... and its degree is ....

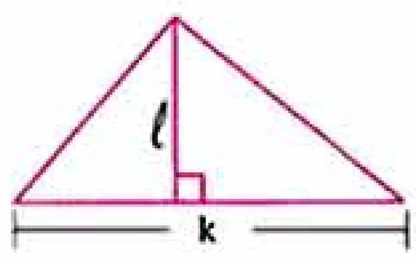
4 Choose the correct answer from the given ones :

1. The coefficient of the algebraic term $2\,x^3y^4z^5$ is ....
   (a) 2 (b) 3 (c) 4 (d) 5
2. The degree of the algebraic term $x^4\,y$ equals the degree of the algebraic term ....
   (a) $x^3\,y^2$ (b) $x^4\,y^2$ (c) $x^2\,y^2$ (d) $y^4\,x^2$
3. The degree of the algebraic expression $5\,x^3 - 3\,x\,y + 2\,y^2$ equals the degree of the algebraic expression ....
   (a) $5\,a^2 - 2\,a\,b + 3$ (b) $2\,x^2\,y^2 - 3\,x^2\,y + 5\,y^3$
   (c) $2\,x + 5\,x^2\,y + y^2$ (d) $a^3 + 2\,a^2\,b - b^4$
4. The algebraic term b^3 = ...
   (a) $3 \times b \times b$ (b) $b + b + b$ (c) $b \times b \times b$ (d) $3 \times b$
5. The algebraic term that represents the length of $\overline{AB}$ in the opposite figure is ....

   (a) x^3 (b) $3\,x$ (c) x (d) $\frac{x}{3}$
6. The algebraic term which expresses the area of the opposite figure is ....

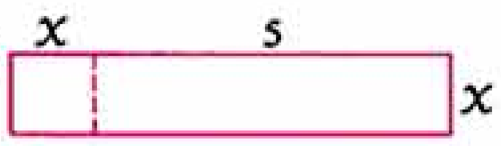


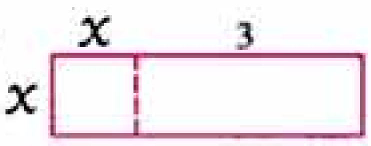
   (a) $x + y$ (b) $2\,x + 2\,y$
   (c) $x\,y$ (d) $x^2\,y^2$
7. The algebraic term which expresses the area of the opposite figure is ....

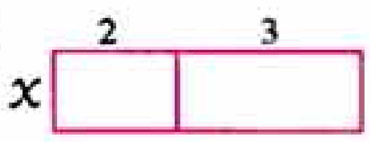


   (a) $2\,k\,\ell$ (b) $\frac{1}{2}\,k\,\ell$
   (c) $\frac{1}{2}\,k + \ell$ (d) $k\,\ell$
8. Which of the following represents the expression $3\,x + 2\,x$?

   (a) 

   (b) 

   (c) 

   (d) 

UNIT 2

5 1 Arrange the terms of the algebraic expression $7ab + 5a^5b^3 - 3a^2b^5$ according to the descending order of the indices of a

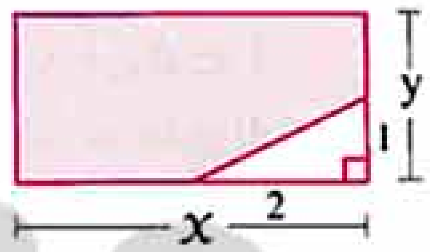
2 Arrange the terms of the algebraic expression $5x + x^2 - 7 + x^3$ according to the ascending order of the indices of x

6 **Arrange the following expression ascendingly one time according to the indices of a and descendingly second time according to the indices of b :**

$2a^2b^2 - 5ba^3 + 3b^3a + 6a^4b^4 - 4$

Geometric Applications

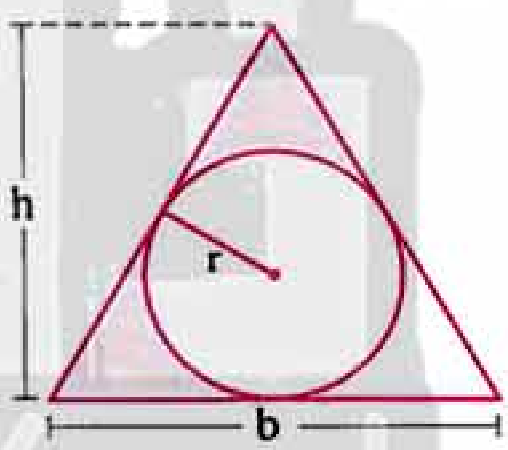
7 Write the algebraic expression which represents the area of the coloured part in the opposite figure and determine its degree.



8 **In the opposite figure :**

Write the algebraic expression which expresses the area of the coloured region , then state its degree

(The area of the circle = πr^2)



For excellent pupils

9 **Complete the following :**

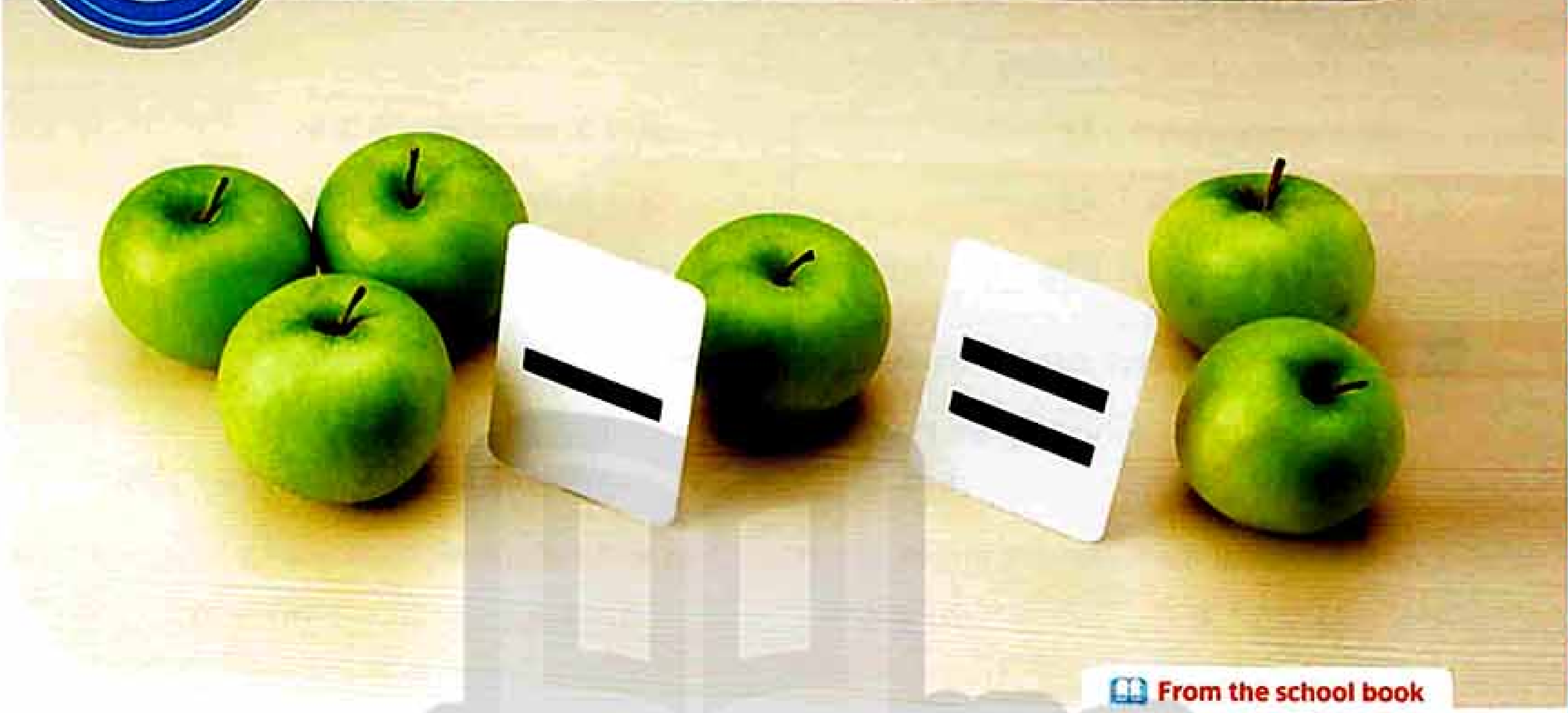
1 If the degree of the algebraic term $5x^n y^2$ is 5 , then n = ...

2 If the degree of the algebraic term y^{2m} is the degree of the algebraic term $5x^2y^4$, then m = ...

3 If the algebraic expression $x^4 + 3x^{n+1} - 2x^2 + 5$ is arranged according to the descending order of the indices of x where $n \in \mathbb{Z}$, then n = ...

4 If the algebraic expression $2xy^2z^3 + 3x^2yz^n$ is of the sixth degree where n is a natural number , then n ∈ {..........}

EXERCISE 7 Like Algebraic Terms



From the school book

1 **Find the result of each of the following :**

1. $3x + 2x$
2. $5x - 2x$
3. $4x - 11x$
4. $-7x - 3x$
5. $-5a^2 + 3a^2$
6. $-2x^2y + 3yx^2$
7. $2a + 3a - 4a$
8. $3ab - 2ba + 5ba - 6ab$
9. $\frac{5x}{4} + \frac{3x}{4}$
10. $\frac{3x}{7} - \frac{x}{7}$

2 **Answer each of the following :**

1. Subtract : y^2 from $-3y^2$
2. Subtract : $-6x^2y$ from $9x^2y$
3. What is the increase of : $-2x$ than $-5x$?
4. What is the increase of : $3a^2b$ than a^2b ?
5. What is the decrease of : $-3ab$ than $2ab$?
6. What is the decrease of : $6x^2y$ than $-7x^2y$?

3 **Complete each of the following :**

1. The result of subtracting 3 a from 7 a is ....
2. The result of subtracting $-3x^2$ from $5x^2$ is ....

UNIT 2

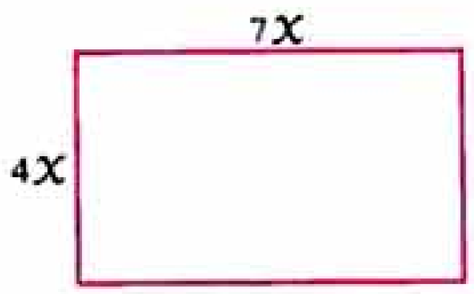
3. The result of subtracting 2 m from zero is ....
4. The result of subtracting $2x$ from $-3x$ is ....
5. 5 a increases 3 a by ....
6. $7x$ increases $-3x$ by ....
7. $4x$ decreases $7x$ by ....
8. $5x$ decreases $3x$ by ....
9. $2x$ decreases $4x$ by .... while $2x$ increases $4x$ by ....

4 **Choose the correct answer from the given ones :**

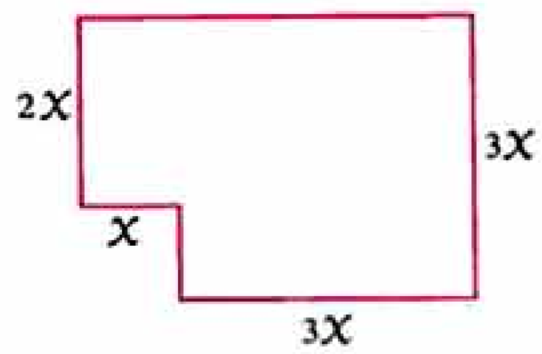
1. Which of the following are two like algebraic terms ?
   (a) $x^2, 2x$ (b) $7x^2, 2x^7$ (c) $3b^2a, -ab^2$ (d) $2a^2, 2b^2$
2. Which of the following algebraic terms is like to the algebraic term $2x^2y$?
   (a) $2y^2x$ (b) yx^2 (c) $2x^2$ (d) x^2y^2
3. $7x^2 - 2x^2 =$ ....
   (a) 5 (b) $5x^2$ (c) $5x$ (d) $9x^2$
4. $2xy - 2yx =$ ....
   (a) xy (b) $2xy$ (c) $4yx$ (d) zero
5. $\frac{1}{2}x^2a + \frac{1}{2}ax^2 =$ ....
   (a) $\frac{1}{4}x^2a$ (b) $\frac{1}{2}ax^2$ (c) $2ax^2$ (d) x^2a

5 **Complete each of the following :**

1. .... $+ 2a^2 = 7a^2$
2. $3x^2 -$ .... $= x^2$
3. $2m^2 +$ .... $=$ zero
4. $5a^2b -$ .... $= 7a^2b$
5. $3a^2b + 2a^2b =$ .... $- 2a^2b$
6. If $4x - y = 11$, $y = 3x$, then $x =$ ....
7. The perimeter of the opposite rectangle equals .... length units.



8. The perimeter of the opposite figure equals .... length units.



Exercise 7

6 If the sum of two terms is $12x^2y$ and one of them is $4x^2y$, find the other term.

7 Reduce to the simplest form :

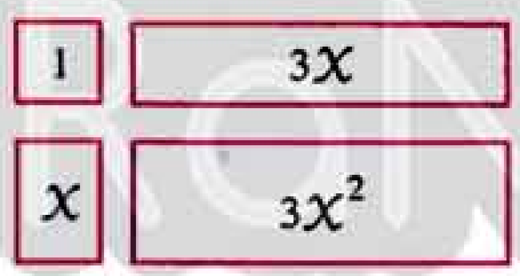
1. $3a + 2b + 5a + 4b$
2. $3x - 5y - x + 2y$
3. $2x - 4y - 9x - 3y$
4. $19m - 4n + 11m - 17n + 9n$
5. $2a + 7 - 5a - 4 - a$
6. $5a + 2b - 8a - 7b + 3a$
7. $2y - 3x - 7y - 5x - y + x$
8. $4a + 9b + 5a - 2b + 6b - 3a$

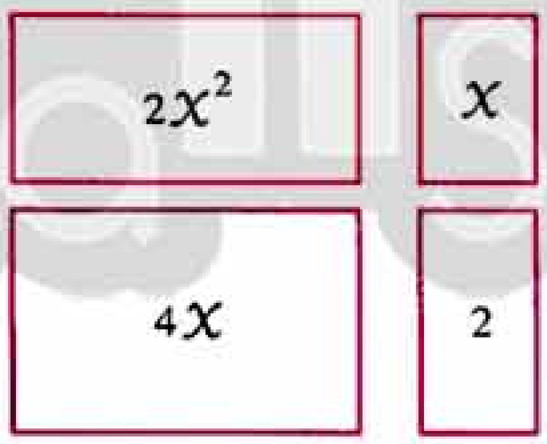
8 Reduce each of the following algebraic expressions :

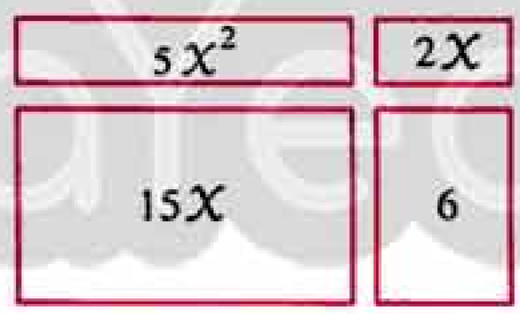
1. $5x - 3x^2 + 4 - 7x^2 - 6x - 1$
2. $6x^2y - 3xy^2 + 2xy^2 - 5x^2y + 2x^2y^2$
3. $a^2 + 4a - 5 + 3a^2 - 6a + 1$
4. $5x^2 - 2x + 8 - 7x - 3 + x^2$

Geometric Applications

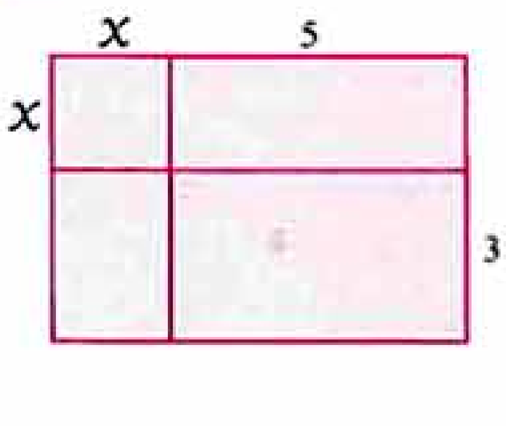
9 Write the sum of the areas of the rectangles as an algebraic expression :

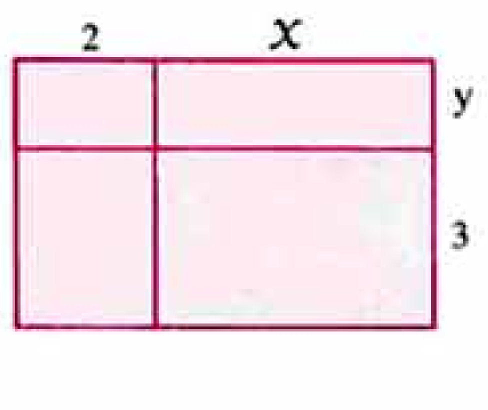
1


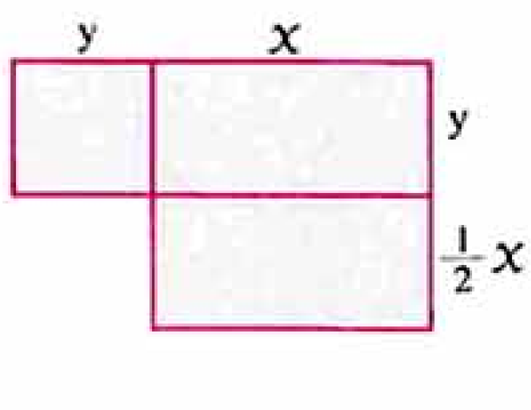
2


3


10 Write the algebraic expression which expresses the perimeter of the coloured part in each of the following rectangles :

1


2


3


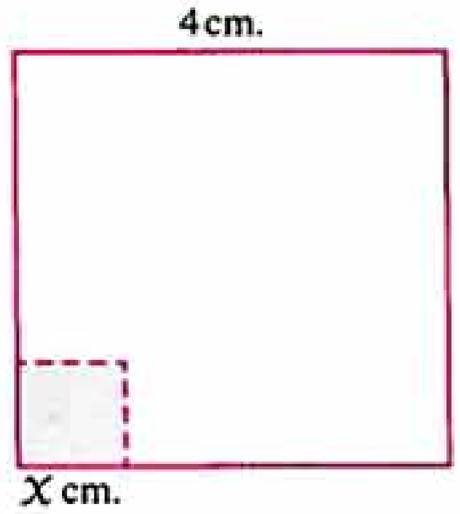
UNIT 2

11 **In the opposite figure :**

A square whose side length is x cm. was cut

from a square with side length 4 cm.

Find the perimeter of the remained part.



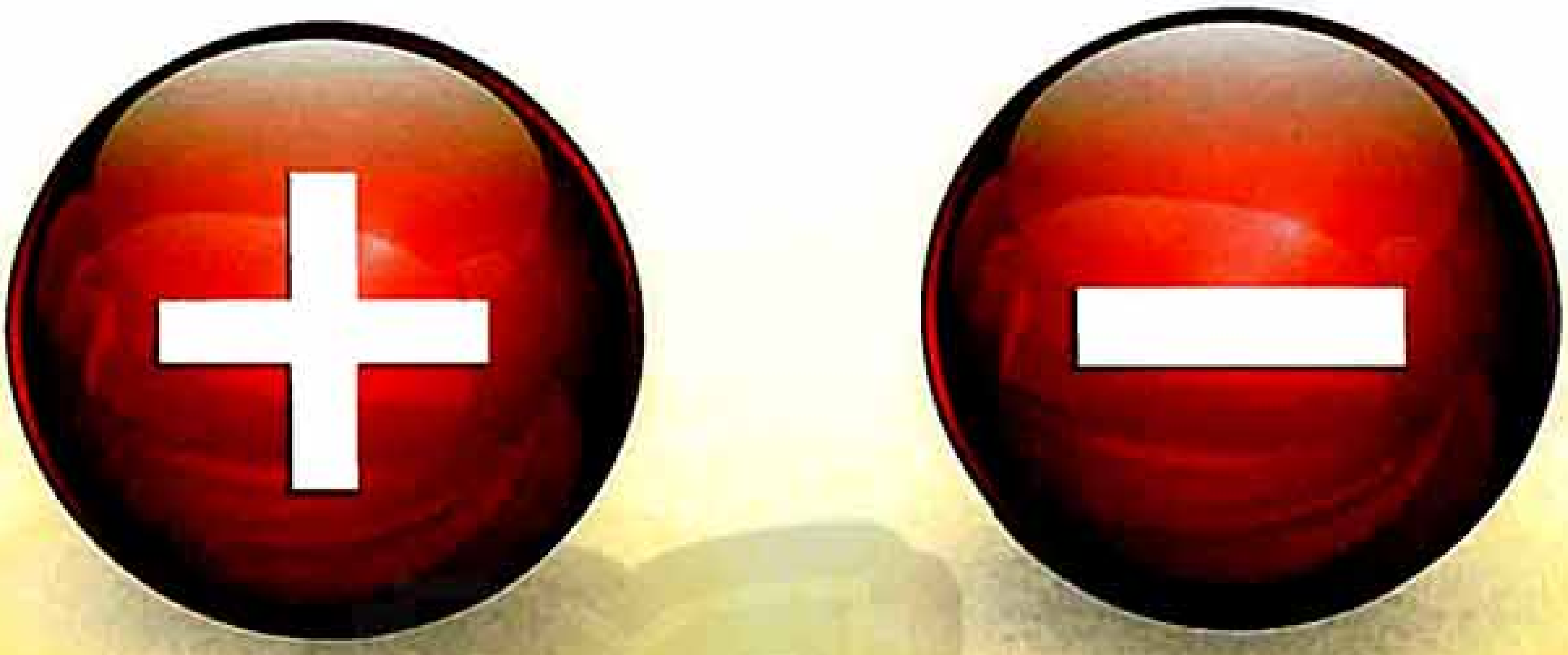
For excellent pupils

12 **Complete the following :**

1 If the two algebraic terms $2\ a^2\ b^{n+2}$ and $5\ a^2\ b^5$ are like terms , then n = ...

2 If the two algebraic terms $9\ x^m\ y^{m+n}$ and $4\ x\ y^3$ are like terms , then m = ... and n = ...

EXERCISE 8 Adding and Subtracting Algebraic Expressions



From the school book

1 **Find the sum of each of the following :**

1.
$$\begin{array}{rrr} 3a & -4b & +6c \\ 5a & +6b & -2c \\ \hline \end{array}$$
....

2. (From the school book)
$$\begin{array}{rrrr} 3a & -7b & -5c & +2 \\ -a & +4b & +c & -5 \\ 2a & & +3c & +3 \\ \hline \end{array}$$
....

3. (From the school book)
$$\begin{array}{rrrr} 5x & +2y & -z & +2 \\ 7x & +y & -3z & +3 \\ -2x & -5y & +4z & -1 \\ \hline \end{array}$$
....

4.
$$\begin{array}{rrrr} -2a^3 & +3a^2b & & -b^3 \\ & -5a^2b & +3ab^2 & -2b^3 \\ 5a^3 & & -4ab^2 & +3b^3 \\ \hline \end{array}$$
....

2 **Find the sum of each of the following :**

1. (From the school book) $3x-2y+5$, $x+2y-2$
2. $3\ell-4m+5n$, $4m-5n-\ell$
3. (From the school book) $3n^2+5n-6$, $-n^2-3n+3$
4. $5m^2+2\ell m$, $\ell^2-3m^2-2\ell m$
5. $2a^2b-3ab^2+b^3$, $-a^2b+b^3$
6. $3a^3-2ab^2+b^3$, $a^3+4a^2b-b^3$

3 **Find the sum of each of the following :**

1. $3a+2b-5$, $2a-7b+4$, $5b-4a+3$
2. $3x+3y-z$, $3x+3z-2y$, $x+2y+z$
3. $5x^2-3x+9$, x^2+2x-5 , $x-3-6x^2$
4. $3x-4x^2+2$, x^2+x-5 , $3+3x^2-4x$
5. $3x-4x^2+x^3$, $2x^2-6x+5$, $4+7x-x^3$
6. $2x^2-3xy+y^2$, $xy-2y^2+x^2$, $3xy-2x^2$

UNIT 2

4 **Subtract :**

1. $x - 2$ **from** $2x - 5$
2. $2x + 6y - 7$ **from** $2x - 5y + 2$
3. $3x^2 - 1 - 5x$ **from** $1 - 5x + 6x^2$
4. $3ab^2 - 4a^2b - b^3$ **from** $a^3 - 2ba^2 + 2b^3$

5 **What is the increase of :**

1. $5a + 7b$ **than** $3a - 2b$
2. $x^2 - 5x - 1$ **than** $3x^2 + 2x - 3$
3. $2x^2 - 3x + 1$ **than** $5x + 2x^2 - 1$
4. $3x^2y - 5x$ **than** $3x - 4x^2y$

6 **What is the decrease of :**

1. $2a + 3b$ **than** $5b - 3a$
2. $3y^2 - 2xy + x^2$ **than** $3x^2 - 5xy + y^2$
3. $2a^2 - 3ab - 5b^2$ **than** $4b^2 + 3a^2 + ab$
4. $5x^2 + 2x$ **than** $7x^2 - x + 3$

7 What is the expression which should be added to $2x - 3x^2 + 5$ to get $6 + x^2 - x$?

8 What is the expression which should be subtracted from $2x - 3y + 6z - \ell$ to get $5z - 4y + 3x - 2\ell$?

9 What is the expression which should be added to $3a^2 - 5ab + 2b^2$ to get zero ?

10 If the sum of two algebraic expressions is $5x - 7y + 9$ and if one of the two expressions is $2y + 3x - 4$, find the other expression.

11 Subtract $2b + 5a$ from $6a + 7b - 2$, then find the numerical value of the result when $a = 2$ and $b = 1$ « 5 »

12 Add $7x - 6y - z$ and $y - 3x - 5z$, then subtract the result from $5x + 5y - z$

13 What is the decrease of $2a - 8b - c$ than the sum of $3a - 3b + c$ and $2a - 4b - 8c$?

14 Add the expressions $3\ell - 2m + 7n$, $5m - 4\ell - 2n$ and $2\ell - 3n - m$, then subtract the result from $2\ell - 4m + 5n$

Exercise 8

15 By what expression is $3x^2 - 5 + 2x$ increased than the sum of $x + 5x^2 + 1$ and $2x^2 - 24 - 2x$?

16 Add $3x^2 + 2xy - 5$ and $-2x^2 - 3xy + x$ and calculate the result when $x = -1$ and $y = 2$ « – 3 »

17 If $x = a - 2b + c$, $y = 2a + 3b - 4c$ and $z = b - 4a + c$, find the expression $x + y - z$ in terms of a , b and c

Geometric Applications

18 **In the following figure, calculate the total surface area of the two solids together :**

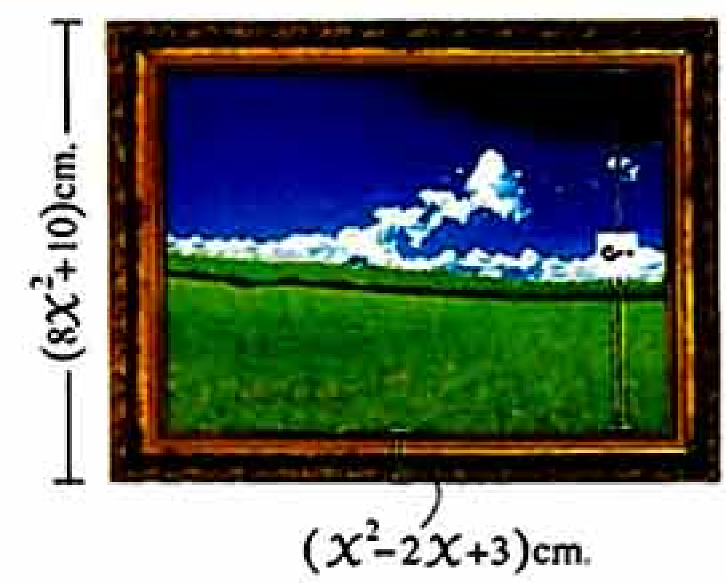
First solid Second solid

19 The perimeter of a triangle is $(2x^4 - 3x^2 + 5x - 4)$ cm. and the lengths of two of its sides are $(x^3 - 3x^2 + 2x - 3)$ cm. and $(x^4 - 2x^2 + 4x + 1)$ cm. Find the length of the third side in terms of x

Life Applications

20 The distance between two cities is $(3x^3 - 4x^2 + 2x - 5)$ km. A passenger covered a distance $(2x^3 + x^2 - 3x + 1)$ km. from it. Find the remainder distance in terms of x

21 The opposite figure shows a picture inside a frame. Using the given data as shown on the figure , find the width of the picture.



For excellent pupils

22 $a + b = \frac{5}{4}$, $b + c = \frac{3}{4}$, $a + c = \frac{1}{2}$

, then find the value of :

1. $a + 2b + c$ « 2 »
2. b « $\frac{3}{4}$ »

EXERCISE 9 Multiplying and Dividing Algebraic Terms

From the school book

1 Multiply :

1. $(5x) \times (3y)$
2. $(-3a) \times (7c)$
3. $(2x) \times (-3x)$
4. $-8y^5 \times (-7y^4)$
5. $(2xy) \times (-3x^2)$
6. $5x^3y^4 \times 2xy^2$
7. $5ab^2 \times (-2a^2b)$
8. $(x) \times (x) \times (2x)$
9. $(5)(-2a) \times (4a)$
10. $ab \times (-3a) \times (-2b)$
11. $(2x^3) \times (-3x^2) \times (-5x^4)$
12. $(4x^3y) \times (-2xy^2) \times (-3x^2y^5)$

2 If the symbols represent non-zero integers , find the quotient of each of the following :

1. $6a \div 2$
2. $12x \div (-x)$
3. $10c \div 2c$
4. $-14x^2 \div 7x$
5. $-25a^6 \div (-5a^2)$
6. $24c^5 \div (-24c^5)$
7. $9x^5y^4 \div 6x^3y$
8. $-32a^3b^6 \div (-4a^3b^2)$
9. $8m^4n^3 \div (-4mn^2)$
10. $-18x^5y^6z^3 \div (-6x^3y^3z^3)$

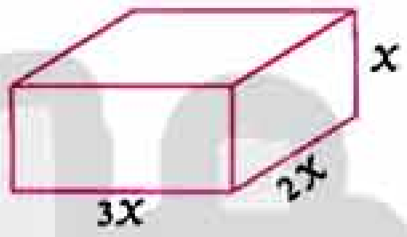
3 Simplify :

1. $\frac{2}{3}t^4 \times \frac{3}{2}t^4$
2. $\frac{2}{7}a^2 \times 21a^5$
3. $\frac{15a^3b}{2} \times \frac{8ab^2}{10}$
4. $(3x^3) \times (\frac{1}{6}x^2)$
5. $\frac{4h^3k^3}{7} \times \frac{21hk^5}{2}$
6. $4m^3 \times \frac{1}{4}m^2 \times (-7m)$

Exercise 9

4 **Choose the correct answer from the given ones :**

1. $(2x) \times (5x) = \dots\dots$
   (a) $10x$ (b) $7x$ (c) $7x^2$ (d) $10x^2$
2. $3a^4 b \times 5a^2 b^2 \times 2a^3 = \dots\dots$
   (a) $60a^{11} b^3$ (b) $30a^{10} b^2$ (c) $150a^{10} b^3$ (d) $30a^9 b^3$
3. $-6x^3 y \div 2xy = \dots\dots$
   (a) $-3x^3$ (b) $-3x^2 y$ (c) $-3x^4 y^2$ (d) $-3x^2$
4. If $2b$ is the edge length of a cube , then its volume is $\dots\dots$
   (a) $4b^2$ (b) $2b^3$ (c) $4b^3$ (d) $8b^3$
5. If the area of a rectangle is $24x^3$ and its length is $8x^2$, then its width is $\dots\dots$
   (a) $3x^5$ (b) $3x$ (c) $3x^2$ (d) 3
6. The volume of the opposite cuboid equals $\dots\dots$



   (a) $6x^3$ (b) $6x$
   (c) $5x^3$ (d) $6x^2$
7. If the price of 4 shirts is x pounds , then the price of 40 shirts of the same kind equals $\dots\dots$ pounds.
   (a) $10x$ (b) $\frac{x}{40}$ (c) $\frac{5x}{2}$ (d) $\frac{40}{4}$
8. You drove 200 km. in 3 hours. Which expression represents your average speed if "d" represents distance and "t" represents time ?
   (a) dt (b) $\frac{d}{t}$ (c) $\frac{3t}{200d}$ (d) $d+t$

5 **Complete the following if the symbols represent non-zero integers :**

1. $\frac{y^5}{y^3} + y^2 = \dots\dots$
2. $(6x^3 \div 2x) - 2x = \dots\dots$
3. $(10x^2 + 5x^2) \div 5x = \dots\dots$
4. $(5a \div a) + \dots\dots = \text{zero}$
5. $81 \ell^4 \div \dots\dots = 27 \ell^3$
6. $\dots\dots \div 7a^3 = -5a^2$
7. $15x^2 y^3 \div \dots\dots = 3xy^2$
8. $\dots\dots \div (-4x^3 y^2) = 16x^4 y^4$

6 **Complete :**

1. $36a^5 b^8 = 12a^3 b^2 \times \dots\dots$
2. $9a^5 = 3a \times \dots\dots$
3. $-4c^3 d^3 = 2cd^2 \times \dots\dots$
4. $98a^7 b^4 = \dots\dots \times 14a^7 b$
5. $36a^8 b^5 = 6ab^2 \times 3a^4 b \times \dots\dots$
6. $42x^4 y^5 = 3x^2 y \times 2xy \times \dots\dots$

UNIT 2

7 **If $x \neq$ zero , y $\neq$ zero and n is a positive number , simplify :**

1 $\frac{27\,y^{2n+4}}{3\,y^{2n+3}}$

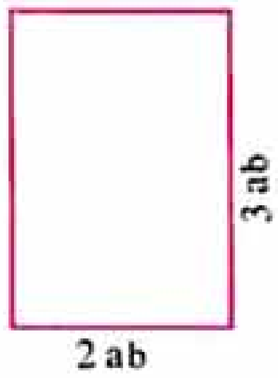
2 $\frac{-24\,x^{5n+1}\,y^{2n}}{36\,x^{5n}\,y^{n}}$

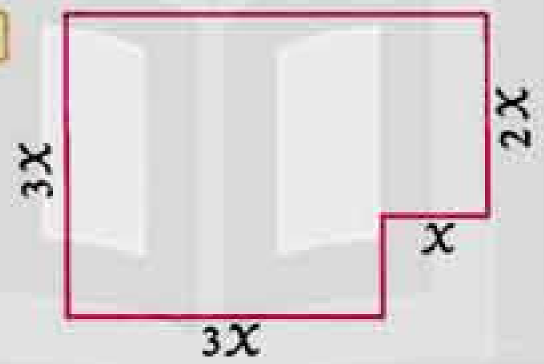
Geometric Applications

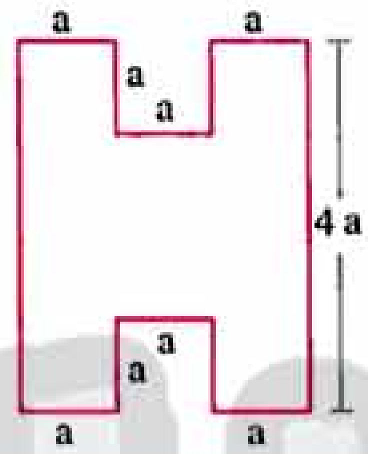
8 A cuboid of dimensions x cm. , $2x$ cm. and $4x$ cm. was melted to make small cubes with edge length x cm. for each one.

Find the maximum number of the small cubes we can make. « 8 »

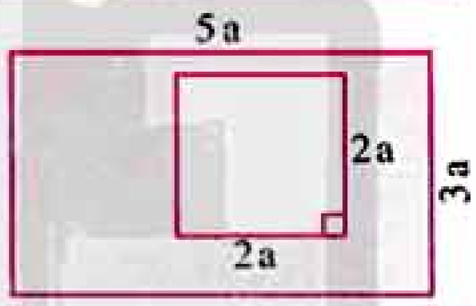
9 **Calculate the perimeter and the area of each figure :**

1 

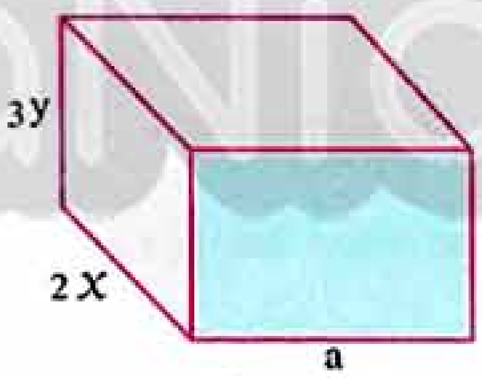
2 

3 

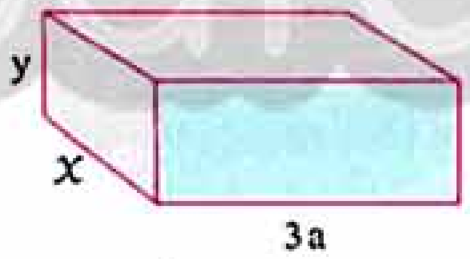
10 Calculate the area of the coloured part in the opposite figure.



11 **Calculate the sum of the total surface areas of the two solids :**



First solid



Second solid

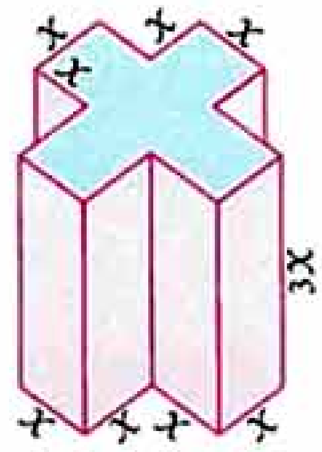
12 Three tennis balls fit into a cuboid box where the balls touch all faces of the box.

Calculate the ratio between the volume of the three balls and the volume of the box.

(Given that : volume of sphere = $\frac{4}{3}\pi r^3$, $\pi \simeq 3.14$)

For excellent pupils

13 Calculate the total surface area and volume of the opposite solid.



EXERCISE 10 Multiplying a Monomial by an Algebraic Expression

From the school book

1 Find the following products :

1. $a(a+1)$
2. 📖 $a(a-2)$
3. $3x(7y-4z)$
4. 📖 $-3(y+3)$
5. 📖 $-2c(7-3c)$
6. $2x(3x^2+4y^2)$
7. $-5x(2x+y-3z)$
8. $3xy(2x^2-5x^2y-4y^2)$
9. 📖 $\ell m^2(\ell^2-3m\ell-4m^2)$
10. 📖 $\frac{1}{3}x^2(6x^2-9xy-3y^2)$

2 Complete the following :

1. 📖
$$\begin{array}{r} 2y^2-y-5 \\ \times 2y \\ \hline \dots\dots\dots\dots \end{array}$$

2.
$$\begin{array}{r} 4xy+3x^2-5 \\ \times(-y^2) \\ \hline \dots\dots\dots\dots \end{array}$$

3.
$$\begin{array}{r} -5x+4y-xy \\ \times 4xy \\ \hline \dots\dots\dots\dots \end{array}$$

4.
$$\begin{array}{r} -2x+y \\ \times \dots\dots \\ \hline 4x^2y+\dots\dots \end{array}$$

3 Complete the following :

1. $x(\dots\dots-2x)=6x-\dots\dots$
2. $3x(\dots\dots+5y)=6x^2+\dots\dots$
3. $2x(\dots\dots-5y)=8x^3-\dots\dots$
4. $3x(\dots\dots-4xy^2)=15x^3y-\dots\dots$
5. $-2ab(\dots\dots+2a^2b)=-6a^2b^3-\dots\dots$
6. $2x(3x-\dots\dots)=\dots\dots-10x$
7. $-4a(2ab-\dots\dots)=\dots\dots+8ab^2$
8. $\dots\dots(3x+y)=6x^2+\dots\dots$

UNIT 2

9 $4\,y\,(\cdots\cdots + \cdots\cdots) = 20\,y^2 + 8\,x\,y$

10 $a\,b\,c\,(\cdots\cdots + \cdots\cdots - \cdots\cdots) = a^2\,b\,c + a\,b^2\,c - a\,b\,c^2$

11 $3\,x\,y\,(\cdots\cdots - \cdots\cdots - 5\,x^2) = 6\,x^2\,y - 12\,x\,y^2 - \cdots\cdots$

12 $\cdots\cdots\,(\cdots\cdots + 3\,m^2\,n) = 10\,m^2\,n^2 + 6\,m^3\,n^3$

4 If $a = 5\,x$, $b = 3\,x\,y$ and $c = x - y$, find the value of abc in terms of x and y

5 **Put in the simplest form :**

1 $3\,a\,(a - b) + 4\,a\,(2\,a + b)$

2 $3\,a\,(4\,a - 2) - 4\,a\,(3\,a - 2)$

3 $3\,a\,(4\,a - 1) + 2\,a\,(a + 3) - 5\,a\,(2\,a - 1)$

4 $2\,x\,(x + y) - y\,(2\,x - y) + 2\,(y^2 - x^2)$

6 Simplify : $2\,a\,(3\,a - 1) + 3\,a\,(a + 2)$, then find the value of the result when $a = 1$ « 13 »

7 Simplify : $2\,a\,(3\,a + b) - 3\,b\,(a + b)$, then find the value of the result when $a = b = 1$ « 2 »

8 Simplify : $x\,(2\,x - y) - 2\,y\,(x - y)$, then find the numerical value of the result when $x = 2$ and $y = -1$ « 16 »

9 Find the sum of : $2\,x\,(3\,x - 2\,y)$, $y\,(x + y)$ and $x^2 - y^2$, then find the value of the result if $x = -2$ and $y = -1$ « 22 »

10 Simplify : $3\,(1 - 2\,x) - (x^2 - 5\,x + 3) + 2\,x\,(x + 3)$, then find the numerical value of the expression when $x = -2$ « – 6 »

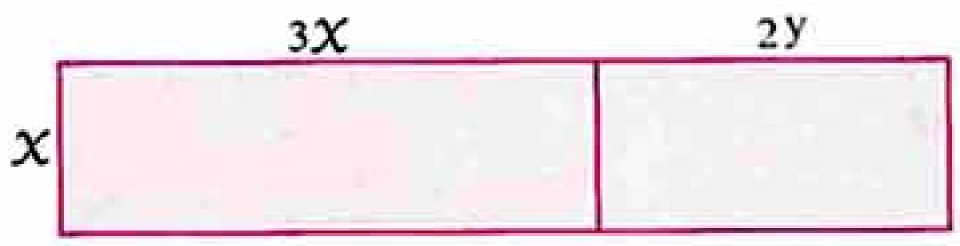
11 Simplify : $a\,b\,(3\,a - 2\,b) - 2\,a\,(a\,b - b^2) + b\,(4\,a\,b - a^2)$, then find the numerical value of the result when $a = 1$ and $b = -3$ « 36 »

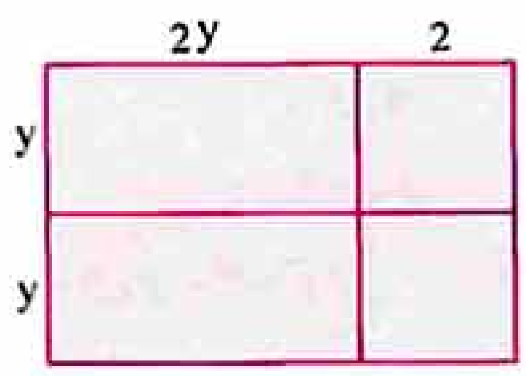
12 Simplify : $2\,x\,[x - 2\,(y - x)] - 3\,y\,[y - 2\,(x - y)]$, then find the numerical value of the result when $x = y = 1$ « – 1 »

Geometric Applications

13 The dimensions of a rectangle are $(2\,a + b)$ cm. and $(4\,a - 2\,b)$ cm. Find its perimeter.

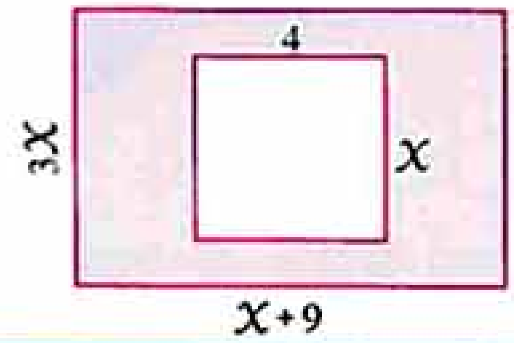
14 **Find the algebraic expression which expresses the area of the coloured part in each of the following :**

1 

2 

Exercise 10

3



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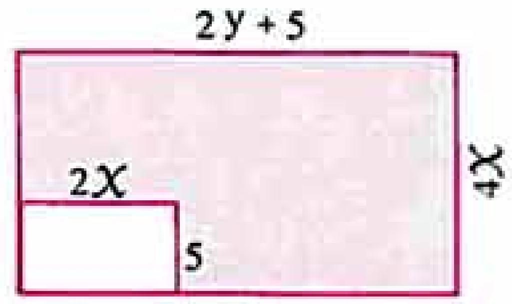

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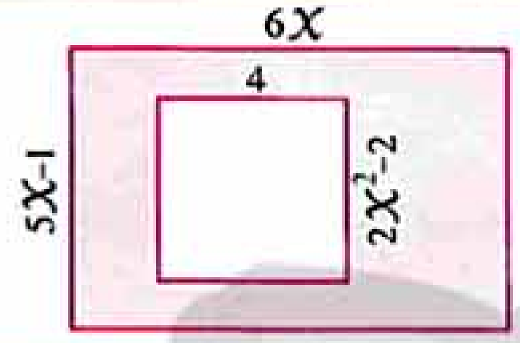

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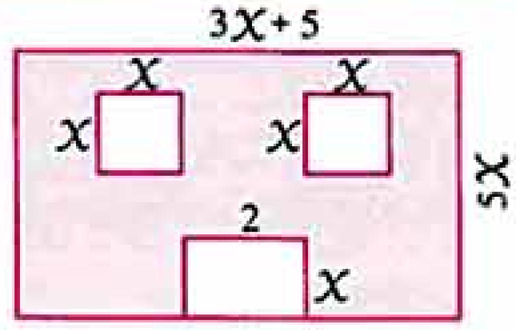

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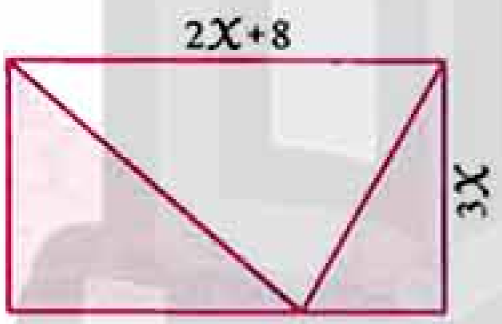

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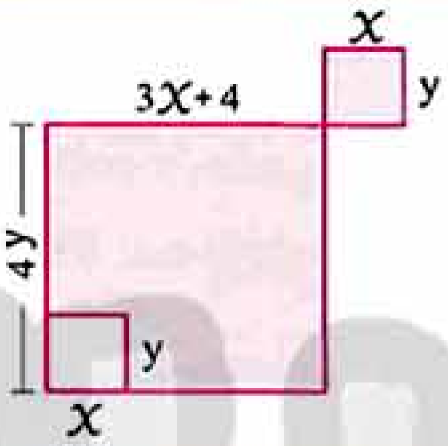

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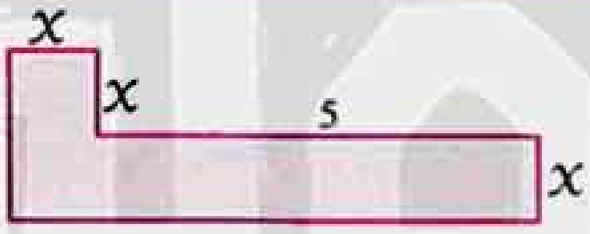

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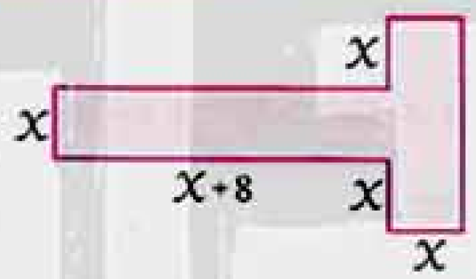


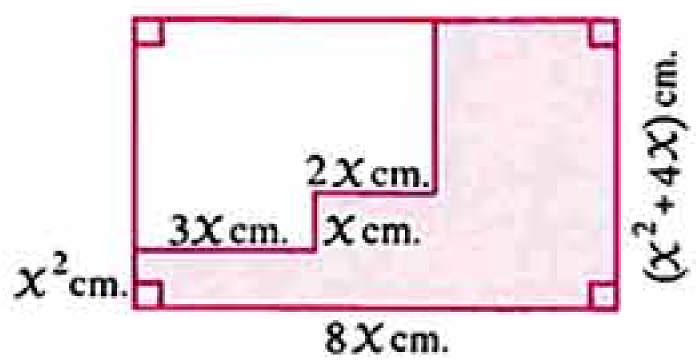
For excellent pupils

15 The width of a rectangle is x cm. and its length exceeds twice its width by 3 cm. Find its area in terms of x

16 A cuboid of a square base of side length $3x$ cm. If its height is $(2x^2 + 3)$ cm. , find its volume in terms of x

17 A cuboid whose base dimensions are $3x$ cm. and $(4x + y)$ cm. If its height is $5y$ cm. , find its lateral surface area and its volume in terms of x and y

18 In the opposite figure , find the area of the coloured part in terms of x



Summary of the first part of unit 2
"From lesson 1 to lesson 5"

- The algebraic term consists of the product of two factors or more.
- The degree of the algebraic term is the sum of the indices of the algebraic factors in this term.
- The algebraic expression consists of one or more terms connected by the sign + or –
- The degree of the algebraic expression is the highest degree of the terms forming it.
- We can add or subtract the like algebraic terms , but we cannot add or subtract the unlike algebraic terms.
- We can reduce the algebraic expression in the simplest form by adding or subtracting the like terms using the commutative and the associative properties.
- We can add or subtract the algebraic expressions by adding or subtracting the like terms in each of them together.
- When multiplying the like bases , we add the indices , but when dividing the like bases , we subtract the indices.
- When multiplying a monomial by an algebraic expression , we have to multiply this monomial by each term of the algebraic expression using the distribution property.

Exams on the first part of unit two from lesson (1) to lesson (5)

Model 1

Answer the following questions :

1 Choose the correct answer from the given ones :

1. The algebraic expression : $x^3 - 3x^2 + 4$ is of the .... degree.
   (a) first (b) second (c) third (d) fourth
2. $-8x^3y \div 2xy =$ .... (where $xy \neq 0$)
   (a) $4x^2y$ (b) $-4x^3$ (c) $-4x^2$ (d) $-16x^2$
3. $5x$ increases $-3x$ by ....
   (a) $2x$ (b) $-8x$ (c) $-2x$ (d) $8x$
4. The degree of the algebraic term : $7x^2y^3z$ equals the degree of the algebraic term ....
   (a) $7xyz^3$ (b) $3xy^4$ (c) $7x^2yz^2$ (d) $5x^3y^3$
5. $2x \times 7xy =$ ....
   (a) $14x^2y$ (b) $9xy$ (c) $14xy$ (d) $9x^2y$
6. The base length of a triangle is $2x$ cm. and its height is $6y$ cm., then its area is .... cm^2
   (a) $12xy$ (b) $8xy$ (c) $6xy$ (d) $4xy$

2 Complete the following :

1. $\frac{y^3}{y} + y^2 =$ .... (where $y \neq 0$)
2. $3x($ .... + ... $) = 15xy + 12x^2$
3. The degree of the absolute term in any algebraic expression is ....
4. $2ab^2 \times 3a^2b^3 \times 5a =$ ....
5. $(6x^2 + 8x^2) \div 7x =$ .... (where $x \neq 0$)

3 **[a] Find the sum of :** $2x + 7y - 5$ and $2x - 7y - 3$

[b] Simplify : $2a - 5a^2 + 4 + 2a^2 - 6a - 1$

4 **[a] Arrange the algebraic expression :**

$4x + 5x^2 - 9 + x^3$ according to the ascending order of the indices of x

[b] Simplify : $4(1 - 2x) - (x^2 - 3x + 2) + 2x(x + 3)$,

then find the numerical value of the result at $x = -2$

5 **[a] What is the decrease of :** $3x - 5y + z$

than the sum of : $2x + 4y + 5z$ and $3x - 2z - 3y$

[b] Find the result of each of the following :

1 $-8x^4 \times (-2x^3)$ 2 $8a^4b^3 \div (-4ab^2)$

Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

1 If the area of a rectangle is $12x^3$ cm^2 and its length is $3x^2$ cm. , then its width equals .... cm.

(a) $4x^2$ (b) $4x$ (c) $36x^5$ (d) $9x$

2 The remainder of subtracting $2x$ from $-5x$ is ....

(a) $-7x$ (b) $-3x$ (c) $-3x^2$ (d) $3x$

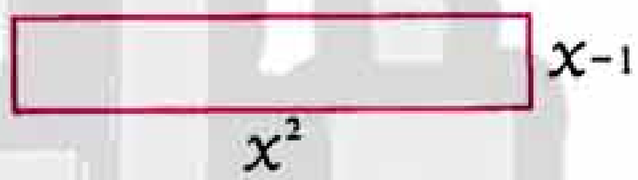
3 The coefficient of the algebraic term : $3xy^2z$ is ....

(a) 2 (b) 3 (c) 4 (d) 1

4 $5x^2 =$ ....

(a) $5 \times x \times x$ (b) $5 \times x$ (c) $5x + x$ (d) $5x \times x^2$

5 The area of the opposite rectangle equals ....



(a) $x^3 - 1$ (b) $x^3 - x$

(c) $x^2 - 1$ (d) $x^3 - x^2$

6 If the edge length of a cube is 3 a cm. , then its volume is .... cm^3.

(a) 6 a (b) $9a^2$ (c) $27a^3$ (d) 9 a

2 Complete the following :

1 $12y^5 = 3y \times$ ....

2 $3a($ .... $- 5ab^2) = 6a^2b -$ ....

3 The degree of the algebraic expression : $3xy + 5x^2y + 7$ is ....

4 $6a^2 +$ .... $=$ zero

5 If the edge length of a cube is $5x$ cm. , then its lateral area equals .... cm^2.

3 [a] Reduce to the simplest form :

$4n(n+5) + n(6-n)$, then find the numerical value of the result when $n = -1$

[b] What is the increase of the algebraic expression : $3x^2 - 5x + 2$ than the sum of the two algebraic expressions : $x + 5x^2 + 1$ and $2x^2 - 4 - 2x$?

4 [a] Simplify : $5x + 10y + 6x - 3y + 7y - 4x$

[b] Find the result of each of the following where the divisor ≠ zero :

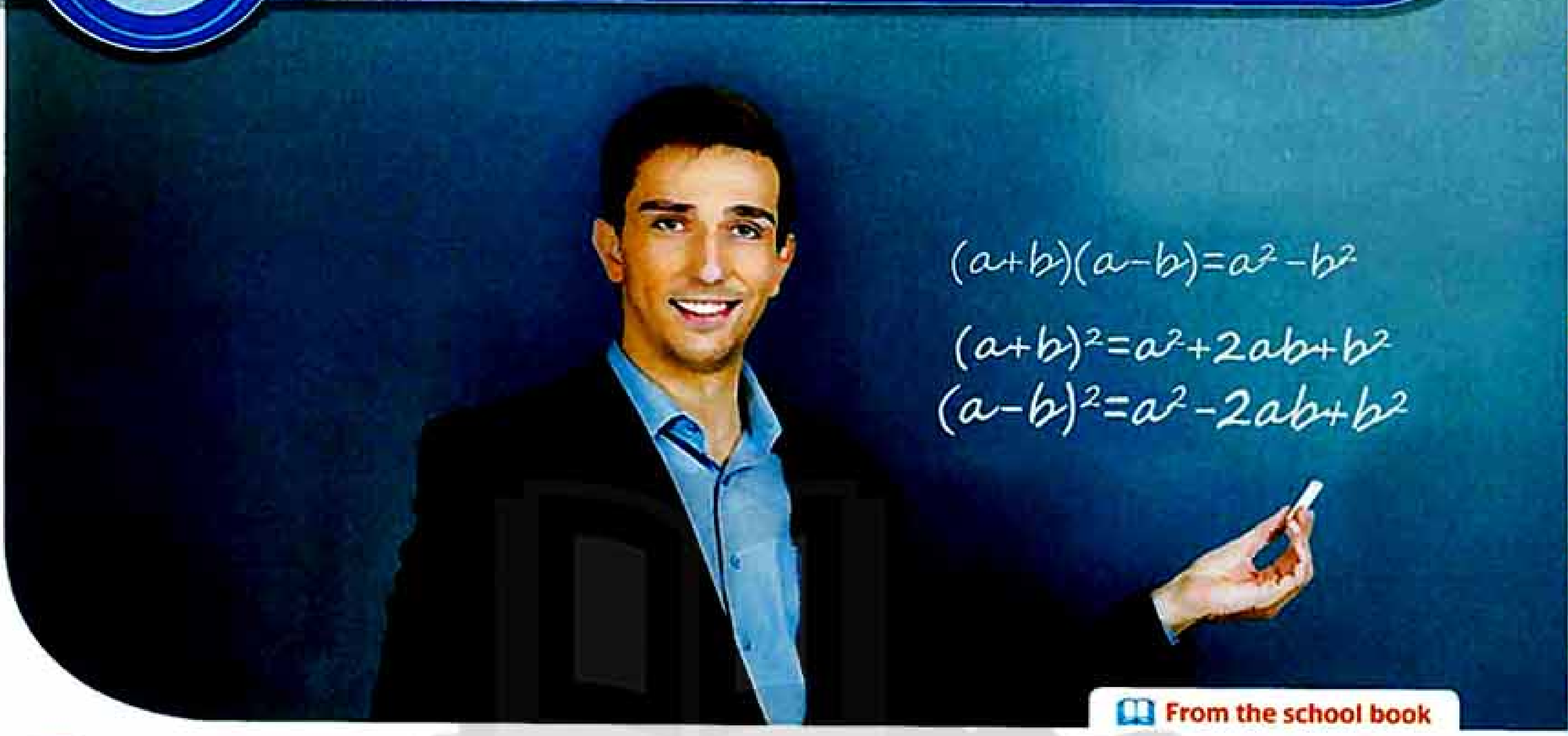
1 $9x^6y^3 \div 6x^4y$ 2 $(10x^2 + 5x^2) \div 3x$

5 [a] Simplify : $\frac{1}{3}x^2(9x^2 - 12xy + 6y^2)$

[b] An equilateral triangle of side length $(2x - 3y + 5z)$ cm. , find its perimeter.

EXERCISE 11

Multiplying a Binomial by an Algebraic Expression



From the school book

1 **Write the missing terms in each of the following products :**

1. $(x+3)(x+2) = \cdots\cdots + 5x + 6$
2. $(x+2)(x-5) = x^2 \cdots\cdots - 10$
3. $(y-4)(y+5) = \cdots\cdots + y - \cdots\cdots$
4. $(a-3)(a-7) = a^2 - \cdots\cdots + \cdots\cdots$
5. $(2x-5)(x+7) = \cdots\cdots + \cdots\cdots - 35$
6. $(4x-3y)(2x+5y) = 8x^2 + \cdots\cdots - 15y^2$

2 **Find by direct product , the result of each of the following :**

1. $(x+2)(x+4)$
2. $(y-5)(y+2)$
3. 📖 $(5m-2)(6m+1)$
4. 📖 $(4x+1)(2x+3)$
5. $(3a+2b)(2a-5b)$
6. $(2x-y)(3x+4y)$
7. $(b^2-4)(b^2+2)$
8. $(3m^2+8)(2m^2-3)$
9. $(x-y)(7y-x)$
10. $\left(\frac{3}{2}a-6b\right)\left(\frac{3}{2}a+4b\right)$

3 **Find by inspection the expansion of each of the following :**

1. $(a+3)^2$
2. $(2y+3)^2$
3. $(4m-7)^2$
4. 📖 $(3x+y)^2$
5. 📖 $(x-3y)^2$
6. $(-\ell-m)^2$
7. $(-4a-7)^2$
8. 📖 $(2x+3y)^2$
9. $\left(4x^2-\frac{1}{2}y^2\right)^2$

UNIT 2

4 **Find by the direct product the result of each of the following :**

1. $(a + 3)(a - 3)$
2. $(4m - 7)(4m + 7)$
3. $(6x - 2y)(6x + 2y)$
4. $(a^2 + 9)(a^2 - 9)$
5. $(3x^2 - 5y^2)(3x^2 + 5y^2)$
6. $(\ell m + 6n)(\ell m - 6n)$
7. $(\frac{1}{2}x - \frac{1}{3}y)(\frac{1}{2}x + \frac{1}{3}y)$
8. $(2x - 3y)(3y + 2x)$
9. $(y - 3)(y + 3)(y^2 + 9)$
10. $(x - 2y)(x + 2y)(x^2 + 4y^2)$

5 **Find the following products :**

1. $(x + 3)(x^2 + x + 1)$
2. $(x + 1)(x^2 - x + 1)$
3. $(2y + 1)(y^2 + y + 5)$
4. $(2x + 3)(4x^2 - 6x + 7)$
5. $(2x - y)(2x^2 - 3xy + y^2)$
6. $(a^2 - 3b^2)(3a^4 - 2a^2b^2 + 5b^4)$
7. $(2a + a^2 - 5)(2a^2 - 1)$
8. $(4 + 2a + 3a^2)(2 - a)$
9. $(x + 4)^2(3x + 2)$
10. $(3x + 2y)^3$

6 **Choose the correct answer from the given ones :**

1. The middle term in the expansion of $(3x - 1)^2$ is ....
   (a) $3x$ (b) $-6x$ (c) $6x$ (d) $6x^2$
2. The middle term in the expansion of $(2a + 3b)^2$ is ....
   (a) $12ab$ (b) $-12ab$ (c) $6ab$ (d) $-6ab$
3. If $x = -1$, then the numerical value of the expression $(x + 1)^2$ is ....
   (a) zero (b) 1 (c) 2 (d) 3
4. If $x = \frac{4}{3}$, then $(x - 2)(x + 2) =$ ....
   (a) $\frac{4}{3} - 2$ (b) $(\frac{4}{3})^2 - 2$ (c) $(\frac{4}{3})^2 - 4$ (d) $(\frac{4}{3})^2 + 4$
5. If $x - y = 3$ and $x + y = 5$, then $x^2 - y^2 =$ ....
   (a) 2 (b) -2 (c) 8 (d) 15
6. If $(x + y)^2 = 26$ and $x^2 + y^2 = 20$, then $xy =$ ....
   (a) 3 (b) 6 (c) 9 (d) 12
7. If $x^2 = 16$, $y^2 = 9$ and $xy = 12$, then $(x - y)^2 =$ ....
   (a) 49 (b) 165 (c) -1 (d) 1
8. If $x + y = 7$, then the numerical value of the expression $x^2 + 2xy + y^2 =$ ....
   (a) 7 (b) 14 (c) 49 (d) 28
9. If $(2x + y)^2 = 4x^2 + kxy + y^2$, then $k =$ ....
   (a) 2 (b) 4 (c) 8 (d) 6

Exercise 11

10 If $(x-3)(x+3) = x^2 + k$, then $k = \dots\dots$

(a) 9 (b) 6 (c) -9 (d) -6

11 If $(x-y)(2x+y) = 2x^2 + k\,xy - y^2$, then $|k| = \dots\dots$

(a) -1 (b) 1 (c) 3 (d) 4

7 Complete the following :

1 $(2x-1)^2 = \dots\dots - 4x + 1$

2 $(x-5)(\dots\dots) = x^2 - 25$

3 $(3x + \dots\dots)(\dots\dots - y) = 9x^2 - y^2$

4 $(x+5)(x + \dots\dots) = x^2 + \dots\dots + 15$

5 $(a + \dots\dots)^2 = \dots\dots + \dots\dots + 16$

6 $(2x+5)(3x + \dots\dots) = \dots\dots + \dots\dots + 10$

7 $(4x + \dots\dots)(\dots\dots - 5) = 8x^2 - \dots\dots - 5$

8 $(2a + \dots\dots)(\dots\dots - 5b) = 8a^2 + \dots\dots - 15b^2$

9 $(\dots\dots + 4)(x + \dots\dots) = x^2 + 7x + \dots\dots$

10 $(\dots\dots - 3b)^2 = \dots\dots - 24ab + \dots\dots$

8 Reduce to the simplest form :

1 $(x-3)^2 - 9$

2 $2a(5a+4b)(5a-4b)$

3 $3(m-5)(m+2)$

4 $(x-2)(x+2) - x(x+1)$

5 $(2x+3)(2x-3) - 2(2x^2+1)$

6 $(2x+3)^2 + (x-2)(x+5)$

7 $(x+1)^2 - x(x+2)$

8 $(x-2)^2 - (x^2-4)$

9 $(2x-y)(2x+y) - (x-2y)^2$

10 $2a(3a+b) + (a-b)^2$

11 $(5x-2y)^2 - (5x+2y)^2$

12 $2(3x-5)(2x+1) - 3(4x+1)(x-7)$

9 Multiply, then find the numerical value of the expression when $x = 1$ and $y = -2$:

1 $(x-5y)(x+5y)$

2 $(3x+y)(x+3y)$

3 $(x+4)(3x+2)$

4 $(2y+7)(3y+4)$

5 $|(x+2y)(x-2y)|$

10 Reduce : $(x-y)^2 + 2xy$, then find the numerical value of the result when $x = -1$, $y = 2$ « 5 »

11 Reduce : $(2x-2)^2 + (x-2)(x+2)$, then find the numerical value of the result when $x = -1$ « 13 »

12 Subtract : $(x-3)^2$ from $(2x+1)(x+9)$

UNIT 2

13 If $a = 3X - 4$ and $b = X + 2$ and $c = 2X - 3$,

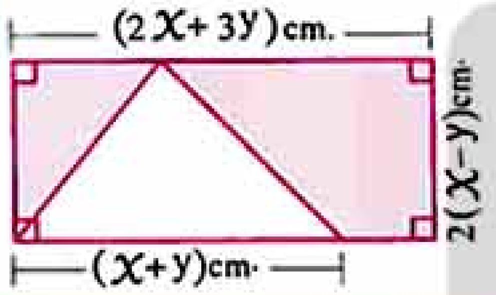
find the value of the expression $a\,b - c^2$, when X = zero « −17 »

14 If $a = 4X - 3$, $b = 2X + 1$ and $c = 3X - 2$,

find the value of the expression $2a^2 - 3b^2 + b\,c$ in terms of X

Applications on multiplying of algebraic expressions

15 **Find the area of the coloured part in each of the following figures :**

1


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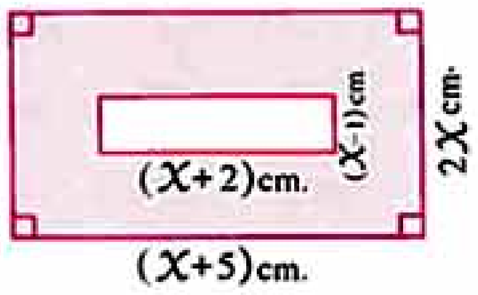
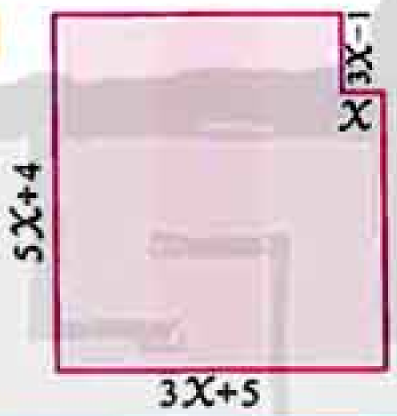


16 **Write an expression for the perimeter and area of each coloured region :**

1


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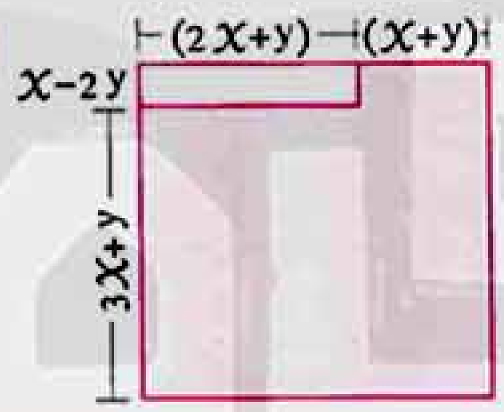

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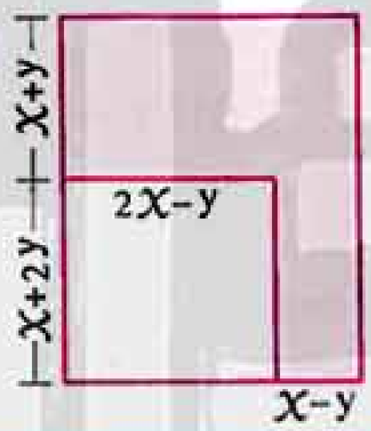


17 **Use the multiplication by inspection to find the following easily :**

1. $(101)^2$
2. $(10\frac{1}{2})^2$
3. $(99)^2$
4. 64×56
5. 98×102
6. 19×21
7. 201×199
8. $(49)^2$
9. $(41)^2$

For excellent pupils

18 If $X = 2a - 5b$, $y = 3a + 4b$ and $z = a - 3b$,

find in terms of a and b the value of the expression $y\,(3X - 2z)$

19 If $(2 - y)^3 = 8 - 12y + 6y^2 - y^3$, **find :** $(2 - y)^4$

20 If $a = 5X + 2y$, $b = 2X + y$ and $c = X - 3y$, **prove that :** $a\,(b + c) = a\,b + a\,c$

21 A square whose side length is $(2X + 5)$ cm. Find its area in terms of X

If two opposite sides of it increases by $(X - 1)$ cm. and the two other sides decreases by the same value , find in terms of X the area of the resulting rectangle.

EXERCISE 12

Dividing an Algebraic Expression by a Monomial

From the school book

1 **If the symbols , in the following expressions are non-zero numbers , find the quotient in each case :**

1. $5a - 10$ by 5
2. $12x + 15y$ by -3
3. $4a^2 + 6a$ by $2a$
4. $24x^3 - 18x^2$ by $-6x^2$
5. $12a^2b + 20ab^2$ by $4ab$
6. $16a^3b^2 - 24a^2b^2$ by $4a^2b$
7. $60x^6 - 48x^{10} - 12x^3$ by $-12x^3$
8. (From the school book) $32x^5 - 48x^3 + 72x^7$ by $-8x^3$
9. $3a^2b - 6ab^2 + 12ab$ by $-3ab$
10. $2a^3b^2 - 4a^2b^3 + 6a^2b^2$ by $2b^2a^2$

2 **If the symbols represent non-zero integers , find the quotient of each of the following :**

1. (From the school book) $\dfrac{26x^2 + 14x^4}{2x}$
2. (From the school book) $\dfrac{18m^4 + 32m^2}{-2m^2}$
3. (From the school book) $\dfrac{48x^3 - 80x^2}{8x^2}$
4. (From the school book) $\dfrac{9\ell^3m^4 - 18\ell m^2}{3\ell m^2}$
5. $\dfrac{16a^3 - 12a^2 + 8a}{-4a}$
6. $\dfrac{15x^3y^2 + 6xy^2 - 3xy}{-9xy}$
7. $\dfrac{5\ell^2m^3n - 20m^2n^3 - 15mn^5}{-5n}$
8. $\dfrac{18x^4y^2 - 42x^5y^4 + 30x^6y^5}{-6x^2y^2}$

UNIT 2

3 Choose the correct answer from those given :

1. $(x^2 + x) \div x = \cdots\cdots$ $(x \neq 0)$

(a) zero (b) x (c) $2x + 1$ (d) $x + 1$

2. $(15a + 5) \div 5 = \cdots\cdots$

(a) $3a$ (b) $10a$ (c) $3a + 1$ (d) $4a$

3. $(4a^3 - 2a) \div (-2a) = \cdots\cdots$ $(a \neq 0)$

(a) $-2a^2$ (b) $-2a^2 + 1$ (c) $2a^2 + 1$ (d) -1

4. $(15x^4 + 5x^3) \div 5x^3 = \cdots\cdots$ $(x \neq 0)$

(a) $3x^2 + x$ (b) $5x^2 + 1$ (c) $3x + 1$ (d) $4x^4$

5. $(3x^2y - \cdots\cdots) \div 3xy = x - 2y$ $(xy \neq 0)$

(a) $6x$ (b) $6xy^2$ (c) $6y^2$ (d) $-6xy^2$

6. If $(6x^2y^3 + kxy) \div 6x = xy^3 - 12y$ where $(x \neq 0)$, then $|k| = \cdots\cdots$

(a) -72 (b) -2 (c) 2 (d) 72

4 Complete the following :

1. $\dfrac{15n^3 - 9m^4n^2}{-3n^2} = \dfrac{15n^3}{-3n^2} + \dfrac{-9m^4n^2}{-3n^2} = \cdots\cdots + \cdots\cdots$

2. $(4a^2 + 2a) \div 2a = \cdots\cdots$

3. $\dfrac{4x^2y - 2xy^2}{-2xy} + 2x - y = \cdots\cdots + 2x - y = \cdots\cdots$

4. $\dfrac{16x^4y^2 - 12x^3y^3 + 24x^2y^4}{8x^2y} = \dfrac{16x^4y^2}{8x^2y} - \dfrac{\cdots\cdots}{8x^2y} + \dfrac{\cdots\cdots}{8x^2y}$

$= \cdots\cdots - \cdots\cdots + \cdots\cdots$

5. $(12a^3b^3 - 3a^5b^4) \div \cdots\cdots = 4ab - \cdots\cdots$

6. $\dfrac{\cdots\cdots - 6ab^2 + 12ab}{\cdots\cdots} = ab + 2b - 4$

7. If $\dfrac{4x^2y - 8xy^4}{-4xy} = kx + \ell y^3$, then $|k| + \ell = \cdots\cdots$

8. If $x = -1$, then $\left|\dfrac{-4x^2 + 20x^3}{-4x}\right| = \cdots\cdots$

Exercise 12

5 **Multiply :** $4x^2$ **by** $(3x^3y^3 - 6x^2y^3)$, then divide the result by $12x^4y^2$

6 Add the quotient of : $(x^3y - 3xy + 2x^2y^2 - 4xy)$ **by** $-xy$

to the expression $2xy - 5x^2 + 3y^2$

7 **Divide :** $(12y^3 - 8y^2)$ **by** $4y$, then find the absolute value of the result when $y = \frac{1}{2}$ « $\frac{1}{4}$ »

8 **Divide :** $(12x^3y^2 - 4x^2y^3)$ **by** $4x^2y^2$,

then find the numerical value of the result when $x = 1$ and $y = -1$ « 4 »

9 **Divide :** $(16x^3 + 8x - 12x^2)$ **by** $4x$, then add the result to $3x - x^2 + 7$,

and find the numerical value of the result when $x = 1$ « 12 »

Geometric Applications

10 The area of a rectangle is $(24x^3 + 18x^2 + 42x)$ cm^2 and its width is $6x$ cm.

Find the length of the rectangle in terms of x

11 The area of a rectangle is $(8a^4b^3 + 12a^3b^4 - 8a^2b^2)$ cm.^{2} and its length is $(4a^2b^2)$ cm.

Find its width if $a = 1$ and $b = 2$ « 14 cm. »

12 The area of a triangle is $(12x^2 + 9x)$ cm^2 and the length of its base is $3x$ cm.

Find the height of the triangle corresponding to this base.

For excellent pupils

13 The volume of a cuboid is $(12x^3 + 8x^2y)$ cm^3 and its base is a square of side length

$2x$ cm. Find its height when $x = 1$ and $y = 2$ « 7 cm. »

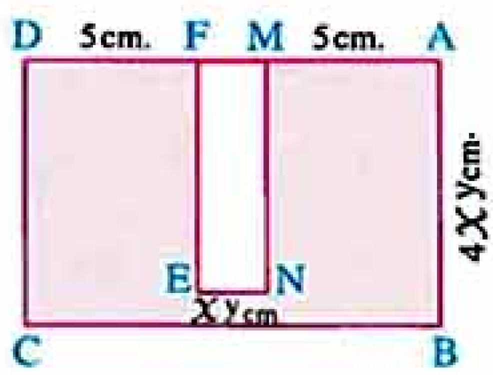
14 **In the opposite figure :**

ABCD and MNEF are two rectangles.

Use the given data on the figure to find

the length of $\overline{FE}$ given that the area of the

coloured part is $(3x^2y^2 + 35xy)$ cm^2



EXERCISE 13 Dividing an Algebraic Expression by Another One

From the school book

1 **Find the quotient of dividing of each of the following expressions , given the divisor in each $\neq 0$:**

1. x^2+5x+6 by $x+2$
2. $y^2-9y+20$ by $y-4$
3. $x^2-5x-14$ by $x-7$
4. $2x^2+13x+15$ by $x+5$
5. $3x^2+2x-8$ by $3x-4$
6. x^2-6-x by $x+2$
7. $14-17x-6x^2$ by $7+2x$
8. $8x^2+6xy-9y^2$ by $4x-3y$
9. $4x^2-16xy+16y^2$ by $2x-4y$
10. x^2-1 by $x+1$
11. $16y^2-4x^2$ by $4y-2x$

2 **Find the quotient of dividing of each of the following expressions , given the divisor in each $\neq 0$:**

1. x^3+5x^2+7x+2 by x^2+3x+1
2. $6x^3+7x^2-18x+5$ by $3x^2-4x+1$
3. $2x^3-43x-9x^2-20$ by x^2-4-7x
4. $3x^2+x^3-x-3$ by x^2-1
5. $8x^3-20x^2-10+4x$ by $4x^2+2$
6. x^4+3x^2+2 by x^2+1
7. x^3-x by $x-1$
8. $8x^3-1$ by $4x^2+2x+1$

Exercise 13

3 **Find the quotient of dividing of each of the following expressions, given the divisor in each $\neq 0$:**

1. $x^3 + 5x^2 + 7x + 2$ **by** $x + 2$
2. $x^3 - x^2 - 9x - 12$ **by** $x - 4$
3. $6x^3 - 5x^2 - 14x + 12$ **by** $2x - 3$
4. $9x + 6x^3 + 10 - 5x^2$ **by** $2 + 3x$
5. $15 - 7x^2 + 3x - 4x^3$ **by** $5 - 4x$
6. $3x^3 - 4x + 1$ **by** $x - 1$
7. $x^3 - 27$ **by** $x - 3$
8. $27a^3 - 8$ **by** $3a - 2$
9. $x^4 + 49 - 18x^2$ **by** $2x - 7 + x^2$
10. $37x^2 - 4 - 9x^4$ **by** $3x^2 - 2 + 5x$

4 **Find the quotient of dividing of each of the following expressions, given the divisor in each $\neq 0$:**

1. $13xy + 6(x^2 + y^2)$ **by** $2x + 3y$
2. $a^4 - 16ab^3 - 6a^2b^2 - 15b^4$ **by** $a^2 - 2ab - 5b^2$

5 If $x + 3$ is one factor of $2x^2 + 3x - 9$, find the other factor.

6 If $x^2 + 3x + 3$ is one of the factors of $x^3 - x^2 - 9x - 12$, find the other factor.

7 Find the sum of $3x^3 - 5x^2 + 7x + 1$ and $3x^3 - x + 7$,
then divide the result by $3x + 2$

8 Find the quotient of dividing $2x^3 - x^2 - 2x + 6$ by $2x + 3$,
then find the numerical value for the quotient when $x = 1$ « 1 »

9 Find the value of m that makes the expression $2x^2 - 7x + m$ divisible by $x - 2$ « 6 »

10 Find the value of k that makes the expression $x^3 - 3x^2 - 25x + k$ divisible by $x^2 + 4x + 3$ « – 21 »

11 Find the value of k that makes the expression $6x^3 - 13x^2 - 13x + k$ divisible by $3x - 5$ « 30 »

12 What is the expression that if multiplyed by $x^2 + x + 2$
the result will be $x^3 + 2x^2 + 3x + 2$?

UNIT 2

Geometric Applications

13 If the area of a rectangle is $(15 x^2 + 11 x - 14)$ cm^2 and its width is $(3 x - 2)$ cm. , calculate its length $\left(x > \frac{2}{3}\right)$

14 If the area of a rectangle is $(2 x^2 + 7 x - 15)$ square units and its length is $(x + 5)$ length units, then find its width and calculate its perimeter when $x = 3$ « 3 , 22 »

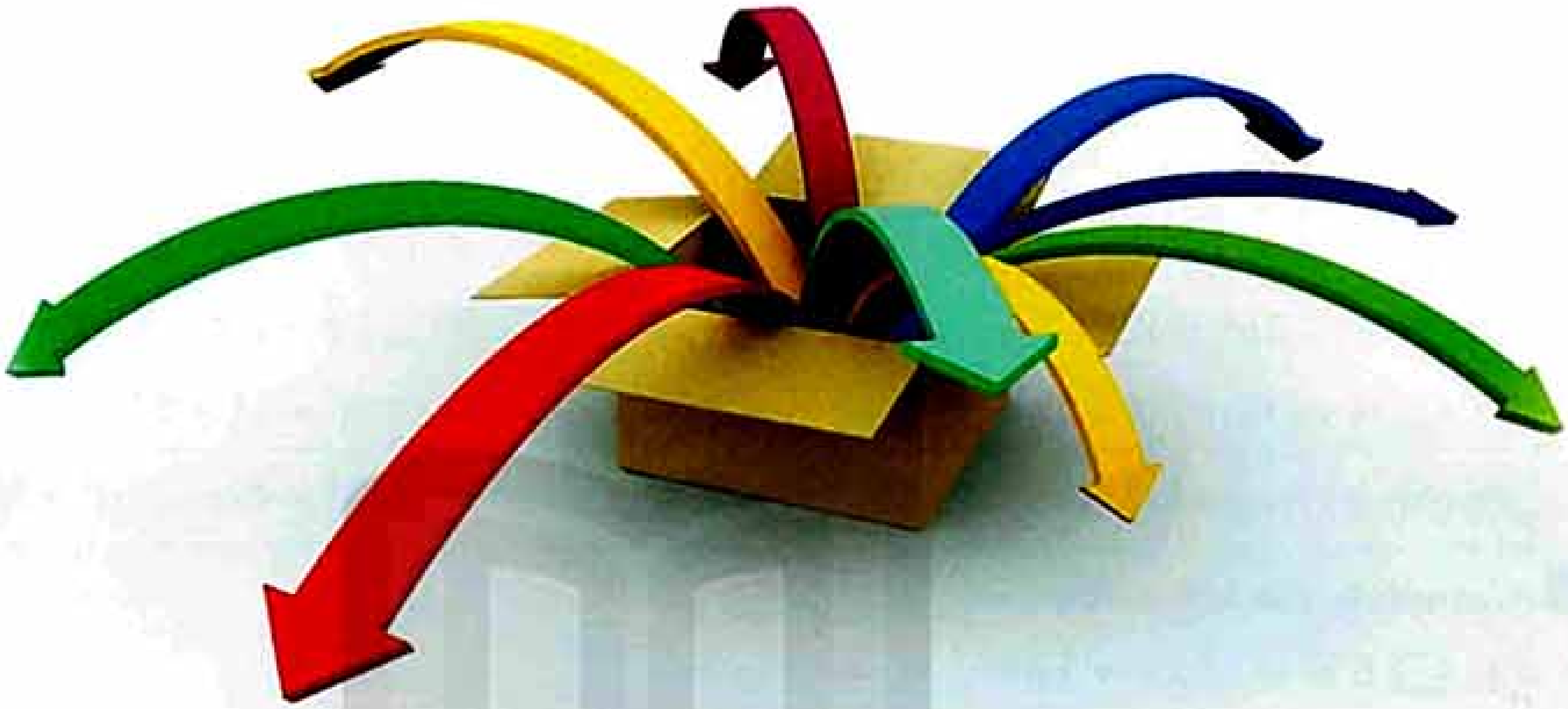
For excellent pupils

15 Find the value of the number k that makes the expression $x^2 - k x + 12$ divisible by $x - 4$
« 7 »

16 Find the number that if added to the expression $6 x^2 - 11 x - 17$, it will become divisible by $2 x - 5$ « 7 »

17 ABC is a triangle , if its area is $(6 x^2 + 7 x + 2)$ cm^2 and the length of $\overline{BC}$ equals $(2 x + 1)$ cm. , find the corresponding height to the side $\overline{BC}$

EXERCISE 14 Factorization by Identifying the H.C.F.



📖 From the school book

1 **Factorize each of the following by identifying the H.C.F. :**

1. $5a + 5b$
2. $3x - 3y$
3. 📖 $5y - 10$
4. 📖 $8y^3 - 4x^2$
5. $7xy + 7yz$
6. $5ab - 15bc$
7. 📖 $3x^2 + 6x$
8. 📖 $35a + 10a^2$
9. $6a^3 - 4a^2b^2$
10. 📖 $49b^2 - 7b^3$
11. $35x^3y + 5xy^2$
12. $15a^3b - 5a^2b^2$

2 **Factorize each of the following by identifying the H.C.F. :**

1. $5a - 5b + 5c$
2. $6a + 8b + 10c$
3. $x^3 + 2x^2 + 5x$
4. $8a^3 - 4a^2 + 6a$
5. $2x^2y + 6xy^2 - 2y$
6. 📖 $9m^4n^2 - 6m^3n^3 + 12m^2n^4$
7. 📖 $-2x^5 + 4x^2 - 6x + 2x^3$
8. $32x^3y^3 + 16x^2y^2 + 8xy$
9. 📖 $18a^2bc - 6abc + 30abc^2 - 24ab^2c^2$
10. 📖 $15a^3b^4 + 6a^5b^3 - 3a^2b^2$

3 **Factorize each of the following by identifying the H.C.F. :**

1. 📖 $3x(a + b) + 7(a + b)$
2. $a(a + 3) + b(a + 3)$
3. 📖 $(x + 4)x^2 + (x + 4)y^2$
4. $14a(x + y) - 21b(x + y)$
5. $6a^2(x - 1) - 8a(x - 1)$
6. $12x^2(x + 1) - 8xy(x + 1)$
7. $24a^2b^3(a - 2) - 36a^3b^2(2 - a)$
8. 📖 $3x^2(x - 7) + 2x(x - 7) + 5(x - 7)$
9. 📖 $4m^2(2x + y) - 3m(2x + y) - 7(2x + y)$
10. $16a^2b^2(a + b + 2) - 8a^2b(a + b + 2)$

UNIT 2

4 **Find the result by identifying the H.C.F. :**

1. $48 \times 45 + 48 \times 55$
2. $52 \times 43 - 52 \times 33$
3. $7 \times 123 + 7 \times 35 - 7 \times 18$
4. $15 \times 17 + 15 \times 13 - 15 \times 30$
5. $12 \times 5 + 12 \times 4 + 12$
6. $35 + 14 \times 35 - 5 \times 35$
7. $\frac{5}{18} \times 11 + \frac{35}{18}$
8. $(58)^2 + 58 \times 42$
9. $(256)^2 - 256 \times 156$
10. $6 \times (15)^2 + 18 \times 15 - 8 \times 15$
11. $5 \times (48)^2 + 7 \times 48 + 53 \times 48$
12. $(31)^2 + 31 \times 23 - 31 \times 54$
13. $(51 \times 17 + 51 \times 33) + (49 \times 21 + 49 \times 29)$
14. $(49)^2 + 49 + (50)^2 + 50$

5 **Complete the following :**

1. $6a^2 + 12ab = 3a$ (.......... + ...)
2. $a^2 b + b^2 a =$ .... $(a + b)$
3. $12x^2 y - 16xy^2 =$ .... ($3x -$ ....)
4. $x(a + b) + y(a + b) =$ (.......... + ...) $(a + b)$
5. $3(a - b) - 4(b - a) =$ .... $(a - b)$
6. $x(a + 1) - y(a + 1) = (a + 1)$ (.......... − ....)
7. If $a + b = 3$, then $5a + 5b =$ ....
8. If $7x - 7y = 21$, then $x - y =$ ....
9. $20x^2 + \frac{15x^2}{3x} = 5x$ (.......... + ...) , $(x \neq 0)$
10. If $x + y = 5$, then $x(x + y) + y(x + y) =$ ....

6 **Choose the correct answer from those given :**

1. $3x - 9x^2 =$ ....
   (a) $12x$ (b) $-6x$ (c) $-6x^2$ (d) $3x(1 - 3x)$
2. $7x^2 + 14y^2 = 7$ (..................)
   (a) $x^2 + y^2$ (b) $x^2 + 2y^2$ (c) $7x^2 + y^2$ (d) $x + 2y$
3. $4x^2 y^2 - 2xy^2 + 4x^2 y =$ .... $(2xy - y + 2x)$
   (a) $4xy$ (b) $2xy$ (c) $2x$ (d) $2y$
4. The factorization of $6x^2 y - 4x$ by identifying the H.C.F. is ....
   (a) $3xy(x + y)$ (b) $2xy(3y - 2)$ (c) $2xy(3x - 2)$ (d) $2x(3xy - 2)$
5. $(75)^2 + 75 \times 25 =$ ....
   (a) 75 (b) 750 (c) 7500 (d) 75000

6 $8 + 8^2 = 8 \times$ ....

(a) 8 (b) 9 (c) 80 (d) 90

7 The highest common factor of the expression $12\,x^3\,y^4 + 8\,x^2\,y^3$ is ....

(a) $2\,x^2\,y^3$ (b) $4\,x^2\,y^3$ (c) $4\,x^3\,y^4$ (d) $12\,x^3\,y^4$

8 If $2\,a^2\,b - a\,b = a\,b\,(2\,a + k)$, then $|\,k\,| =$ ....

(a) zero (b) 1 (c) -1 (d) 2

7 If $2\,a + b = 3$, find using the factorizing by identifying the H.C.F. the numerical value of the expression $2\,a\,(2\,a + b) + b\,(2\,a + b)$ « 9 »

8 If $a + c = -3$, find using the factorizing by identifying the H.C.F. the absolute value of the expression $2\,a\,(a + c) + 2\,c\,(a + c)$ « 18 »

9 If $x + y = 3$ and $b - a = 4$, find the numerical value of the expression $a\,(x + y) - b\,(x + y)$ « -12 »

10 **Use the factorization by identifying the H.C.F. to find the value of the following easily :**

1 $\dfrac{(19)^2 - 2 \times 19 + 19}{9}$ 2 $\dfrac{5 \times (9)^2 + 11 \times 9 - 9}{45}$ 3 $\left|\dfrac{(36)^2 \times 5 - 3 \times (36)^2}{-2 \times (36)^2}\right|$

11 If $3\,a^2\,b^2$ is one of the factors of the expression : $12\,a^2\,b^2\,c - 6\,b^3\,a^2\,c^3 + 9\,a^2\,b^2$, find the other factor.

Geometric Application

12 **In the opposite figure :**

Write in two different ways the algebraic expression which expresses the area of the rectangle.

For excellent pupils

13 If $b = 8$, find the value of : $b\,(x - 1) + b\,(c - 2\,x) - b\,(c - x)$ « -8 »

14 If $2\,x + 3\,y = 2$ and $m\,(4\,x + 6\,y) + 2\,n\,(2\,x + 3\,y) = 16$, find the value of $m + n$ « 4 »

15 If $a\,b\,c = 12$ and $a + b + c = 8$, find the numerical value of the expression $a^2\,b\,c + a\,b^2\,c + a\,b\,c^2 - a\,b\,c$ « 84 »

Summary of the second part of unit 2
"From lesson 6 to lesson 9"

- $(a + b)(c + d) = ac + [ad + bc] + bd$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- When we divide an algebraic expression by a monomial, we divide each term of the expression by this monomial.
- To operate the division of an algebraic expression by another one, we should arrange each of the dividend and the divisor either in a descending order or in an ascending order according to the powers of the given symbol (It is preferable to arrange in a descending order).
- To factorize an algebraic expression by identifying the highest common factor (H.C F.) :

1. Find H.C.F. of the algebraic terms of the expression.
2. Put H.C.F. outside two brackets.
3. Divide each term of the algebraic expression by the H.C.F. and write the quotients inside the two brackets.

Exams on the second part of unit two from lesson (6) to lesson (9)

Model 1

Answer the following questions :

1 Choose the correct answer from the given ones :

1. If $(x-4)(x+4) = x^2 + m$, then $m =$ ....
   (a) 16 (b) -16 (c) zero (d) 8
2. The highest common factor of the algebraic expression : $3x^2y - 6x$ is ....
   (a) $3x$ (b) $6x$ (c) $3xy$ (d) $xy - 2$
3. $(12a - 3) \div 3 =$ ....
   (a) $4a - 3$ (b) $4a$ (c) $4a - 1$ (d) 4
4. The middle term in the expansion of $(2x - 5)^2$ is ....
   (a) $10x^2$ (b) $-10x$ (c) $20x$ (d) $-20x$
5. If $x - y = 5$, then $6x - 6y =$ ....
   (a) 11 (b) 30 (c) 1 (d) -1
6. If $x^2 = 25$, $y^2 = 9$, $xy = 15$, then $(x+y)^2 =$ ....
   (a) 94 (b) 4 (c) 30 (d) 64

2 Complete the following :

1. $(y-5)(y+3) =$ .... $- 2y -$ ....
2. $(24x^4 + 18x^3) \div 6x^3 =$ .... (where $x \neq 0$)
3. $7x^2 - 21y = 7($ .... $)$
4. If $x + y = 5$, then the numerical value of the expression : $x^2 + 2xy + y^2$ is ....
5. If $(x+4)(x-3) = x^2 + m - 12$, then $m =$ ....

3 **[a] Factorize the following by identifying the H.C.F. :** $15x^2y - 25xy^2 + 10xy$

[b] Find the quotient of dividing : $30x^3y^2 - 15x^2y^3 + 20xy$ **by** $5xy$ (where $xy \neq 0$)

4 **[a] Reduce to the simplest form :** $(x+3)^2 - x(x+9)$

[b] Find the value of k that makes the expression : $x^2 + 5x + k$ divisible by $x + 2$

5 **[a] Reduce to the simplest form :** $(x-5)(x+5) + 25$

[b] Use the multiplication by inspection to find the value of each of the following easily :

1. 202×198
2. $(31)^2$

UNIT 2

Model 2

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1. $4a^2 b^2 - 2ab + 6ab^2 = \cdots\cdots\cdots (2ab - 1 + 3b)$

   (a) $4ab$ (b) $2a^2b^2$ (c) $2ab$ (d) 2

2. If $(3x + y)^2 = 9x^2 + kxy + y^2$, then $k = \cdots\cdots\cdots$

   (a) 3 (b) 9 (c) 4 (d) 6

3. $(10x^2 - 15x^2 y) \div 5x = \cdots\cdots\cdots$ (where $x \neq 0$)

   (a) $2x - 3xy$ (b) $2x - 3y$ (c) $2x + 3y$ (d) $2x - 3$

4. If $(x - 5)(x + 5) = x^2 + k - 16$, then $k = \cdots\cdots\cdots$

   (a) 9 (b) -25 (c) 25 (d) -9

5. The highest common factor of the expression : $12a^2b^3 + 8a^3b^2$ is $\cdots\cdots\cdots$

   (a) $6a^2b^2$ (b) $8ab$ (c) $2a^2b^2$ (d) $4a^2b^2$

6. $(6x^2 y - \cdots\cdots\cdots) \div 2xy = 3x - 4xy^2$

   (a) $2y$ (b) $8xy$ (c) $8x^2y$ (d) $8x^2y^3$

2 **Complete the following :**

1. $(a - 2)(a - 7) = a^2 - \cdots\cdots\cdots + \cdots\cdots\cdots$
2. The last term of the expression : $(4a - 5b)^2$ is $\cdots\cdots\cdots$
3. $x(a + b) - y(a + b) = (a + b)(\cdots\cdots\cdots)$
4. $(3x - 1)^2 = \cdots\cdots\cdots - 6x + 1$
5. If $3x + 4y = 7$, then $9x + 12y = \cdots\cdots\cdots$

3 **[a] Find the quotient of dividing :** $12x^3 + 18x^2 - 6x$ by $6x$ (where $x \neq 0$)

[b] Factorize by identifying the H.C.F : $27x^4 - 18x^3$

4 [a] A rectangle , its area is $(3x^2 + 17x + 10)$ square units , if its length is $(3x + 2)$ length units , then find its width , then calculate its perimeter when $x = 3$

[b] Simplify : $(a + b)^2 - 2ab$, then find the numerical value of the result at : $a = 1$, $b = -2$

5 **[a] Use the factorization by identifying the H.C.F. to find the value of the following easily :**

$(157)^2 - 157 \times 57$

[b] Simplify :

$(2x - 1)^2 + (x + 1)(x - 1)$, then find the numerical value of the result at : $x = -1$

UNIT 3 Statistics

Exercises of the unit :

15. The arithmetic mean.
16. The median.
17. The mode.

- Activities from the school book.
- Summary of unit three.
- Unit exams.

EXERCISE 15

The Arithmetic Mean

From the school book

1 Find the arithmetic mean for each set of the following :

1. 4 , 6
2. 3 , 5
3. 3 , 4
4. 2 , 4 , 6
5. 1 , 3 , 5
6. 1 , 2 , 3 , 4 , 5
7. 6 , 10
8. $\frac{1}{2}$, 1
9. 35 , 50 , 60 , 55

2 If the heights of 5 students in grade 1st prep. in cm. are 124 , 130 , 122 , 126 , 128 , calculate the arithmetic mean of the heights of these students.

3 If the marks of Sherif in 3 consecutive months in maths tests are as the following : 89 , 91 and 96 , calculate the mean of the monthly marks for this student.

4 If the temperature degrees for a week in one of the cities in December are :
25° , 27° , 31° , 23° , 22° , 22° and 18°
, calculate the arithmetic mean of these degrees.

5 If the number of goals registered by Al Zamalek in 6 matches are 3 , 2 , zero , 6 , 1 , 6
, calculate the arithmetic mean of the number of goals.

Exercise 15

6 **If the numbers of studying hours for one of the students during 5 consecutive days are as the following :**

The day	Saturday	Sunday	Monday	Tuesday	Wednesday
Number of studying hours	$3\frac{1}{2}$	3	$2\frac{1}{2}$	3	4

Find the mean of the daily number of studying hours.

7 **Complete the following :**

1. The arithmetic mean of the values 18 , 35 , 24 , 6 is ....
2. The arithmetic mean of the values $2-a$, 4 , 1 , 5 , $3+a$ is ....
3. The arithmetic mean of the values $x+y$, $9-y$, $-x$ is ....
4. If the arithmetic mean of the numbers 3 , 5 , x is 4 , then $x=$ ....
5. If the sum of five numbers is 30 , then the arithmetic mean of these numbers is ....

8 **Choose the correct answer from the given ones :**

1. The arithmetic mean of the values x , $x-y$, $y-x$ is ....
   (a) xy (b) $\frac{y}{2}$ (c) $\frac{x}{2}$ (d) $\frac{x}{3}$
2. If the arithmetic mean of the numbers 9 , 4 , 5 , x is 5 , then $x=$ ....
   (a) 2 (b) 3 (c) 4 (d) 5
3. If the arithmetic mean of the values 3 , 4 , 8 , a , a + 2 is 15 , then a = ...
   (a) 29 (b) 58 (c) 75 (d) 17
4. If the arithmetic mean of the values $x-1$, x , $x+1$ is 6 , then $x=$ ....
   (a) 18 (b) 9 (c) 15 (d) 6
5. If the arithmetic mean of the marks of 5 students is 20 , then the sum of their marks equals .... marks.
   (a) 4 (b) 15 (c) 25 (d) 100
6. If the arithmetic mean of the ages of Hanan and Wesam is 7 years and the age of Hanan is 8 years , then the age of Wesam is .... years.
   (a) 6 (b) 7 (c) 8 (d) 15
7. If the arithmetic mean of the side lengths of a triangle is 8 cm. , then the perimeter of the triangle is ....
   (a) 8 cm. (b) 18 cm. (c) 24 cm. (d) 15 cm.

UNIT 3

9 **Find the rational number which lies at the middle of the distance between each of the following two numbers :**

1 $\frac{1}{3}$, $\frac{2}{3}$ 2 $-\frac{3}{5}$, $-\frac{1}{5}$ 3 $1\frac{1}{2}$, 2

For excellent pupils

10 If the arithmetic mean of the marks of Youssif in three tests in a subject is 16 marks and the arithmetic mean of the marks of the next two successive tests in the same subject is 18 marks.

What is the arithmetic mean of the marks in the 5 tests ? « 16.8 marks »

11 If the arithmetic mean of the marks of Magdi in 4 tests is 16 marks , what is the mark which Magdi should obtain in the 5^{th} test to make the arithmetic mean of his marks in all tests 18 marks ? « 26 marks »

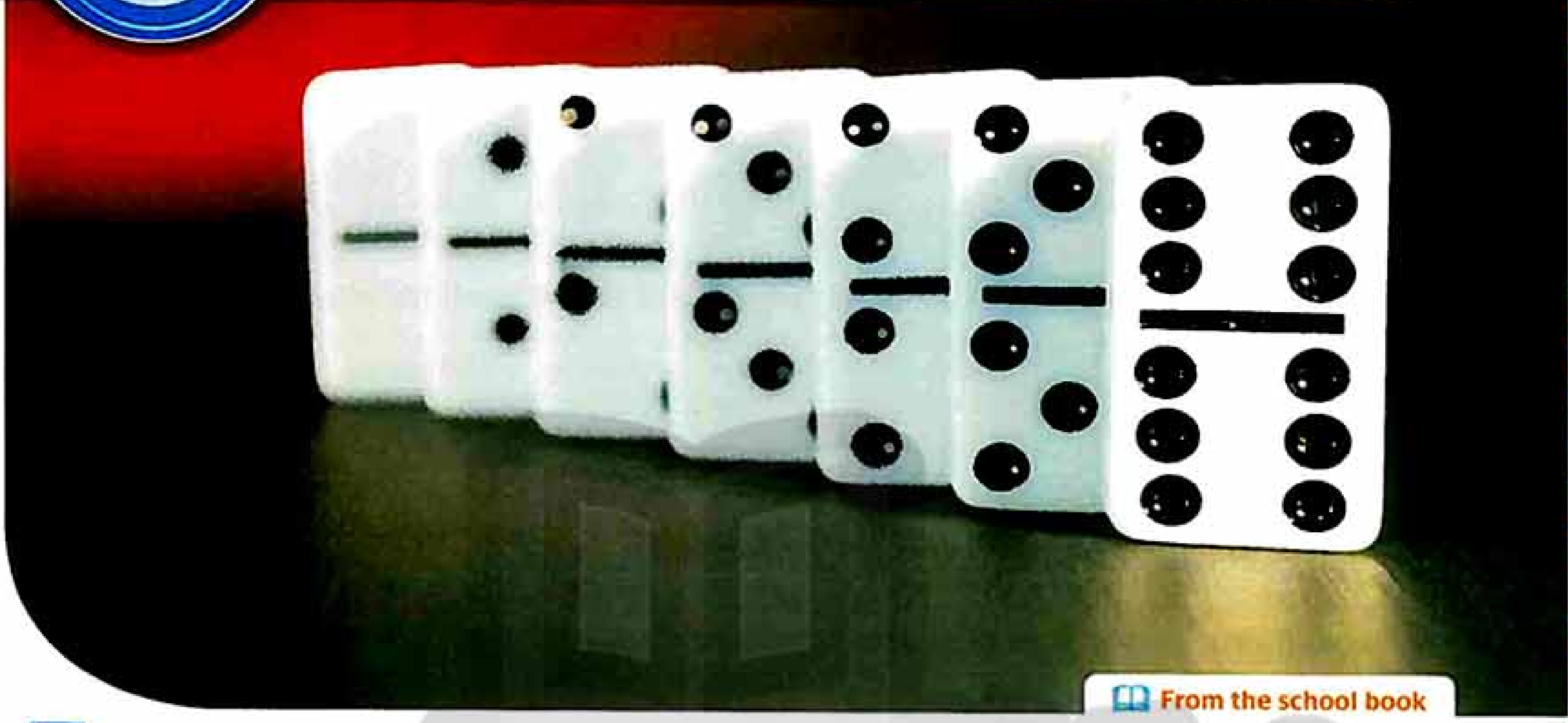
12 **The following table shows the distribution of marks of 30 students in an examination :**

Mark	6	9	12	15	17	Total
Number of students	4	7	8	5	6	30

Find the arithmetic mean of these marks. « 12 marks »

EXERCISE 16

The Median



From the school book

1 Choose the correct answer from those given :

1. The median of the values 4 , 8 , 3 is ....
   (a) 3 (b) 4 (c) 5 (d) 8
2. The median of the values 6 , 5 , 9 , 8 is ....
   (a) 5 (b) 6 (c) 7 (d) $7\frac{1}{2}$
3. The median of the values 4 , 8 , 3 , 5 , 7 is ....
   (a) 3 (b) 4 (c) 5 (d) 7
4. The median of the values 3 , 7 , 2 , 9 , 5 , 11 is ....
   (a) 5 (b) 6 (c) 7 (d) 12
5. The median of the marks 25 , 32 , 28 , 40 , 50 , 58 , 50 is ....
   (a) 40 (b) 45 (c) 50 (d) 58
6. The order of the median of the values 6 , 2 , 5 , 4 , 1 is ....
   (a) 1 (b) 2 (c) 3 (d) 4
7. If the order of the median for a set of ordered values is the fourth , then the number of these values equals ....
   (a) 3 (b) 5 (c) 7 (d) 9
8. If the order of the median for a set of orderd values is the fourth and the fifth , then the number of these values equals ....
   (a) 4 (b) 5 (c) 8 (d) 9
9. If the median of the values a + 3 , a + 2 , a + 4 is 8 , then a = ...
   (a) 2 (b) 3 (c) 4 (d) 5
10. If the median of the values a − 1 , a + 1 , a − 2 , a + 2 , a + 4 is 6 , then a = ...
    (a) 2 (b) 4 (c) 5 (d) 7

UNIT 3

2 **Find the median of each set of the following :**

1. $-2, 0, -1, 1, 5$
2. $10, -2, -2, 8, -12, 18$
3. $\frac{1}{2}, \frac{1}{4}, 1$
4. $\frac{5}{6}, \frac{3}{10}, \frac{7}{15}, \frac{2}{5}$
5. $2.3, 3.2, 2.8, 0.2, 2.9$
6. $0.8, \frac{3}{5}, \frac{1}{2}, 0.4, \frac{5}{25}, 0.25$

3 **The following table shows the weekly absence of one of studying classes :**

Day	Sunday	Monday	Tuesday	Wednesday	Thursday
Number of pupils	6	7	10	8	6

Find the median of the number of absent pupils.

4 **The following table shows the number of daily studying hours of two friends in 1st preparatory in six days :**

Sally	3	2	4.5	7	3.5	5
Basma	4	3	6	2	4.5	3

Find the median of the number of studying hours for each friend.

5 **The following table shows the heights of a group of 20 pupils in 1st preparatory in cm. Find the median height of these pupils.**

128	121	116	120
122	124	123	131
125	118	127	126
120	133	128	135
134	135	133	117

6 **The following table shows the marks of Gehad in maths tests in 6 months :**

The month	October	November	December	February	March	April
The mark	41	35	47	37	44	48

Find :

1. The median of the previous marks.
2. The mean of the previous marks.

For excellent pupils

7 **Complete each of the following :**

1 If 3 , 7 and 2 x are three values such that : $3 < 2x < 7$ and the median of these values is 4 , then $x =$ ....

2 If 9 , 10 , 5 and x are four values such that : $5 < x < 9 < 10$ and the median of these values is 8 , then $x =$ ....

EXERCISE 17 The Mode

From the school book

1 Complete each of the following :

1. The mode of a set of data is ....
2. The mode of the values 2 , 3 , 8 , 2 , 9 is ....
3. The mode of the values 14 , 11 , 12 , 11 , 14 , 15 , 11 is ....
4. The mode of the values 8 , 11 , 5 , 8 , 4 , 5 , 4 , 11 , 4 is ....
5. The mode of the colours red , yellow , red , white , black , red , white is the .... colour.
6. The mode of the tools pen , ruler , pen , rubber , ruler , pen , rubber , rubber , pen , pen is ....
7. If the mode of the values 4 , a , 5 , 3 is 3 , then a = ...
8. If the mode of the values $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{5}$, $\frac{1}{7}$ is $\frac{1}{x}$, then x = ...
9. If the mode of the values 15 , 9 , x + 1 , 9 , 15 is 9 , then x = ...
10. If the mode of the values a + 2 , a + 1 , a + 3 , a + 2 equals 12 , then a = ...

2 The following frequency table represents the marks of 40 pupils in an examination :

The mark	15	16	17	18	19	20
Number of pupils (frequence)	4	5	8	12	7	4

Find the mode mark.

Exercise 17

3 **The following frequency table shows the number of studying hours of 30 pupils in a week :**

The number of studying hours	25	26	27	28	29	30
The number of pupils (frequence)	3	5	12	6	3	1

Find the mode number of studying hours.

4 **The following frequency table shows the maximum temperature degrees registered in some Arabic capitals :**

Temperature degree	18	19	20	21	22	23
No. of capitals	3	2	4	6	2	1

Find the mode of temperature degrees.

5 **Find the mean , the median and the mode for each of the following sets :**

1. 2 , 5 , 8 , 12 , 13 , 5 , 4
2. 5 , 4 , 10 , 3 , 3 , 4 , 7 , 4 , 6 , 5

For excellent pupils

6 **The following table shows the marks of pupils in a class in a mathematics examination. (The full mark is 10 marks)**

Mark	5	6	7	8	9	10
Frequency	4	8	10	6	3	2

1. How many pupils did obtain a mark more than the mode mark ? « 11 »
2. How many pupils did obtain a mark less than the mode mark ? « 12 »

Activities

on unit three from the school book

1 Which of the following numbers is the arithmetic mean for the other numbers ?

(a) 26 (b) 28 (c) 29 (d) 30 (e) 37

2 If the mean of Karem's marks in five tests is 84 and the mean of his marks in the first three tests is 80 , then what is the mean of his marks in the last two tests ?

3 **Calculate the mean and the median of each set of the following sets of numbers :**

1. 1 , 2 , 3 , ... , 8 , 9 , 10
2. 1 , 2 , 3 , ... , 9 , 10 , 11
3. 1 , 2 , 3 , ... , 99 , 100
4. 1 , 2 , 3 , ... , 100 , 101
5. 0 , 2 , 4 , 6 , 8 , 10
6. 1 , 3 , 5 , ... , 99

* Does each of the previous sets have a mode or not ?

Summary of Unit 3

- The arithmetic mean of a set of values = $\frac{\text{Sum of these values}}{\text{Number of these values}}$
- The median of a set of values is the value which lies exactly in the middle of the set if it is arranged ascendingly or descendingly.
- If the number of values (n) is odd, then the median equals the value which lies in the middle of the values after arranging them , which is the value whose order is $\frac{n+1}{2}$
- If the number of values (n) is even, then the median equals the arithmetic mean of the two values which lie in the middle of the values after arranging them and the orders of these values are $\frac{n}{2}$ and $\frac{n}{2}+1$
- The mode of a set of data is the most common data.
- If all of the data are different, then these data have not a mode.
- Some of data have more than one mode.

Exams on Unit Three

Model 1

Answer the following questions :

1 Complete the following :

1. The mode of the values 1 , 3 , 7 , 3 , 6 , 7 , 3 is ....
2. The order of the median of the values 8 , 5 , 7 , 12 , 3 is ....
3. The mode of the degrees : excellent , good , good , very good , good , excellent is ....
4. If the arithmetic mean of six values is 12 , then the sum of these values is ....
5. The arithmetic mean of the values 5 – a , 2 , 3 , 4 + a , 1 is ....

2 Choose the correct answer from the given one :

1. The arithmetic mean of the values 19 , 32 , 27 , 6 , 6 is ....
   (a) 90 (b) 32 (c) 18 (d) 6
2. If the mode of the values 7 , 5 , y + 3 , 5 , 7 is 7 , then y = ...
   (a) 3 (b) 4 (c) 5 (d) 7
3. The median of the values 34 , 24 , 25 , 20 , 22 , 4 is ....
   (a) 22 (b) 23 (c) 24 (d) 25
4. If the arithmetic mean of the values 27 , 8 , 16 , 24 , 6 , k is 14 , then k = ...
   (a) 3 (b) 6 (c) 27 (d) 84
5. If the order of the median of a set of values is the fifth , then the number of these values is ....
   (a) 5 (b) 6 (c) 9 (d) 10

3 Sameh recorded the number of minutes that the bus took for going to the school for 10 days as the following : 15 , 18 , 22 , 15 , 25 , 20 , 16 , 20 , 14 , 15

Find each of the following :

1. The mode number of minutes.
2. The arithmetic mean of the number of minutes.
3. The median number of minutes.

4 The following table shows the number of sleeping hours for each of Mahmoud and Mohamed during a week :

Mahmoud	7	5	8	9	8	6	6
Mohamed	8	9	7	9	9	5	5

1 Find the arithmetic mean of the number of sleeping hours of Mahmoud.

2 Determine the median number of hours for each of them.

3 Determine the mode of the number of sleeping hours of Mohamed.

Model 2

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 The median of the values 7 , 5 , 13 , 3 , 2 is ....

(a) 3 (b) 5 (c) 7 (d) 13

2 The arithmetic mean of the values 3 , 5 , 10 , 2 , 10 is ....

(a) 30 (b) 5 (c) 10 (d) 6

3 The mode of the values 7 , 5 , 3 , 7 , 5 , 4 , 5 is ....

(a) 5 (b) 7 (c) 3 (d) 4

4 The order of the median of the values 5 , 9 , 7 , 4 , 8 is ....

(a) second (b) third (c) fourth (d) first

5 If the arithmetic mean of the numbers 5 , 6 , 4 , 8 , x is 7 , then $x =$ ....

(a) 6 (b) 8 (c) 7 (d) 12

2 **Complete the following :**

1 If the arithmetic mean of the marks of four pupils is 12 , then the sum of their marks equals .... marks.

2 The median of the values 7 , 8 , 5 , 11 , 6 , 13 is ....

3 The mode of a set of data is ....

4 If the order of the median of a set of arranged values is the fifth and the sixth , then the number of these values equals ....

5 If the arithmetic mean of the values $x + 2$, x , $x - 2$ is 5 , then $x =$ ....

UNIT 3

3 **The following table shows the marks of Sara in maths in 6 months :**

Month	October	November	December	February	March	April
Mark	33	42	37	45	41	48

Find :

1 The median of the previous marks.

2 The arithmetic mean of the previous marks.

4 **The following frequency table shows the ages of some friends in years :**

Age	20	21	22	23	24
Frequency	3	2	4	3	1

Find the mode.

TIMSS Problems

Accumulative basic skills

1 Complete the following :

1. If three times a number is 6 , then quarter of this number is ....
2. If $x \in \mathbb{Z}$, $-2 < 2x < 2$, then the S.S. = ...
3. The smallest number whose prime factors are 2 , 5 and 7 is ....
4. Three consecutive natural numbers , the smallest is $x - 1$, then their sum is ....
5. Two consecutive even numbers , the greater is $x + 3$, then the smaller is ....
6. A number , if it is added to its double , the result is 12 , then the number is ....
7. If the ratio between the length and the width of a rectangle is 2 : 1 , then the ratio between the length and the perimeter is ....
8. If 15% of a number equals 30 , then the number is ....
9. There are 54 kg. of apple in two boxes. If the second box weighs 12 kg. more than the first, then the number of kilograms in each box is ....
10. The value of x which makes the two numbers x , $x + 41$ prime numbers is ....
11. If $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, ... , then the next term in this pattern is .... , and the term whose order is 50 is ....
12. 1 , 1 , 2 , 3 , 5 , 8 , ... (in the same pattern)

2 Choose the correct answer from the given ones :

1. The value of 3 in the number 0.1432 is ....

(a) $\frac{3}{10}$ (b) $\frac{3}{100}$ (c) $\frac{3}{1000}$ (d) $\frac{3}{10000}$

Basic Skills

2 Hany is taller than Gamal 8 cm. Hossam is shorter than Hany 12 cm. If the height of Gamal is 125 cm. , then the height of Hossam is .... cm.

(a) 105 (b) 113 (c) 121 (d) 129

3 A baker makes 8 cakes using 2 kg. of butter , 3 kg. of sugar and 4 kg. of flour. How many cakes of the same kind can he make if he has 14 kg. of butter, 15 kg. of sugar and 16 kg. of flour ?

(a) 32 (b) 40 (c) 44 (d) 56

4 **In the opposite figure :**

The sum of numbers in each row , column and diagonal are equal. When we supply the missing numbers , which of the following numbers is not used ?

		13
	10	
7		9

(a) 6 (b) 8 (c) 12 (d) 15

5 Half of $99\frac{1}{2}$ is ....

(a) $45\frac{1}{4}$ (b) $45\frac{3}{4}$ (c) $49\frac{1}{4}$ (d) $49\frac{3}{4}$

6 Which of the following is the closest to $(11)^2 + (9)^2$?

(a) 20 + 20 (b) 20 + 80 (c) 120 + 20 (d) 120 + 80

7 If k is a negative number , which of the following is positive ?

(a) k^2 (b) k^3 (c) $2k$ (d) $\frac{k}{2}$

8 Sound moves through air with velocity 330 metres each second approximately. An explosion sound takes 28 seconds to reach someone. Which of the following is the best estimation of the distance between this person and the explosion place ?

(a) 12000 m. (b) 9000 m. (c) 8000 m. (d) 6000 m.

9 Quarter of 4^{20} equals ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

10 The smallest fraction of the following is ....

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{8}$ (d) $\frac{7}{16}$

11 Which of the following is the best estimation of the result of $\frac{32 \times 2.7}{14.7}$?

(a) 0.6 (b) 3 (c) 6 (d) 60

12 The next number in the pattern $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, ... is ....

(a) 0 (b) 1 (c) 10 (d) 100

Second Geometry

Geometry and Measurement

UNIT 4

Exercises of the unit :

1. Geometric concepts - The relations between the angles.
2. The relations between the angles "follow".
3. Congruence.
- Summary of the first part of unit four.
- Exams on the first part of unit four.
4. Congruent triangles.
5. Parallelism.
6. Geometric constructions.
- Summary of the second part of unit four.
- Exams on the second part of unit four.

EXERCISE 1 Geometric Concepts – The Relations between the Angles

From the school book

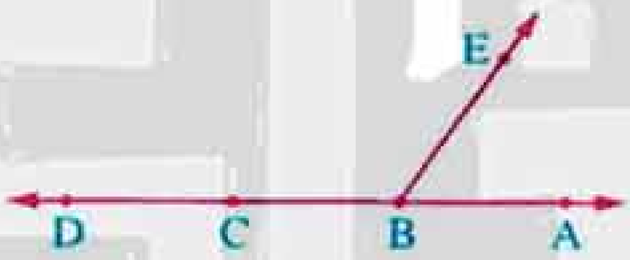
1 **In the opposite figure :**

A , B , C and D are points lying on one line ,

$\overleftrightarrow{AD} \cap \overrightarrow{BE} = \{B\}$

Complete each of the following by using

$\in$, $\notin$, $\subset$ or $\not\subset$:



1. A .... $\overrightarrow{DC}$
2. D .... $\overline{AC}$
3. C .... $\overleftrightarrow{AB}$
4. A .... $\angle EBC$
5. $\overline{DC}$ .... $\overleftrightarrow{AB}$
6. $\overline{BC}$ .... $\overrightarrow{BA}$
7. $\overrightarrow{BA}$ .... $\overleftrightarrow{DC}$
8. $\overline{AC}$ .... $\overline{AD}$

2 **Mention the type of the angle whose measure is as the following :**

1. 57°
2. 117°
3. 90°
4. 200°
5. 180°
6. $43\frac{1}{2}^\circ$
7. $89^\circ\ 60'$
8. $179^\circ\ 62'$

3 **Write the measure of the angle which complements each of the angles whose measures are as follows :**

1. 30°
2. 60°
3. (From the school book) 48°
4. (From the school book) $22\frac{1}{2}^\circ$
5. $53\frac{1}{4}^\circ$
6. 90°
7. $25^\circ\ 60'$
8. 0°

4 **Write the measure of the angle which supplements each of the angles whose measures are as follows :**

1. 20°
2. 90°
3. 152°
4. (From the school book) 10°
5. (From the school book) $92\frac{1}{2}^\circ$
6. 0°
7. 180°
8. $141^\circ\ 60'$

UNIT 4

5 **Complete the following table :**

m (∠ ABC)	50°	....	105°	....	179°	....
m (reflex ∠ ABC)	....	330°	....	237°	....	350°

6 **Complete the following :**

1. The angle is ....
2. The measure of the straight angle = ... ° and the measure of zero angle = ... °
3. The measure of the right angle = ... °
4. The acute angle is the angle whose measure is less than .... and more than ....
5. The two complementary angles are the two angles whose sum of measures is ....
6. The two supplementary angles are the two angles whose sum of measures is ....
7. The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are ....
8. If the two outer sides of two adjacent angles are perpendicular , then these two adjacent angles are ....
9. If the two outer sides of two adjacent angles are on the same straight line, then these two adjacent angles are ....
10. If the two adjacent angles are supplementary , then their outer sides are ....
11. The measure of the angle which is equivalent to two right angles equals .... and it is called .... angle.
12. The angle whose measure is 50° complements an angle of measure .... and supplements an angle of measure ....
13. The angle whose measure is .... complements the angle whose measure is 30° and supplements the angle whose measure is ....
14. The angle whose measure is .... complements the angle whose measure is .... and supplements the angle whose measure is 150°
15. The acute angle complements .... angle and supplements .... angle.
16. Zero angle is complemented by .... angle and is supplemented by .... angle.
17. The right angle is complemented by .... angle and is supplemented by .... angle.
18. The obtuse angle supplements .... angle.

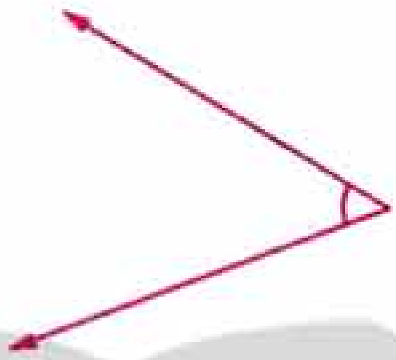
Exercise 1

7 **Draw the angles whose measures are as follows showing the type of each of them :**

1 115° 2 80° 3 195° 4 245° 5 180°

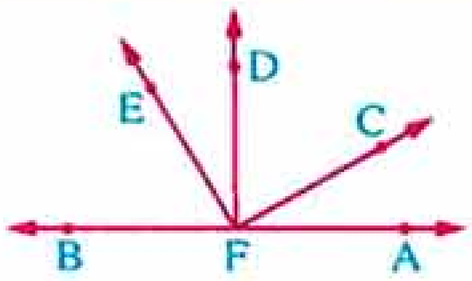
8 **For each of the following angles , write the closest measure from the following 80° , 120° , 240°**

1


2


3


9 **In the opposite figure :**



$F \in \overleftrightarrow{AB}$, $\overrightarrow{FD} \perp \overleftrightarrow{AB}$ and m (∠ CFE) = 90°

Complete the following :

1 $\overrightarrow{FA} \cup \overrightarrow{FC}$ = ...

2 $\overrightarrow{FC} \cup \overrightarrow{FB}$ = ...

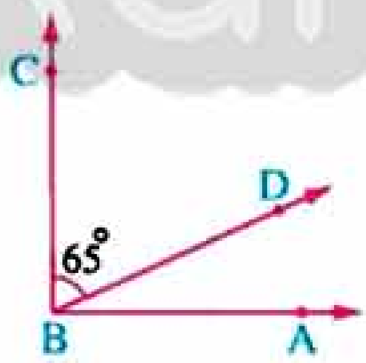
3 ∠ AFC supplements ∠ ....

4 ∠ DFC complements each of ∠ .... and ∠ ....

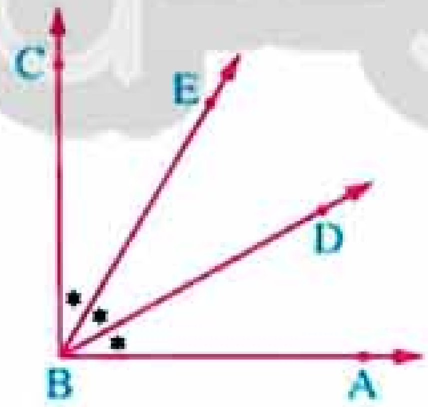
5 ∠ AFB is .... angle , and ∠ DFB is .... angle.

6 m (∠ DFE) = m (∠ ....) because each one of them complements ∠ ....

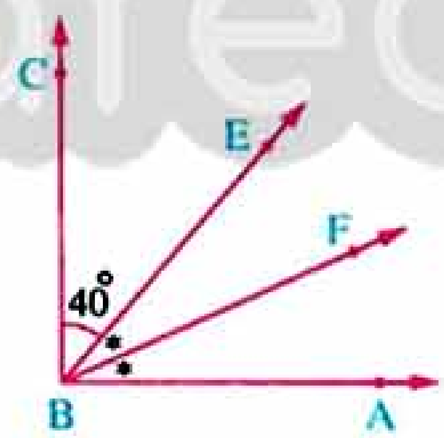
10 **In each of the following figures, if $\overrightarrow{BA} \perp \overrightarrow{BC}$, find the measure of the required angle under each figure :**

1


m (∠ ABD) = ...°

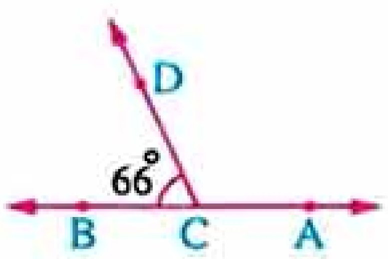
2


m (∠ DBC) = ...°

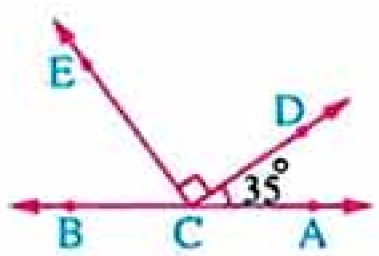
3


m (∠ ABF) = ...°

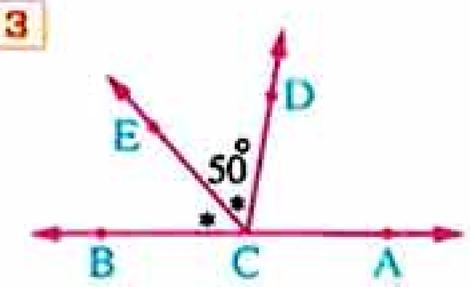
11 **In each of the following figures, if $C \in \overleftrightarrow{AB}$, find the measure of the required angle under each figure :**

1


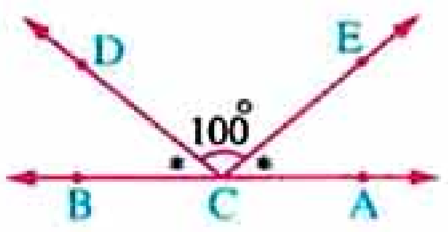
m (∠ ACD) = ...°

2


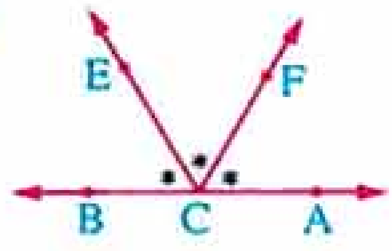
m (∠ ECB) = ...°

3


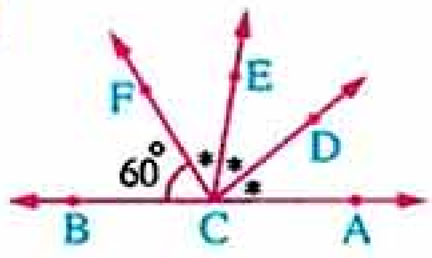
m (∠ ACD) = ...°

4 

m (∠ DCB) = ...°

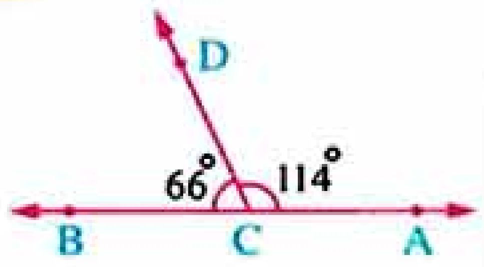
5 

m (∠ FCB) = ...°

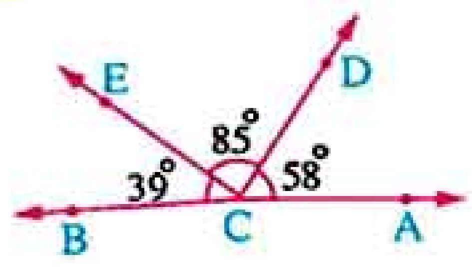
6 

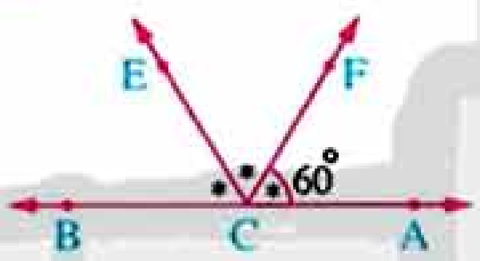
m (∠ DCB) = ...°

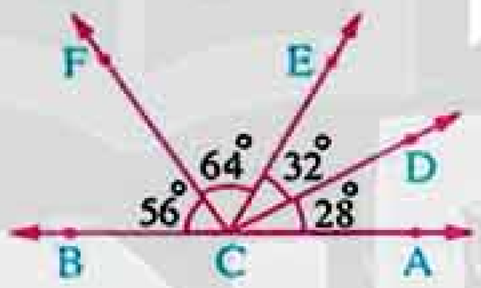
12 **In each of the following figures, state if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line or not, and why :**

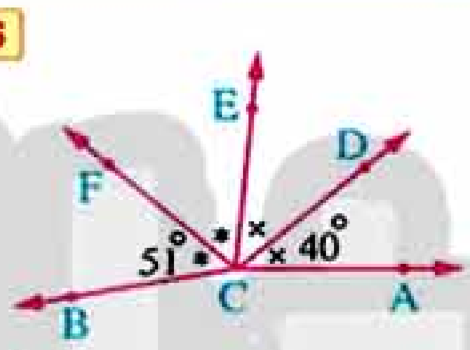
1 

2 

3 

4 

5 

6 

13 **Choose the correct answer from the given ones :**

1 Between any two distinct points we can draw .... straight line passing through them.

(a) zero (b) 1 (c) 2 (d) 3

2 If m (∠ A) + m (∠ B) = 180°, then ∠ A and ∠ B are two .... angles.

(a) equal in measure (b) complementary
(c) supplementary (d) adjacent

3 If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then m (∠ ABC) = ...

(a) 40° (b) 90° (c) 180° (d) 360°

4 If ∠ A supplements ∠ B, ∠ A supplements ∠ C, then ∠ B and ∠ C are ....

(a) equal in measure. (b) complementary.
(c) supplementary. (d) adjacent.

5 If m (∠ X) = 15°, then the two angles whose measures are 2 m (∠ X), 4 m (∠ X) are ....

(a) complementary. (b) supplementary.
(c) equal in measure. (d) obtuse angles.

6 If m (∠ A) = 2 m (∠ B), ∠ A supplements ∠ B, then m (∠ B) = ...

(a) 30° (b) 60° (c) 120° (d) 90°

7 $\overline{AB}$ .... $\overrightarrow{AB}$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

8 If m (∠ X) = 2 m (∠ Y) and ∠ Y is an obtuse angle, then ∠ X is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

14 **Complete the following :**

1 If ∠ X complements ∠ Y , ∠ Z complements ∠ Y , then ∠ Z and ∠ X are ....

2 If ∠ X complements ∠ Y , m (∠ X) = m (∠ Y) , then m (∠ X) = ...°

3 If ∠ A and ∠ B are two supplementary angles and m (∠ A) = m (∠ B)
, then m (∠ A) = ... °

4 If m (∠ X) = $\frac{1}{2}$ m (∠ Y) , m (∠ X) = 30° , then the two angles X and Y are ....

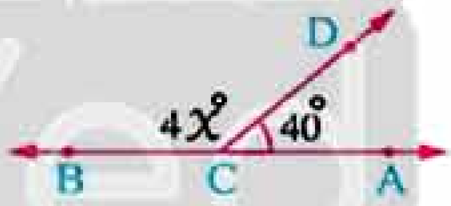
5 If the ratio between the measures of two supplementary angles is 2 : 7
, then the measure of the greater angle equals .... °

6 If m (∠ A) = $\frac{1}{2}$ m (∠ B) , m (∠ C) = $\frac{1}{2}$ m (∠ D) , ∠ B supplements ∠ D
, then m (∠ A) + m (∠ C) = ... °

7 If ∠ A complements ∠ B and ∠ B supplements ∠ C , m (∠ A) = 32°
, then m (∠ C) = ... °

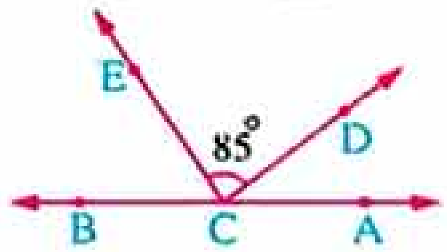
8 **In the opposite figure :**

If C ∈ $\overleftrightarrow{AB}$, then x = ...



9 **In the opposite figure :**

If C ∈ $\overleftrightarrow{AB}$, m (∠ DCE) = 85° ,
m (∠ ACD) : m (∠ ECB) = 2 : 3
, then m (∠ ACE) = ...° , m (∠ DCB) = ...°



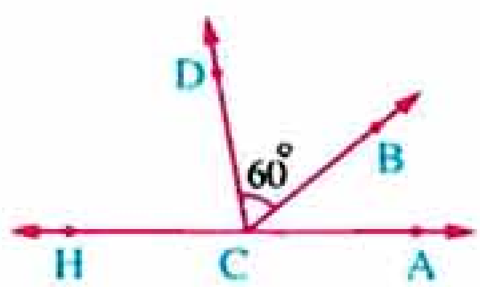
For excellent pupils

15 **In the opposite figure :**

m (∠ DCB) = 60°

and m (∠ ACB) : m (∠ BCD) : m (∠ DCH) = 2 : 3 : 4

Are $\overrightarrow{CA}$ and $\overrightarrow{CH}$ on the same straight line or not ? Why ?



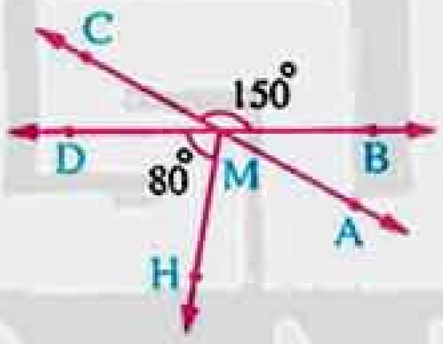
EXERCISE 2

The Relations between the Angles (Follow)

From the school book

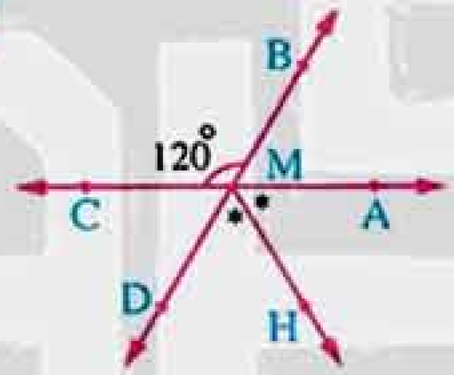
1 **In each of the following figures , find the measure of the required angle under each figure :**

1



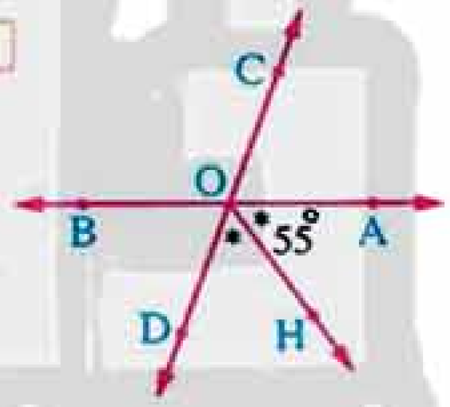
m (∠ AMH) = ...°

2



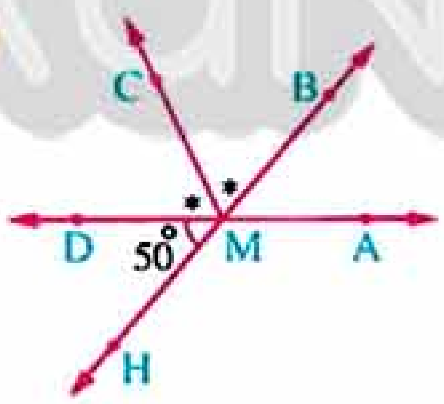
m (∠ HMD) = ...°

3



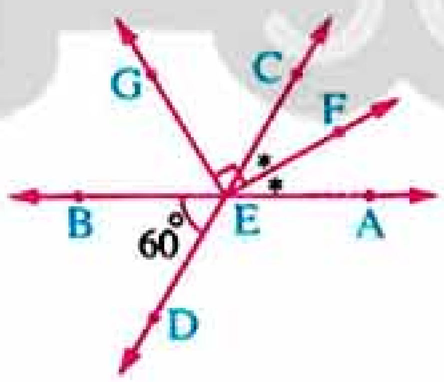
m (∠ COB) = ...°

4



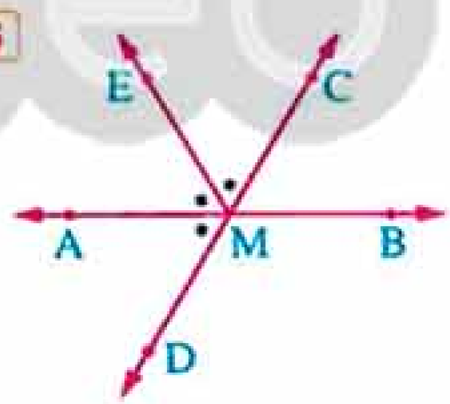
m (∠ AMC) = ...°

5



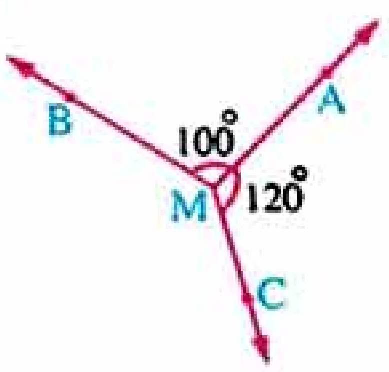
m (∠ GEB) = ...°

6



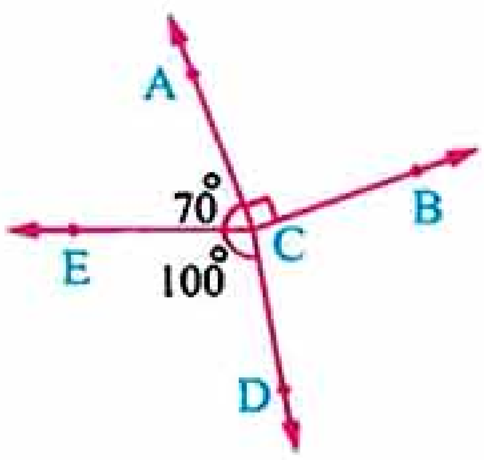
m (∠ DMB) = ...°

7



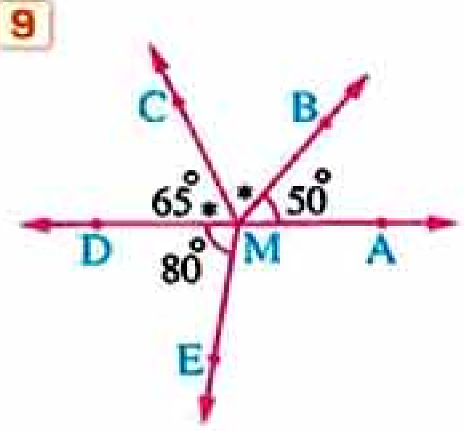
m (∠ BMC) = ...°

8



m (∠ BCD) = ...°

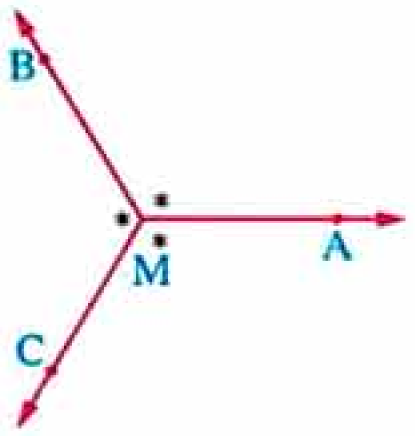
9



m (∠ AME) = ...°

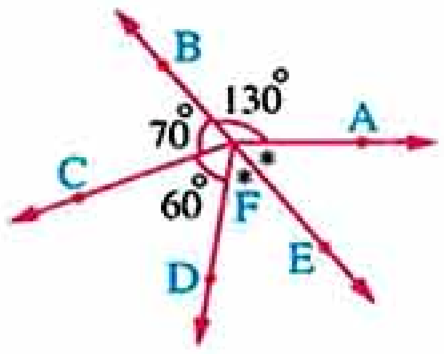
Exercise 2

10



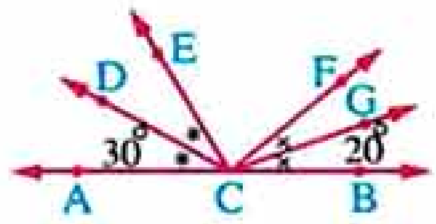
m (∠ AMC) = ...°

11



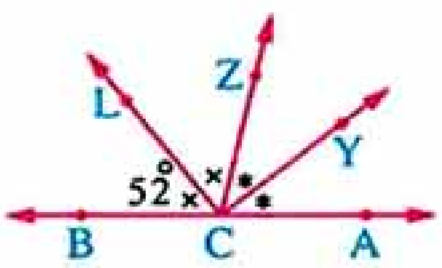
m (∠ EFD) = ...°

12



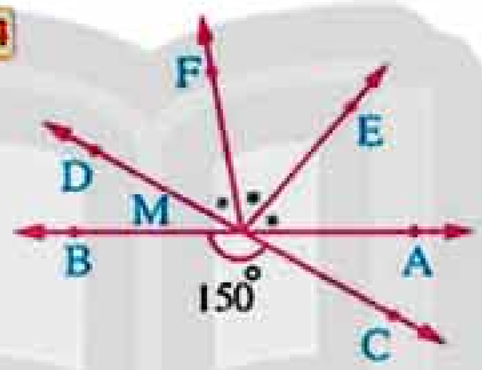
m (∠ FCE) = ...°

13



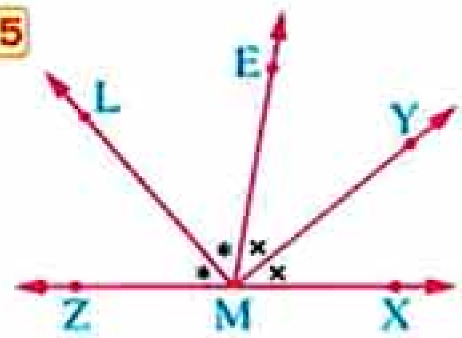
m (∠ YCA) = ...°

14



m (∠ CMF) = ...°

15



m (∠ YML) = ...°

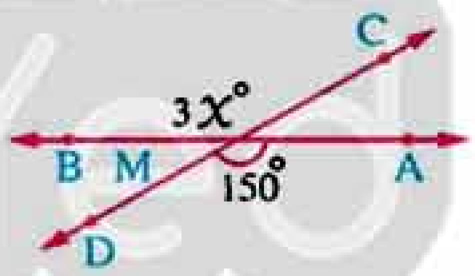
2 Complete the following :

1 If two straight lines intersect , then each two vertically opposite angles are ....

2 The sum of the measures of the accumulative angles at a point equals ....°

3 **In the opposite figure :**

If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then x = ...°

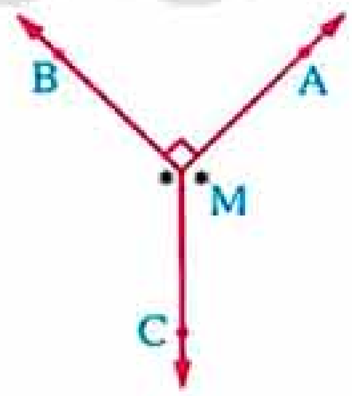


4 **In the opposite figure :**

If $\overrightarrow{MB} \perp \overrightarrow{MA}$

and $\overrightarrow{MC}$ bisects the reflexed angle AMB

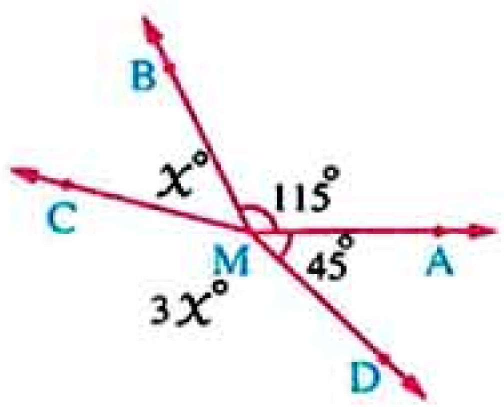
, then m (∠ AMC) = ...°



5 If $\overrightarrow{BD}$ bisects ∠ ABC and m (∠ ABD) = 35° , then m (∠ ABC) = ...°

6 **In the opposite figure :**

x = ...°



UNIT 4

3 **Choose the correct answer from the given ones :**

1 The sum of measures of the accumulative angles at a point equals the sum of measures of .... angles.

(a) 2 right (b) 3 right (c) 4 right (d) 5 right

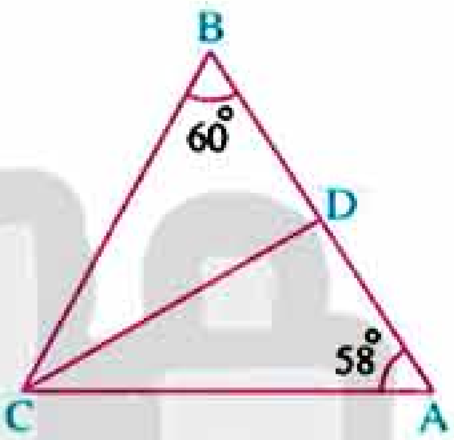
2 The sum of measures of 4 accumulative angles at a point .... the sum of measures of 5 accumulative angles at a point.

(a) = (b) < (c) > (d) ≠

3 The two bisectors of two adjacent supplementary angles ....

(a) are perpendicular. (b) are parallel.
(c) are coincident. (d) included an acute angle between them.

4 **In the opposite figure :**



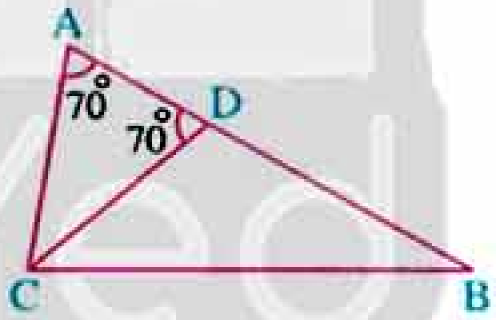
If ABC is a triangle in which $\overrightarrow{CD}$ bisects $\angle ACB$, $m(\angle A) = 58°$,

$m(\angle B) = 60°$

, then $m(\angle ADC) =$ ....

(a) 62° (b) 89° (c) 91° (d) 130°

5 **In the opposite figure :**

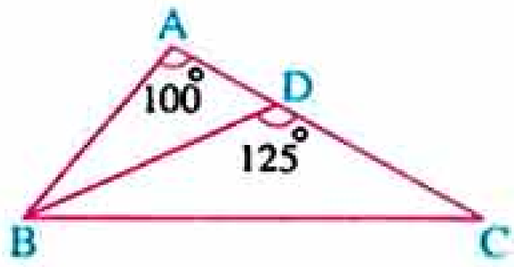


If $\overrightarrow{CD}$ bisects $\angle BCA$, $m(\angle A) = m(\angle ADC) = 70°$

, then $m(\angle B) =$ ....

(a) 70° (b) 30° (c) 80° (d) 40°

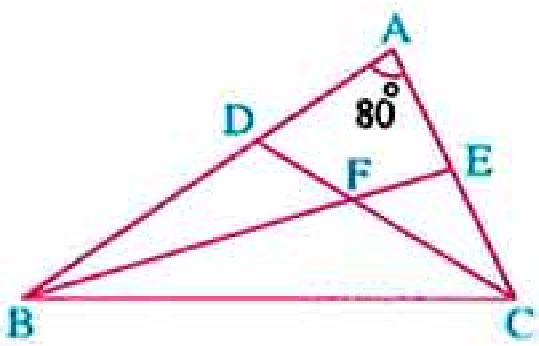
6 **In the opposite figure :**



ABC is a triangle , $D \in \overline{AC}$ and $\overrightarrow{BD}$ is the bisector of $\angle B$, what is the measure of $\angle C$?

(a) 25° (b) 30° (c) 45° (d) 55°

7 **In the opposite figure :**



$m(\angle A) = 80°$, $\overrightarrow{BE}$ is the bisector of $\angle B$,

$\overrightarrow{CD}$ is the bisector of $\angle C$

What is the measure of $\angle BFC$?

(a) 80° (b) 100° (c) 120° (d) 130°

Exercise 2

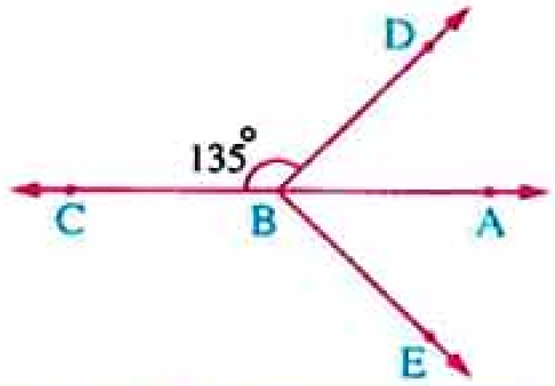
4 **In the opposite figure :**

If $B \in \overleftrightarrow{AC}$, $m(\angle DBC) = 135^\circ$

and $\overrightarrow{BA}$ bisects $\angle DBE$

, find each of :

$m(\angle ABD)$, $m(\angle DBE)$, $m(\angle CBE)$

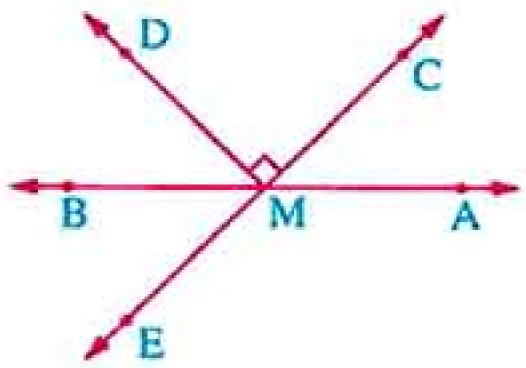


5 **In the opposite figure :**

If $\overleftrightarrow{AB} \cap \overleftrightarrow{CE} = \{M\}$, $\overrightarrow{MD} \perp \overleftrightarrow{CE}$ and $\overrightarrow{MB}$ bisects $\angle DME$

, find the measures of the following angles :

$\angle BME$, $\angle DME$, $\angle AMC$ and $\angle AME$



6 **In the opposite figure :**

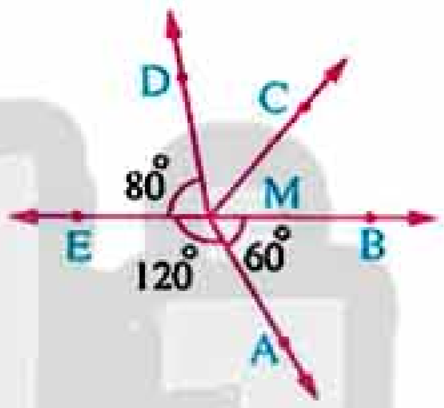
$m(\angle AMB) = 60^\circ$, $m(\angle AME) = 120^\circ$,

$m(\angle EMD) = 80^\circ$ and $\overrightarrow{MC}$ bisects $\angle BMD$

Find :

1 $m(\angle CMD)$

2 $m(\angle AMC)$

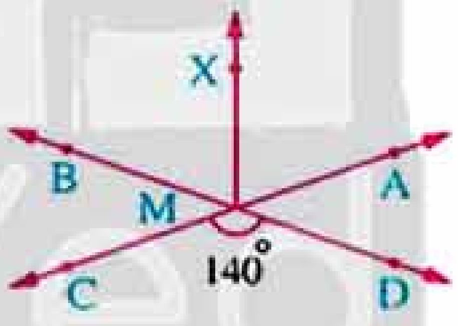


7 **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, $\overrightarrow{MX}$ bisects $\angle AMB$

and $m(\angle CMD) = 140^\circ$

Find : $m(\angle DMX)$



8 **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, $m(\angle CMX) = 90^\circ$,

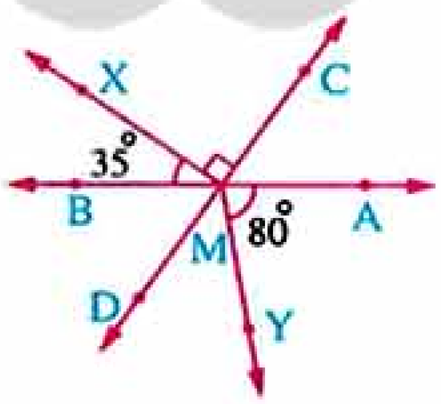
$m(\angle XMB) = 35^\circ$ and $m(\angle AMY) = 80^\circ$

Find :

1 $m(\angle AMD)$

2 $m(\angle DMY)$

3 $m(\angle BMY)$



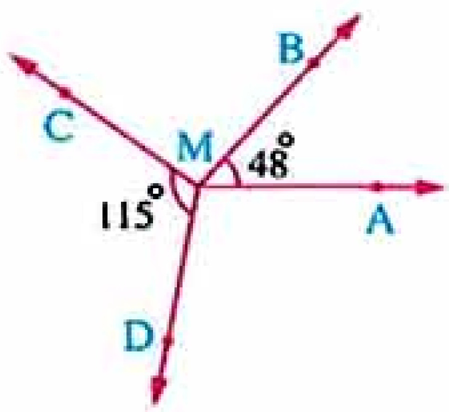
9 **In the opposite figure :**

$m(\angle BMC) = 2\, m(\angle AMB)$,

$m(\angle AMB) = 48^\circ$

and $m(\angle DMC) = 115^\circ$

Find : $m(\angle AMD)$



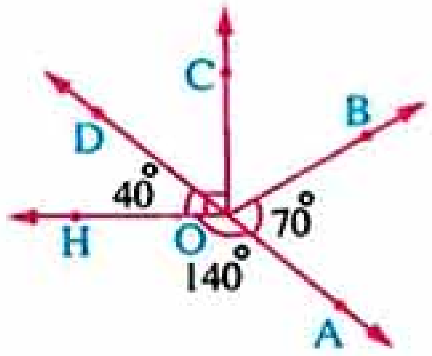
UNIT 4

10 **In the opposite figure :**

$\overrightarrow{OC} \perp \overrightarrow{OH}$

Are $\overrightarrow{OA}$ and $\overrightarrow{OD}$ on the same straight line or not ? Why ?

, then find : m (∠ BOC)



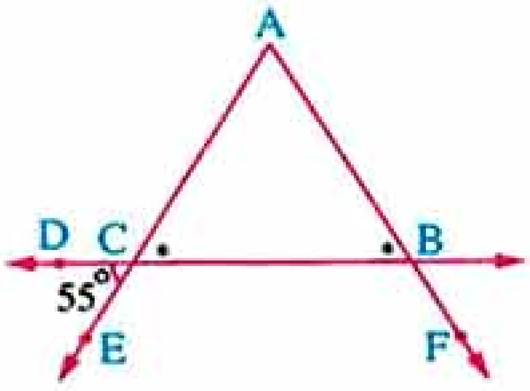
11 **In the opposite figure :**

$D \in \overrightarrow{BC}$, $E \in \overrightarrow{AC}$, $F \in \overrightarrow{AB}$

, m (∠ ABC) = m (∠ ACB)

and m (∠ ECD) = 55°

Find : m (∠ FBC)



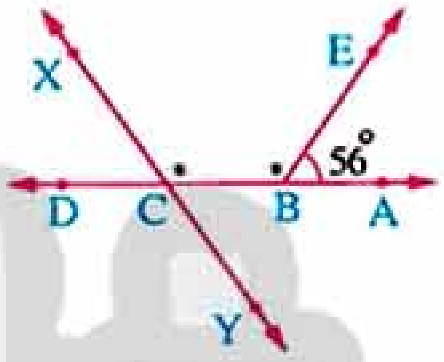
12 **In the opposite figure :**

A , B , C and D are collinear ,

$\overleftrightarrow{XY} \cap \overleftrightarrow{BD} = \{C\}$, m (∠ ABE) = 56°

and m (∠ EBC) = m (∠ BCX)

Find : m (∠ DCY)

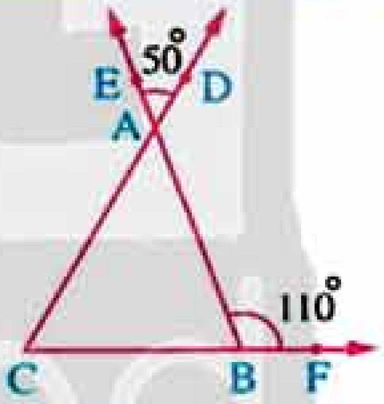


13 **In the opposite figure :**

m (∠ DAE) = 50°

and m (∠ ABF) = 110°

Find : The measures of the angles of the triangle ABC



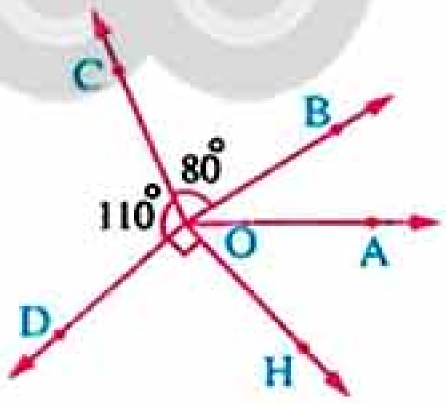
14 **In the opposite figure :**

m (∠ BOC) = 80° , m (∠ COD) = 110° ,

m (∠ DOH) = 90°

and m (∠ AOB) : m (∠ AOH) = 2 : 3

Find : m (∠ AOB) and m (∠ AOH)



For excellent pupils

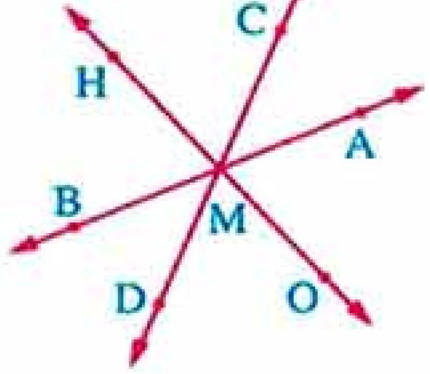
15 **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} \cap \overleftrightarrow{HO} = \{M\}$,

m (∠ AMO) + m (∠ BMH) = 140°

and m (∠ AMC) : m (∠ DMO) = 2 : 3

Find : m (∠ CMH)



EXERCISE 3

Congruence

From the school book

1 **Complete the following :**

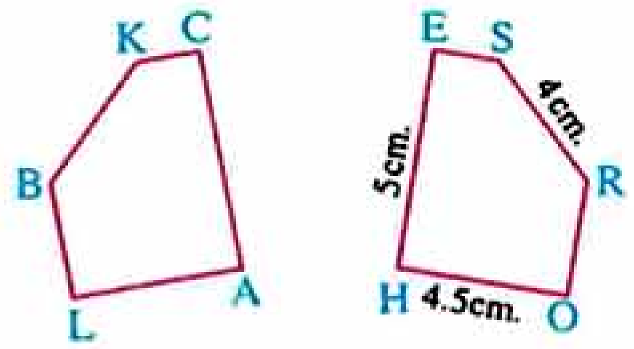
1. Two line segments are congruent if ....
2. Two angles are congruent if ....
3. Two polygons are congruent if there is a correspondence between their vertices such that each .... and each .... in the first polygon is congruent to its corresponding element in ....
4. The axis of symmetry of a polygon divides it into two .... polygons.
5. If $\overline{AB} \equiv \overline{CD}$, then AB = ...
6. If $\overline{AB} \equiv \overline{XY}$, then AB – XY = ...
7. If $\angle A \equiv \angle B$ and $m(\angle A) = 50°$, then $m(\angle B)$ = ...°
8. If $\angle A$ supplements $\angle B$ and $\angle A \equiv \angle B$, then $m(\angle B)$ = ...°
9. If $\angle A$ complements $\angle B$ and $\angle A \equiv \angle B$, then $m(\angle A)$ = ...°
10. If C is the midpoint of $\overline{AB}$, then $\overline{AC}$ .... $\overline{BC}$
11. If the polygon ABCD ≡ the polygon XYZL , then $\overline{DA} \equiv$ ....
, $m(\angle BCD) = m(\angle$ ....)
12. The two squares are congruent if .... are equal in length , and the two rectangles are congruent if .... are equal.

UNIT 4

2 **In the opposite figure :**

The two pentagons shown are congruent.

Complete :



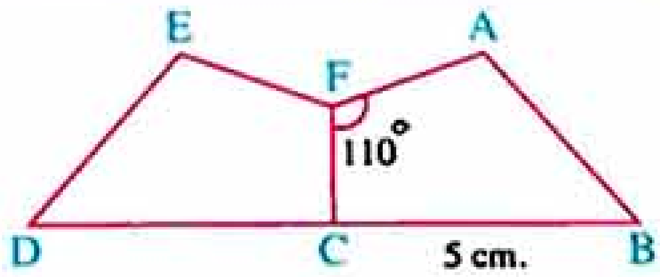
1. B corresponds to ....
2. The polygon BLACK is congruent to the polygon ....
3. KB = ... cm.
4. m (∠ E) = m (∠ ....)
5. CA = ... cm.
6. m (∠ A) = m (∠ ....)

3 **In the opposite figure :**

If $C \in \overleftrightarrow{BD}$, m (∠ AFC) = 110° , BC = 5 cm.

and the polygon ABCF ≡ the polygon EDCF

, complete the following :



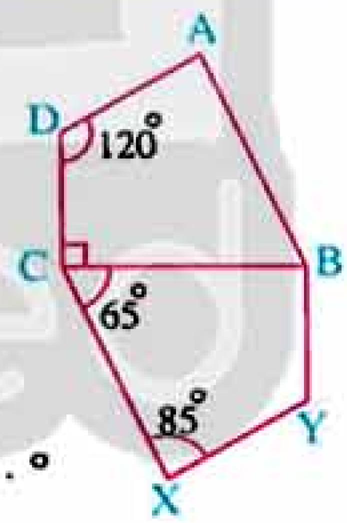
1. AB = ...
2. AF = ...
3. m (∠ E) = m (∠ ....)
4. m (∠ B) = m (∠ ....)
5. m (∠ FCD) = m (∠ ....)
6. m (∠ EFC) = ... °
7. BD = ... cm.
8. m (∠ FCD) = ... °
9. m (∠ AFE) = ... °
10. The axis of symmetry of the polygon ABDEF is ....

4 **In the opposite figure :**

If $\overline{DC} \perp \overline{BC}$, m (∠ ADC) = 120° , m (∠ BCX) = 65° , m (∠ X) = 85°

and the polygon ABCD ≡ the polygon XCBY

, complete the following :



1. AB = ...
2. XY = ...
3. CD = ...
4. $\overline{BC}$ is .... side.
5. m (∠ Y) = ... °
6. m (∠ A) = ... °
7. m (∠ ABC) = ...°
8. m (∠ YBC) = ... °

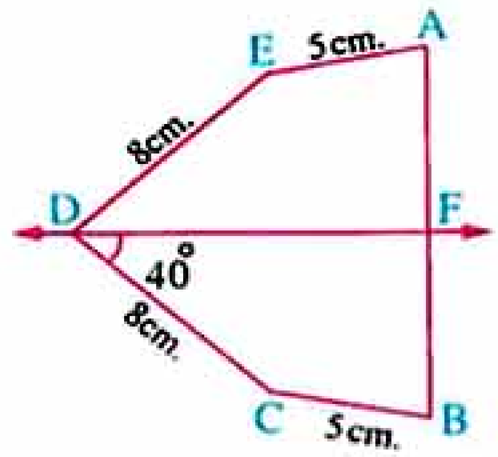
5 **In the opposite figure :**

If m (∠ A) = m (∠ B) , m (∠ C) = m (∠ E) , $\overleftrightarrow{FD}$ bisects ∠ EDC ,

$\overleftrightarrow{FD}$ is the axis of symmetry of $\overline{AB}$, AE = BC = 5 cm. ,

CD = ED = 8 cm. , AB = 12 cm. and m (∠ CDF) = 40°

, complete the following :



1. m (∠ AFD) = ...°
2. m (∠ CDE) = ...°
3. The length of $\overline{BF}$ = ... cm.
4. The two figures .... and .... are congruent.

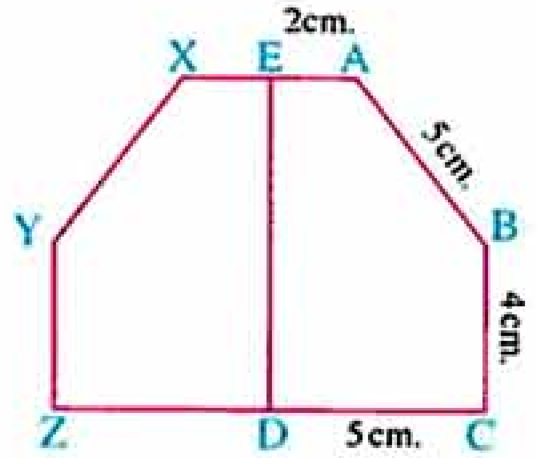
Exercise 3

6 **In the opposite figure :**

If $D \in \overleftrightarrow{CZ}$ and the figure ABCDE ≡ the figure XYZDE ,

AE = 2 cm. , BC = 4 cm. and AB = CD = 5 cm.

, **find :** The perimeter of the figure ABCZYX

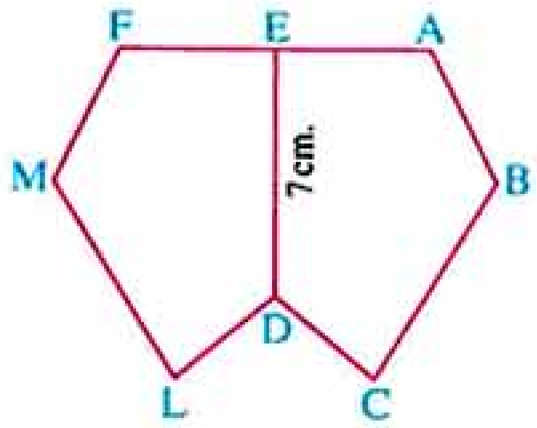


7 **In the opposite figure :**

If $E \in \overleftrightarrow{AF}$, the perimeter of the figure ABCDE = 27 cm. ,

DE = 7 cm. and the polygon ABCDE ≡ the polygon FMLDE

, **find :** The perimeter of the figure ABCDLMF

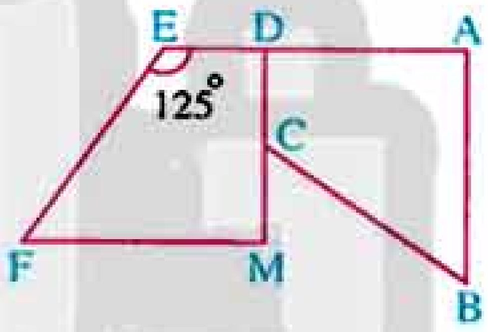


8 **In the opposite figure :**

If the figure ABCD ≡ the figure MFED ,

C is the midpoint of $\overline{MD}$ and MC = 3 cm.

, **complete the following :**



1. m (∠ A) = m (∠ ....)
2. m (∠ ADC) = m (∠ ....)
3. m (∠ B) = m (∠ ....)
4. m (∠ BCD) = ... °
5. m (∠ BCM) = ... °
6. AE = ... cm.

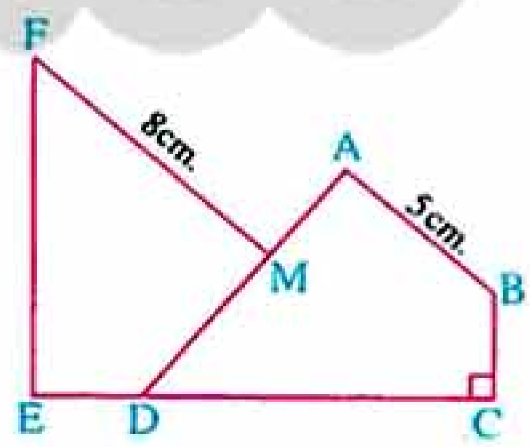
For excellent pupils

9 **In the opposite figure :**

If $D \in \overleftrightarrow{CE}$, $\overline{BC} \perp \overline{CD}$

and the figure ABCD ≡ the figure MDEF

, **complete the following :**



1. m (∠ A) = m (∠ ....)
2. m (∠ CDA) = m (∠ ....)
3. m (∠ E) = ...°
4. AM = ... cm.
5. m (∠ B) + m (∠ F) = ...°

Summary of the first part of unit 4
"From lesson 1 to lesson 3"

- The acute angle , its measure is more than 0° and less than 90°
- The obtuse angle , its measure is more than 90° and less than 180°
- The zero angle , its measure is 0° , and the right angle its measure is 90°
- The straight angle , its measure is 180° , and the reflex angle its measure is more than 180° and less than 360°
- The sum of measures of two complementary angles is 90° and the sum of measures of two supplementary angles is 180°
- Two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are supplementary.
- If the two adjacent angles are supplementary , then their outer sides are on the same straight line.
- If the two adjacent angles are complementary , then their outer sides are perpendicular.
- The two vertically opposite angles are equal in measure.
- The sum of the measures of the accumulative angles at a point is 360°
- The angle bisector is the ray that divides the angle into two equal angles in measure.
- Two line segments are congruent if they are equal in length.
- Two angles are congruent if they are equal in measure.
- Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.
- If the two polygons are congruent , then each side and each angle in one of them is congruent to its corresponding element in the other polygon.

Exams on the first part of unit four from lesson (1) to lesson (3)

Model 1

Answer the following questions :

1 Choose the correct answer from the given ones :

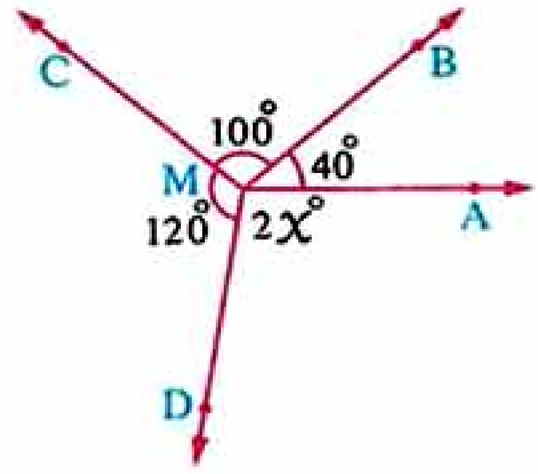
1. Two complementary angles are two angles whose sum of their measures is ....
   (a) 90° (b) 180° (c) 100° (d) 45°
2. The sum of measures of the accumulative angles at a point equals the sum of measures of .... angles.
   (a) 2 right (b) 3 right (c) 4 right (d) 5 right
3. The obtuse angle supplements .... angle.
   (a) an obtuse (b) a right (c) an acute (d) a straight
4. The two bisectors of two adjacent supplementary angles ....
   (a) are perpendicular. (b) are parallel.
   (c) are coincident. (d) included an obtuse angle between them.
5. If $\angle A$ supplements $\angle B$, $\angle A \equiv \angle B$, then m ($\angle B$) = ...
   (a) 45° (b) 90° (c) 180° (d) 360°
6. $\overrightarrow{AB} \cup \overrightarrow{AC}$ = ...
   (a) $\overleftrightarrow{BC}$ (b) $\angle BCA$ (c) $\angle BAC$ (d) $\overleftrightarrow{AC}$

2 Complete the following :

1. If two straight lines intersect , then each two vertically opposite angles are ....
2. The angle whose measure is 50° complements an angle of measure ....° and supplements an angle of measure ....°
3. Two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are ....
4. If $\overline{AB} \equiv \overline{XY}$, then AB – XY = ...
5. Two angles are congruent if they are ....

UNIT 4

3 **[a] In the opposite figure :**

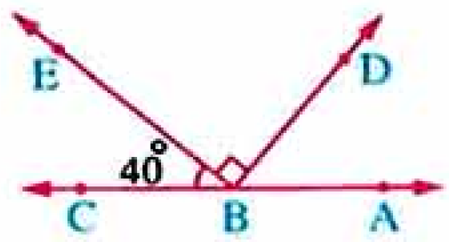


m (∠ AMB) = 40° , m (∠ BMC) = 100°

, m (∠ CMD) = 120° and m (∠ AMD) = 2 x°

Find the value of x

[b] In the opposite figure :

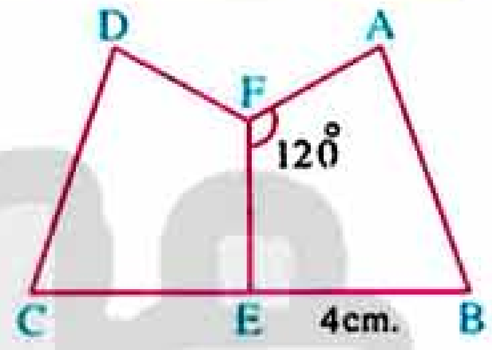


B ∈ $\overleftrightarrow{AC}$, m (∠ EBC) = 40°

and m (∠ DBE) = 90°

Find : m (∠ ABD)

4 **In the opposite figure :**



E ∈ $\overline{BC}$, m (∠ AFE) = 120°

, BE = 4 cm. and

the polygon ABEF ≡ the polygon DCEF

Complete the following :

1 DF = ...

2 m (∠ ABE) = m (∠ ....)

3 m (∠ DFE) = ...°

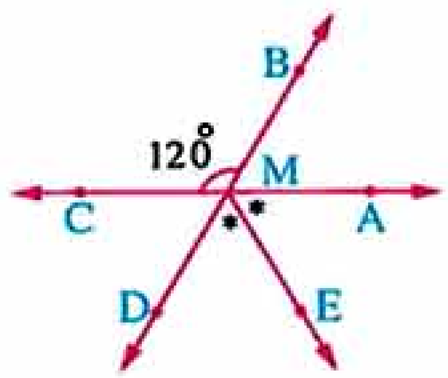
4 $\overline{AB}$ ≡ ....

5 m (∠ FEB) = ...°

6 m (∠ AFD) = ...°

7 The axis of symmetry of the figure ABCDF is ....

5 **[a] In the opposite figure :**



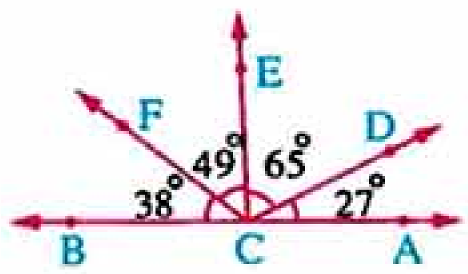
$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$

, m (∠ BMC) = 120°

and $\overrightarrow{ME}$ bisects ∠ AMD

Find : m (∠ EMD)

[b] In the opposite figure :



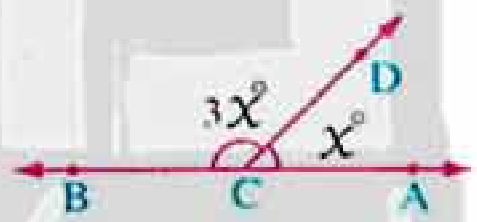
Are $\overrightarrow{CA}$ and $\overrightarrow{CB}$ on the same straight line ? Why ?

Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

1. If m (∠ A) = 70° , then m (reflex ∠ A) = ...
   (a) 20° (b) 110° (c) 290° (d) 70°
2. If m (∠ X) + m (∠ Y) = 180° , then ∠ X and ∠ Y are ....
   (a) equal in measure. (b) complementary. (c) supplementary. (d) adjacent.
3. The sum of measures of 6 accumulative angles at a point .... the sum of measures of 3 accumulative angles at a point.
   (a) = (b) < (c) > (d) ≠
4. If m (∠ A) = 2 m (∠ B) and ∠ A supplements ∠ B , then m (∠ A) = ...
   (a) 60° (b) 120° (c) 45° (d) 90°
5. The type of the angle of measure 89° 60̀ is ....
   (a) acute. (b) straight. (c) right. (d) obtuse.
6. **In the opposite figure :**
   C ∈ $\overleftrightarrow{AB}$, then x = ...
   (a) 135° (b) 90°
   (c) 45° (d) 60°

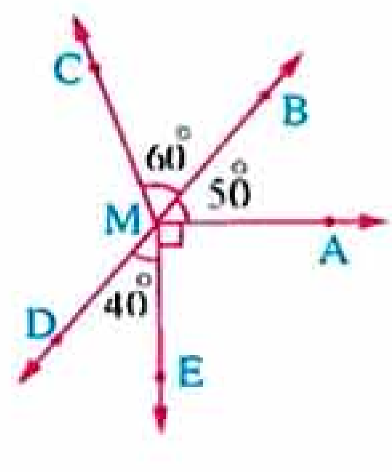


2 Complete the following :

1. Two adjacent angles whose outer sides are perpendicular are ....
2. The right angle is complemented by .... angle.
3. Two line segments are congruent if they are ....
4. The sum of measures of the accumulative angles at a point is ....°
5. If ∠ A complements ∠ B and ∠ A ≡ ∠ B , then m (∠ B) = ...°

3 [a] In the opposite figure :

If m (∠ AMB) = 50°
, m (∠ BMC) = 60°
, m (∠ DME) = 40°
and $\overrightarrow{MA} \perp \overrightarrow{ME}$
, **find :** m (∠ CMD)



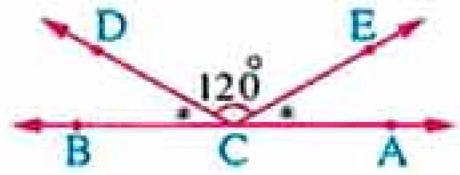
UNIT 4

[b] In the opposite figure :

$C \in \overleftrightarrow{AB}$, m (∠ ECD) = 120°

and m (∠ ACE) = m (∠ BCD)

Find : m (∠ ACE)



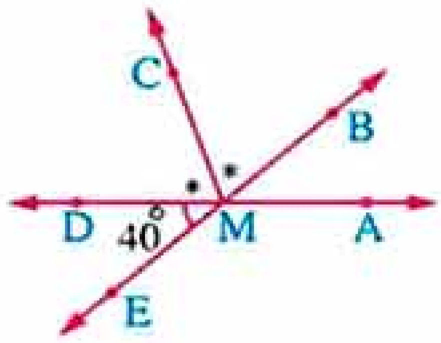
4 **[a] In the opposite figure :**

$\overrightarrow{MC}$ bisects ∠ BMD

, $\overleftrightarrow{AD} \cap \overleftrightarrow{BE} = \{M\}$

and m (∠ DME) = 40°

Find : m (∠ AMC)



[b] In the opposite figure :

The two polygons ABCD , EFGH are congruent ,

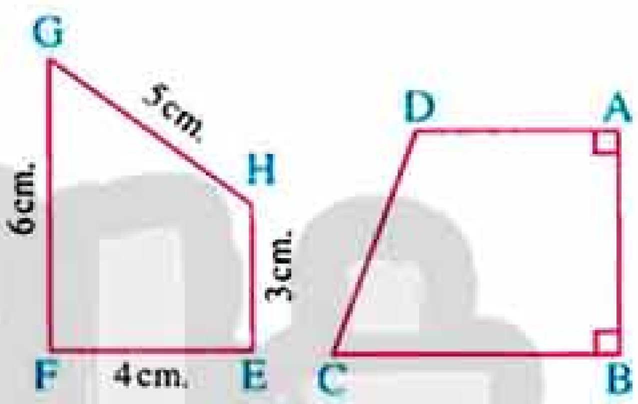
EF = 4 cm. , FG = 6 cm.

, GH = 5 cm. , HE = 3 cm.

, m (∠ A) = m (∠ B) = 90°

Find : 1 The perimeter of the polygon ABCD

2 m (∠ F) , m (∠ E)



5 **[a] In the opposite figure :**

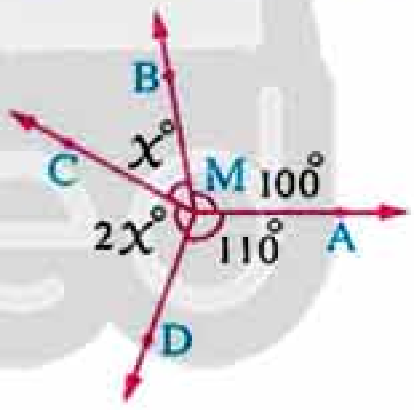
m (∠ AMD) = 110°

, m (∠ AMB) = 100°

, m (∠ BMC) = x°

and m (∠ CMD) = 2 x°

Find the value of x

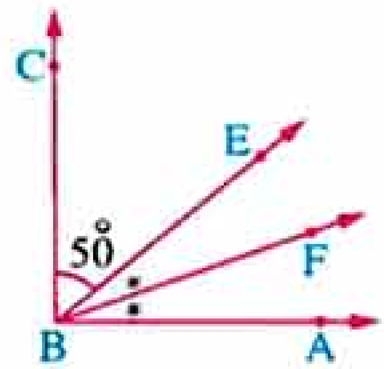


[b] In the opposite figure :

$\overrightarrow{BA} \perp \overrightarrow{BC}$, $\overrightarrow{BF}$ bisects ∠ ABE

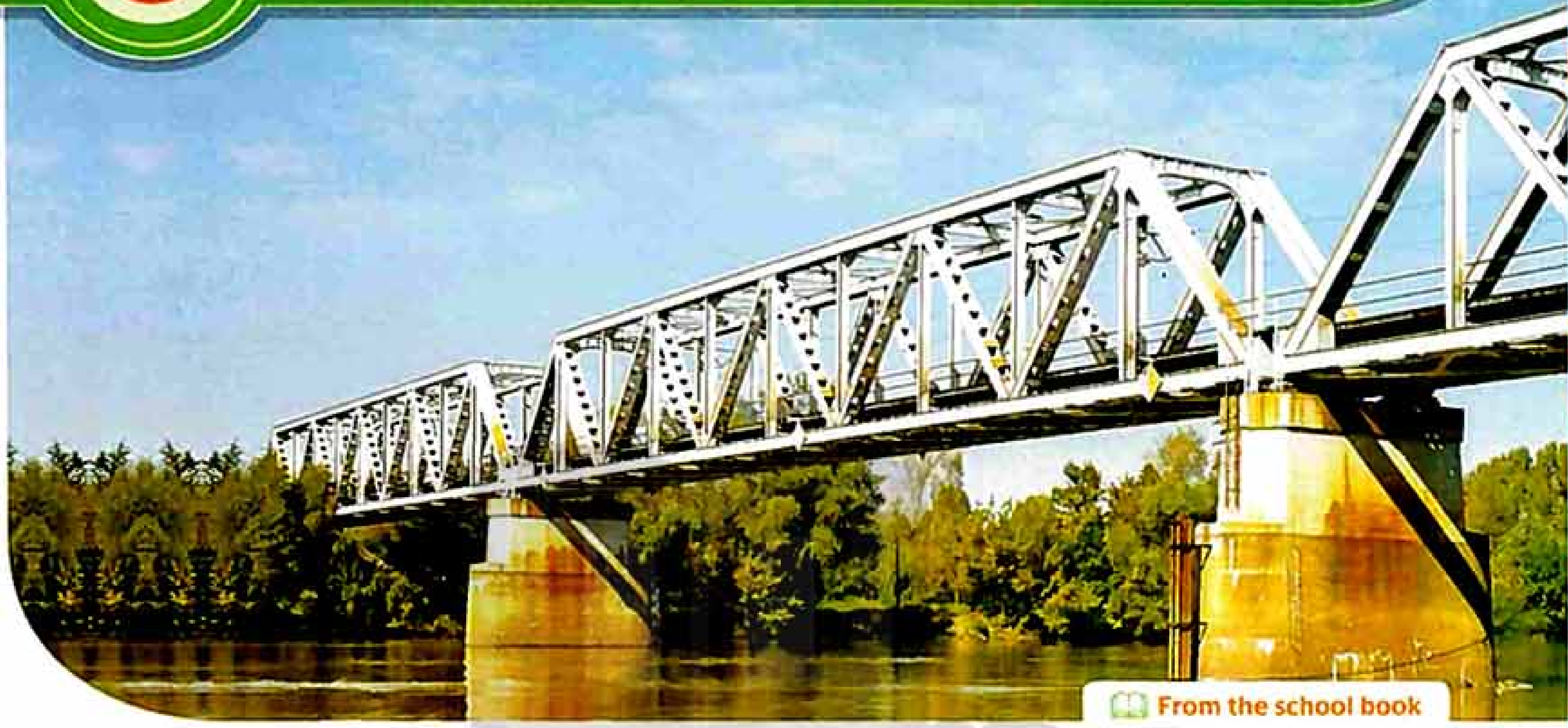
and m (∠ EBC) = 50°

Find : m (∠ FBC)



EXERCISE 4

Congruent Triangles



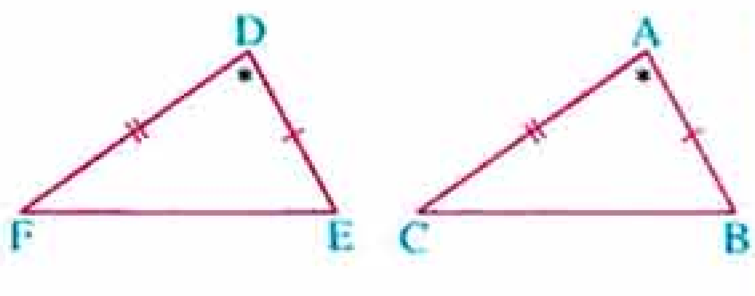
From the school book

1 **Complete the following :**

1. Any two triangles are congruent if two sides and ....
2. Any two triangles are congruent if two angles and .... in one of the triangles are congruent to their corresponding elements in the other.
3. Any two triangles are congruent if each .... is congruent to its corresponding .... in the other triangle.
4. Any two right-angled triangles are congruent if ....
5. The diagonal of the rectangle divides its surface into two .... triangles.
6. If $\Delta ABC \equiv \Delta XYZ$, then AB = ... and m ($\angle$ Z) = m ($\angle$ ....)
7. If AB = LM , BC = MN and m ($\angle$ B) = m ($\angle$ M) , then the two triangles .... and .... are congruent.

2 **In each of the following figures , show if the two triangles are congruent or not.**
If they are congruent , name the case of congruence.
If they are not congruent , give reason "Given that the similar signs denote the congruence of the shown elements labelled by these signs".

1



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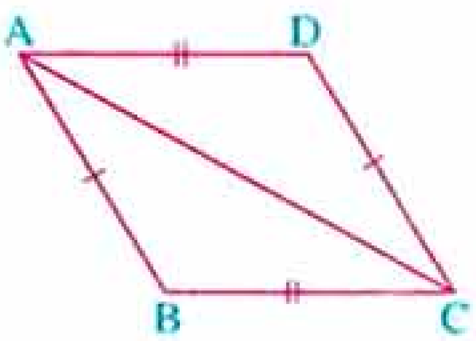

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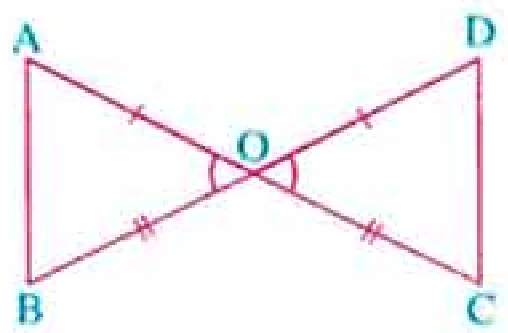

UNIT 4

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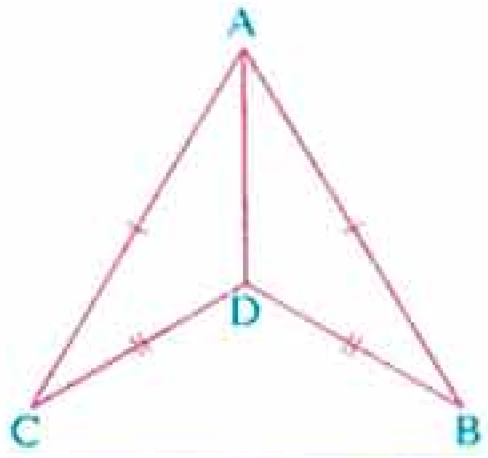

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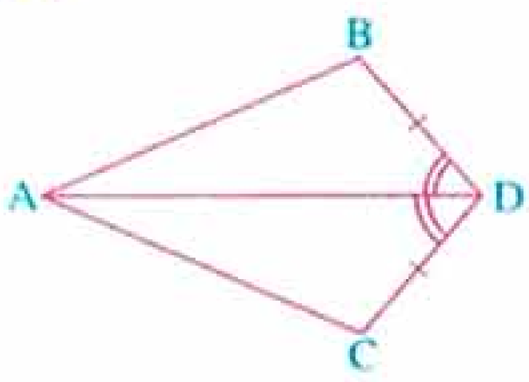

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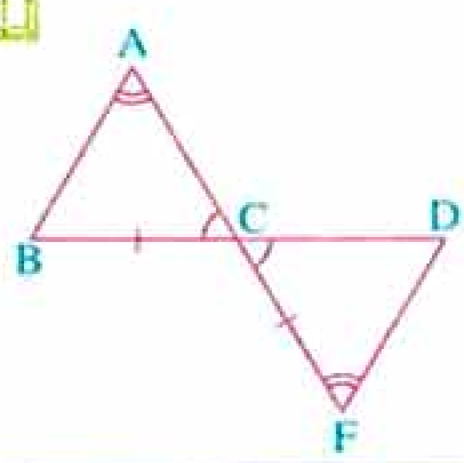

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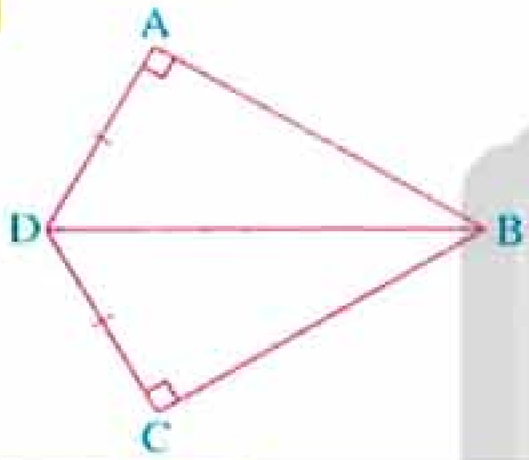

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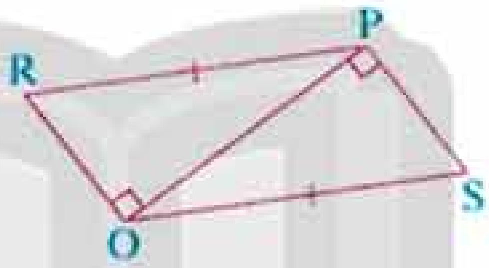

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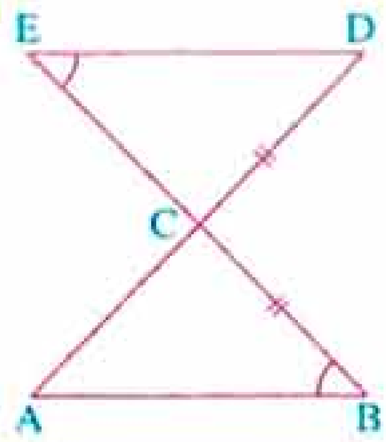

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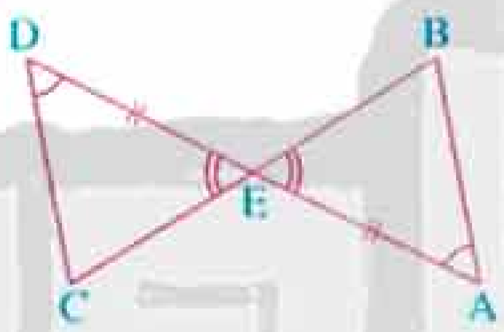

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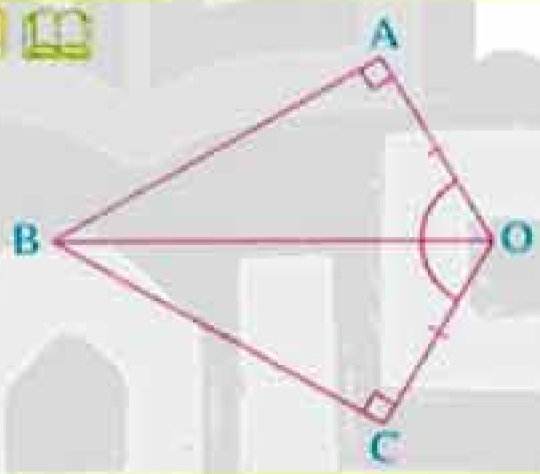

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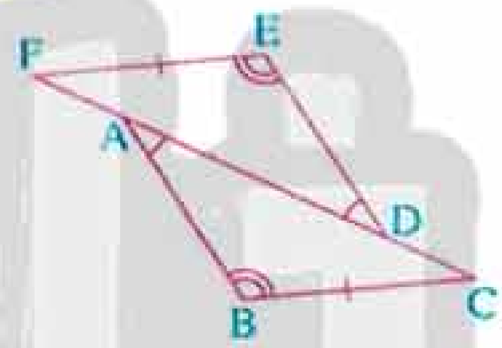

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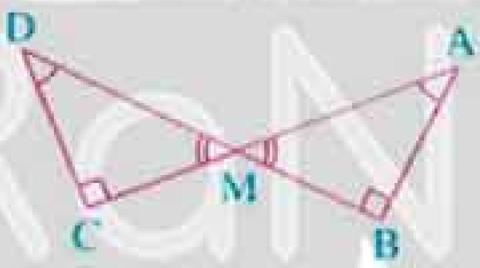

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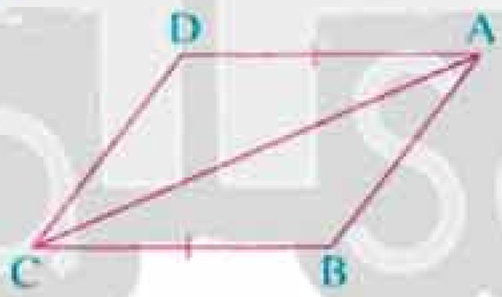

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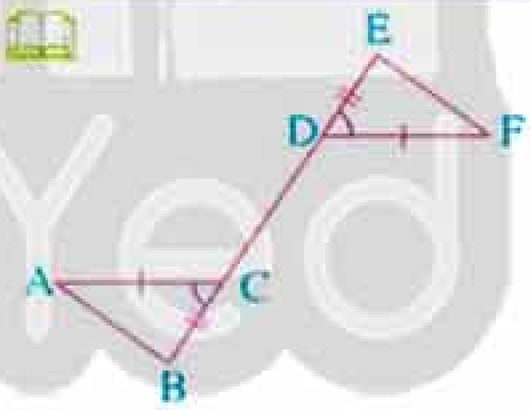

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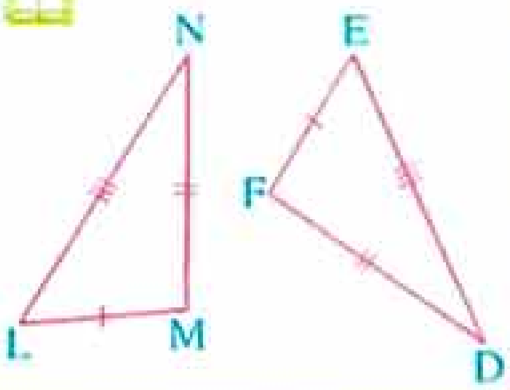

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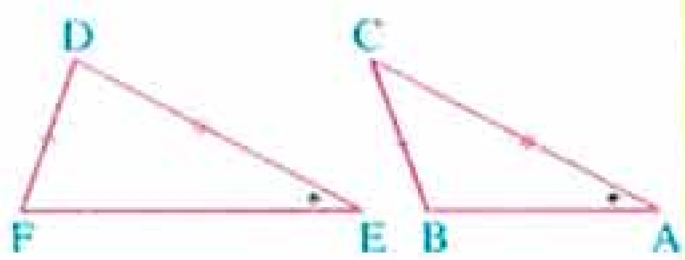

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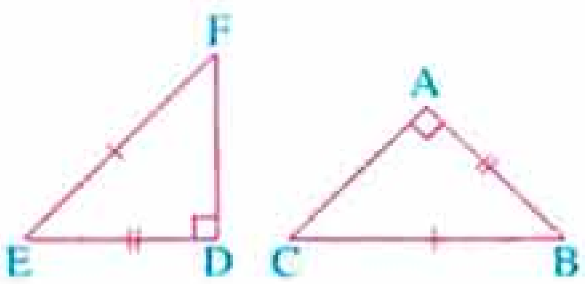

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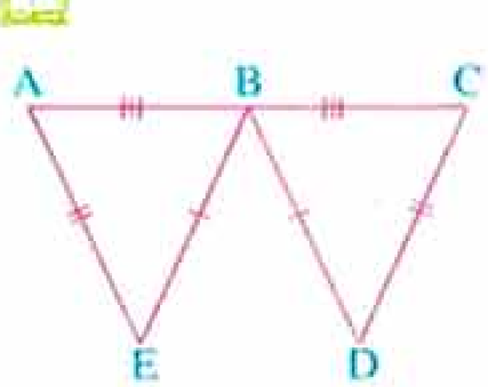

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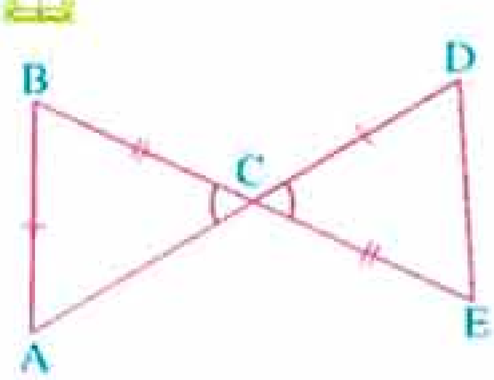

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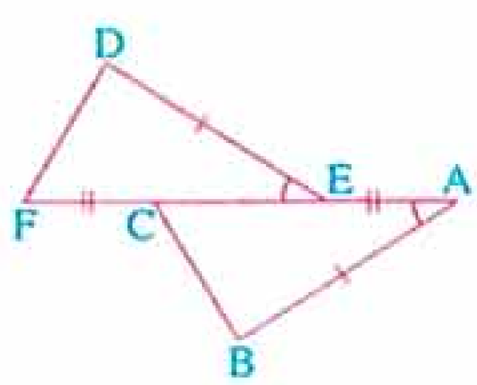
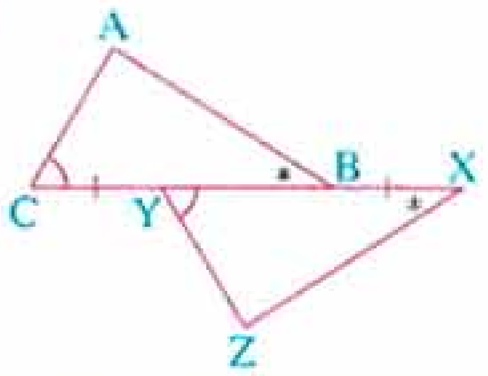


Exercise 4

22


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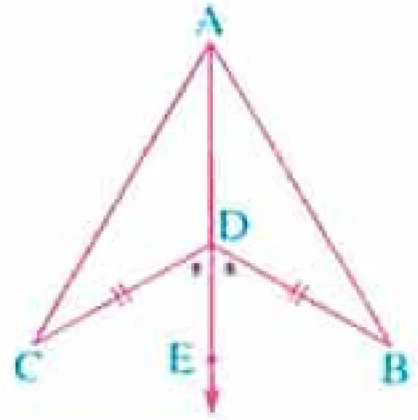

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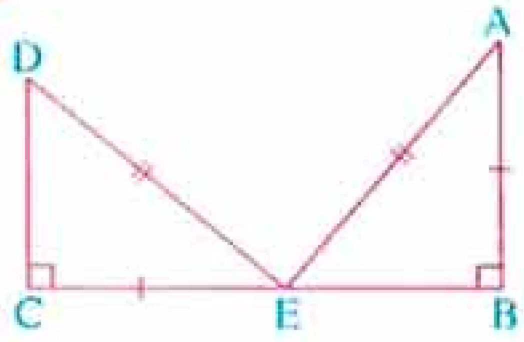

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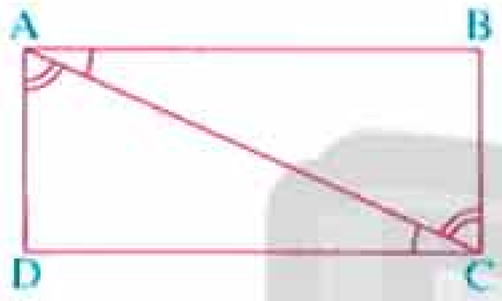

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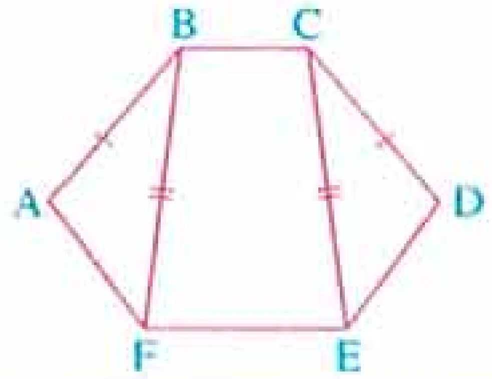
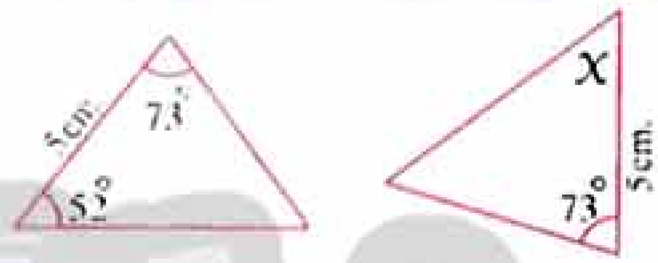


3 **In the opposite figure :**

These triangles are congruent

Complete : $x =$ ....°



4 **In the opposite figure :**

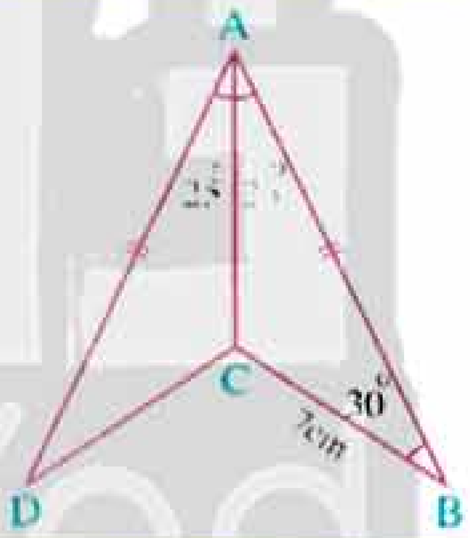
If AB = AD , BC = 7 cm. , m (∠ BAC) = m (∠ DAC) = 25°

and m (∠ B) = 30°

, complete the following :

1 Δ ACB ≡ Δ .... | 2 m (∠ D) = ...°

3 CD = ... cm. | 4 m (∠ ACD) = ...°

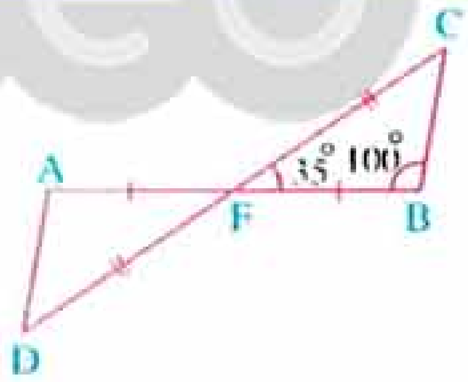


5 **In the opposite figure :**

If $\overline{CD} \cap \overline{BA} = \{F\}$, FA = FB , CF = FD ,

m (∠ CFB) = 35° and m (∠ B) = 100° ,

then complete : m (∠ D) = ...°



6 **In the opposite figure :**

If BC = FD , m (∠ A) = m (∠ E) = 95° ,

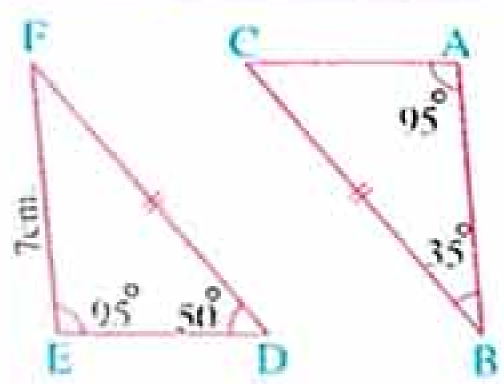
m (∠ B) = 35° , m (∠ D) = 50° and FE = 7 cm.

, complete the following :

1 m (∠ C) = ...° | 2 m (∠ F) = ...°

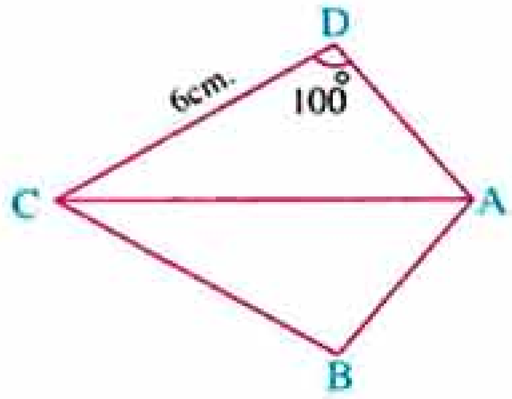
3 Δ ABC ≡ .... | 4 $\overline{AC}$ ≡ ....

5 AB = ... cm.



UNIT 4

7 **In the opposite figure :**



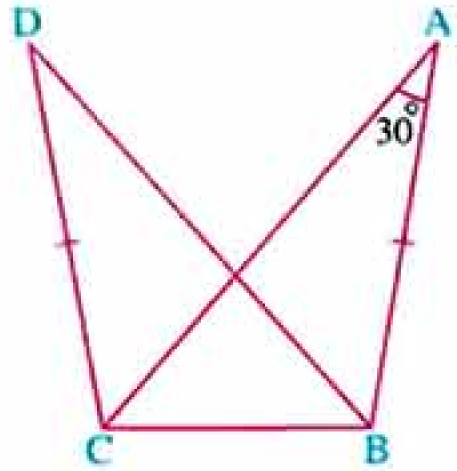
If $\overleftrightarrow{AC}$ bisects ∠ DCB , ∠ DAB , m (∠ D) = 100°

and DC = 6 cm. , **complete the following :**

1 Δ ADC ≡ Δ ....
2 m (∠ B) = ...°

3 BC = ... cm.

8 **In the opposite figure :**



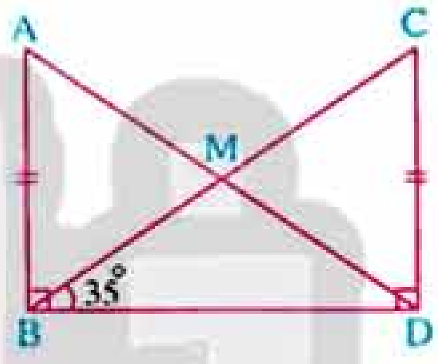
If AB = DC , AC = DB and m (∠ A) = 30°

, **complete the following :**

1 Δ ABC ≡ Δ ....
2 m (∠ D) = ...°

3 m (∠ DBC) = m (∠ ....)

9 **In the opposite figure :**



If AB = CD , m (∠ DBC) = 35° ,

$\overline{AB} \perp \overline{BD}$ and $\overline{DC} \perp \overline{DB}$, **then complete the following :**

1 m (∠ A) = ...°
2 m (∠ ADC) = ...°

3 m (∠ DMB) = ...°

10 **Choose the correct answer from the given ones :**

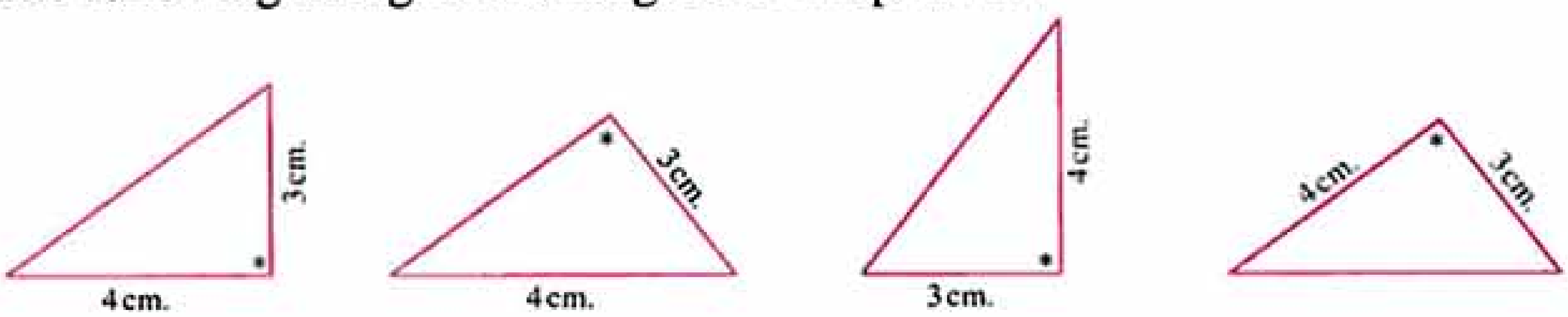
1 The following triangles are congruent except ....

(a) (b) (c) (d)

2 The following triangles are congruent except ....

(a) (b) (c) (d)

3 The following triangles are congruent except ....



(a) (b) (c) (d)

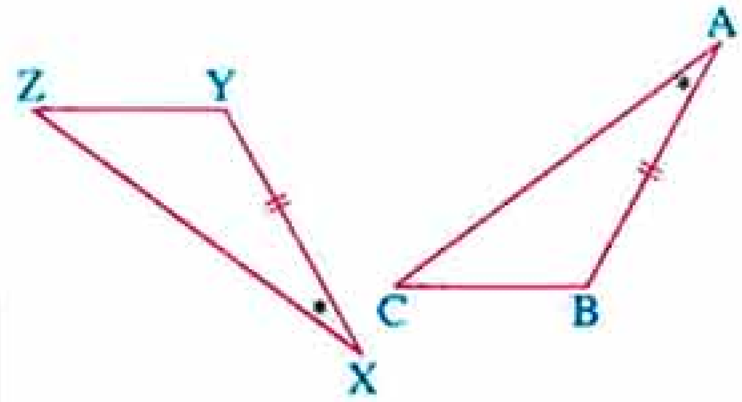
Exercise 4

4 Which pair of the following triangles are congruent ?

(a) (b) (c) (d)

5 **In the opposite figure :**

The necessary and enough condition which makes the two triangles ABC and XYZ be congruent is ....



(a) BC = YZ (b) AC = XZ

(c) m (∠ C) = m (∠ Z) (d) m (∠ B) = m (∠ Z)

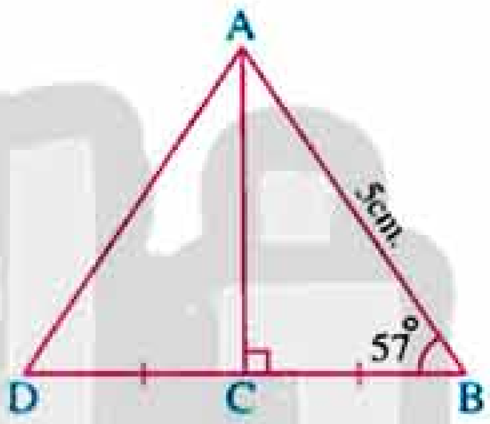
11 **In the opposite figure :**

C is the midpoint of $\overline{BD}$, $\overline{AC} \perp \overline{BD}$,

AB = 5 cm. and m (∠ B) = 57°

Find : 1 The length of $\overline{AD}$

2 m (∠ DAC)



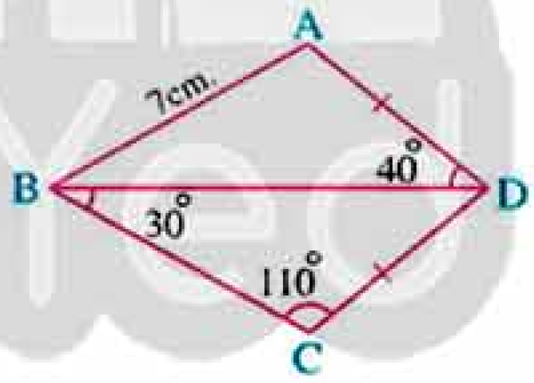
12 **In the opposite figure :**

AD = DC , m (∠ ADB) = 40° , m (∠ DBC) = 30° ,

m (∠ BCD) = 110° and AB = 7 cm.

Find : 1 The length of $\overline{BC}$

2 m (∠ BAD)

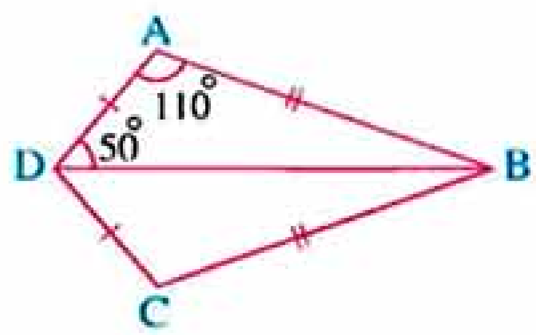


13 **In the opposite figure :**

BA = BC , DA = DC ,

m (∠ ADB) = 50° and m (∠ BAD) = 110°

Find : m (∠ ABC)

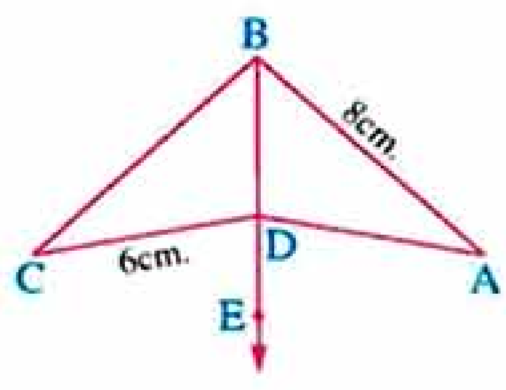


14 **In the opposite figure :**

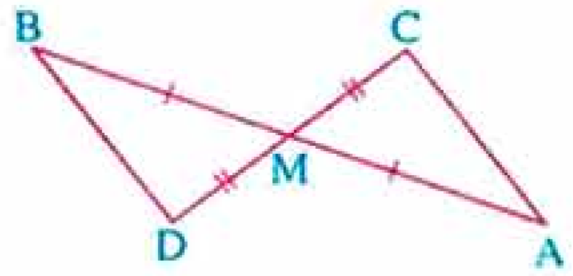
$\overrightarrow{BE}$ bisects ∠ ADC , ∠ ABC , DC = 6 cm. and AB = 8 cm.

Find : 1 The length of $\overline{CB}$

2 The length of $\overline{AD}$



15 **In the opposite figure :**

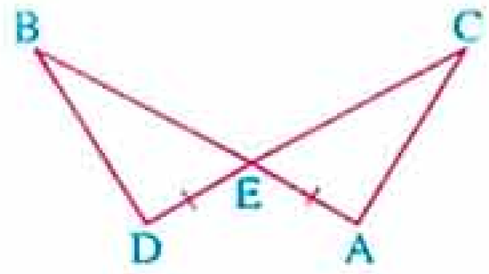


$\overline{AB} \cap \overline{CD} = \{M\}$, AM = BM

and CM = DM

Is Δ AMC $\equiv$ Δ BMD ? Why ?

16 **In the opposite figure :**

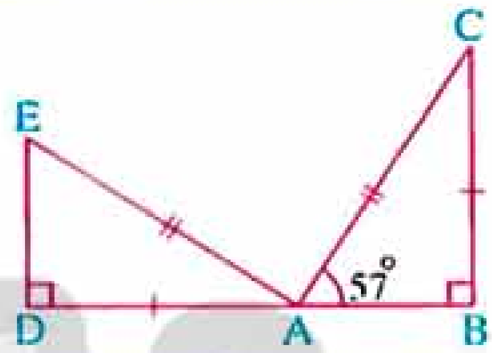


$\overline{AB} \cap \overline{CD} = \{E\}$, AE = ED and $\angle$ A $\equiv$ $\angle$ D

Is Δ ACE $\equiv$ Δ DBE ? Why ?

Then prove that : CE = EB

17 **In the opposite figure :**

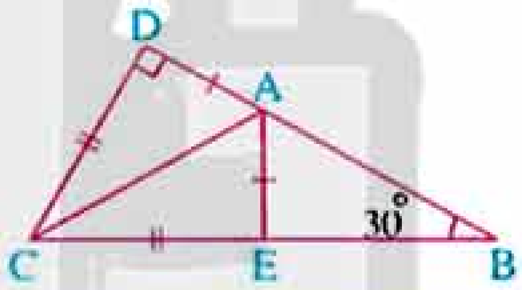


BC = AD , AC = AE

and m ($\angle$ CAB) = 57°

Find the measures of the unknown angles in Δ ADE

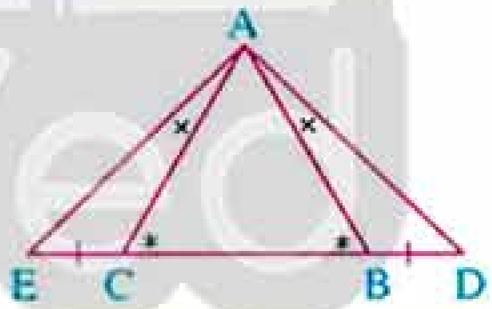
18 **In the opposite figure :**



AD = AE , DC = CE , m ($\angle$ ADC) = 90° and m ($\angle$ B) = 30°

Find : m ($\angle$ BAE)

19 **In the opposite figure :**



BD = CE , m ($\angle$ ABC) = m ($\angle$ ACB)

and m ($\angle$ BAD) = m ($\angle$ CAE)

Is AD = AE ? Why ?

20 **Complete each of the following :**

1. If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ A) = 50° and m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...°
2. If Δ ABC $\equiv$ Δ LMN , m ($\angle$ L) = 40° and m ($\angle$ B) = 90° , then m ($\angle$ C) = ...°
3. If Δ ABC $\equiv$ Δ XYZ and m ($\angle$ A) + m ($\angle$ B) = 120° , then m ($\angle$ Z) = ...°
4. If Δ ABC $\equiv$ Δ DEF and m ($\angle$ C) = 90° , then m ($\angle$ D) + m ($\angle$ E) = ...°
5. If Δ ABC $\equiv$ Δ XYZ , the perimeter of Δ ABC = 12 cm. , XY = 4 cm. and YZ = 5 cm. , then AC = ...

21 **a** Use a protractor to draw a triangle whose angles have measures 50° , 60° and 70°

b Can you draw another triangle whose angles have measures 50° , 60° and 70° but it is not congruent to the first triangle ?

Exercise 4

22 **Study these figures and calculate the values of X and Y :**

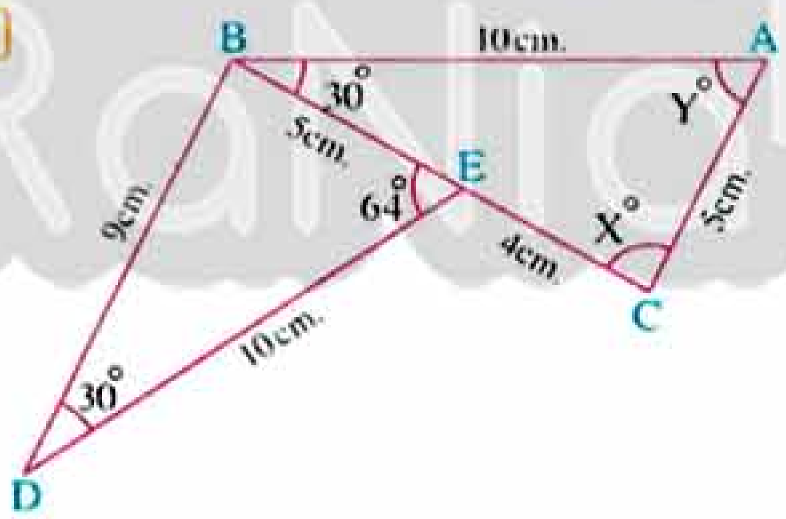
1

2

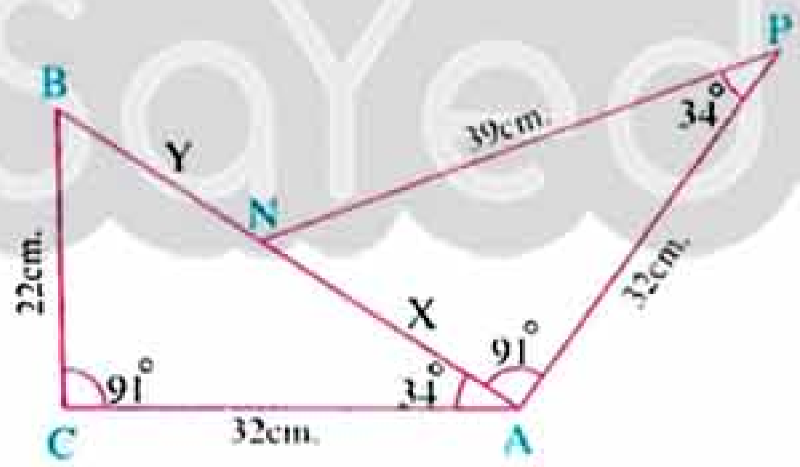
[Hint : The two angles of the base in the isosceles triangle are equal in measure]

3

4



5



23 **Study the data for Δ ABC and Δ XPG Are these triangles congruent ? Write if applicable , a correct statement of congruence and state the test used.**

1 AB = PX , AC = XG , $\angle$ A $\equiv$ $\angle$ X

2 BC = PG , BA = XP , $\angle$ B $\equiv$ $\angle$ G

3 AB = PG , BC = PX , AC = XG

4 AB = XP , CA = GX , $\angle$ B $\equiv$ $\angle$ P

5 $\angle$ B $\equiv$ $\angle$ G , $\angle$ C $\equiv$ $\angle$ X , BC = XG

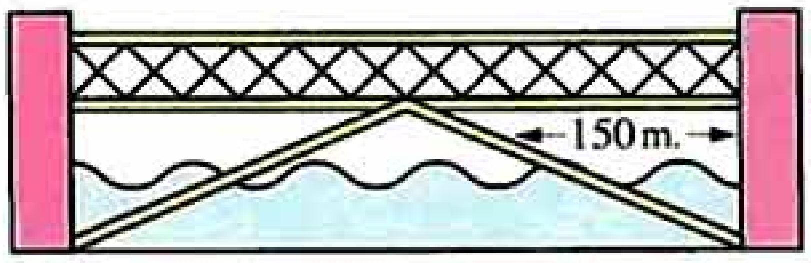
6 $\angle$ A $\equiv$ $\angle$ X , $\angle$ B $\equiv$ $\angle$ P , AC = PG

UNIT 4

Life Applications

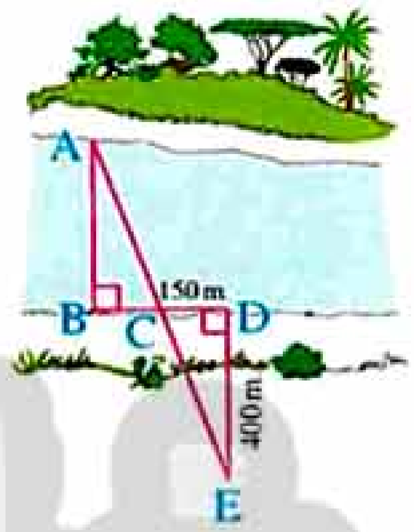
24 **In the opposite figure :**

A horizontal bridge is built above a part of the river on two vertical pillars equal in length and two sloping carriers equal in length.



Using the drawing , find the length of the bridge showing the steps of solution.

25 To find the width of the river AB , put the point C on the shore of the river , then measure the distance between B and C and move the same distance to the point D , then walk perpendicularly to reach the point E such that the points A , C and E are collinear and measure the length of $\overline{DE}$



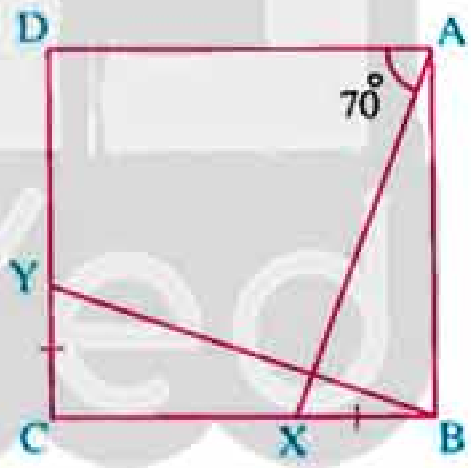
Using the above-mentioned method , and the data on the opposite illustrative drawing , find the width of the river AB

For excellent pupils

26 **In the opposite figure :**

ABCD is a square , BX = CY and m (∠ XAD) = 70°

Find : m (∠ YBC) with showing the steps of the solution.



EXERCISE 5 Parallelism

From the school book

1 Complete the following :

1. The straight line which is perpendicular to one of two parallel straight lines is .... to the other straight line in the plane.
2. If two straight lines are parallel to a third straight line , then they are ....
3. If a straight line cuts two parallel straight lines , then each two alternate angles are ....
4. If a straight line cuts two parallel straight lines , then each two corresponding angles are ....
5. If a straight line cuts two parallel straight lines , then each two interior angles in the same side of the transversal are ....
6. If a straight line cuts two straight lines and there are two corresponding angles having the same measure , then the two straight lines are ....
7. If a straight line cuts two straight lines and there are two alternate angles having the same measure , then the two straight lines are ....
8. If a straight line cuts two straight lines and there are two interior angles in the same side of the transversal are supplementary , then the two straight lines are ....
9. If a straight line cuts several parallel lines and the intercepted parts of this transversal between these parallel straight lines are equal in length , then the intercepted parts for any transversal are ....

UNIT 4

2 In each of the following figures, the straight line L // the straight line M and the straight line K is a transversal to them. Find the measures of the angles marked by « ? »

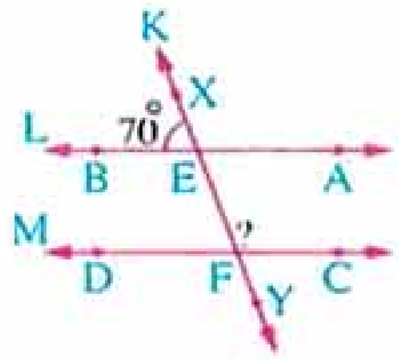


Fig. (1)

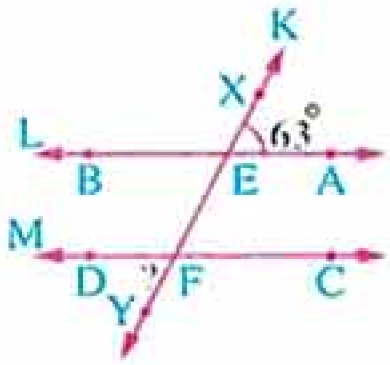


Fig. (2)

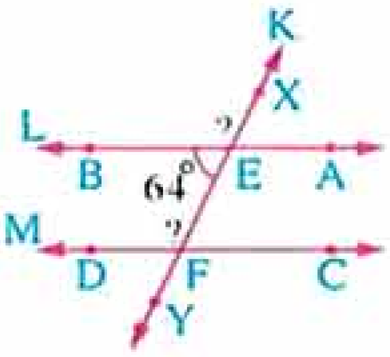


Fig. (3)

3 In each of the following figures, if $\overrightarrow{AC}$ // $\overrightarrow{BD}$ and $\overrightarrow{AB}$ // $\overrightarrow{DE}$, find the measures of the angles marked by « ? »

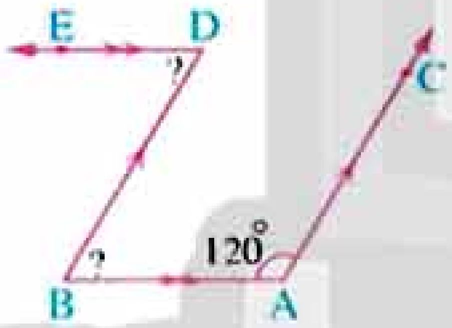


Fig. (1)

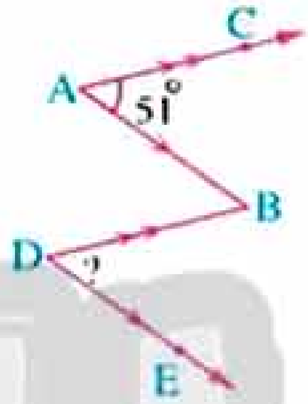


Fig. (2)

4 Complete, using the data shown in each figure :

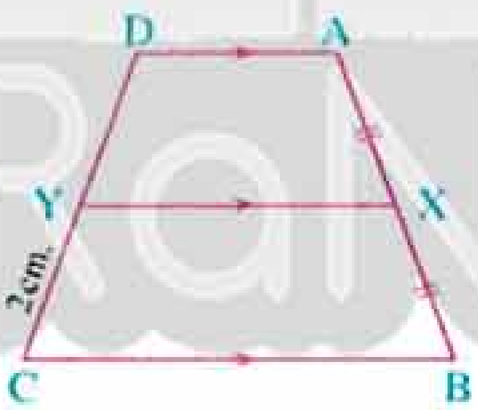


Fig. (1)

DY = ... cm.

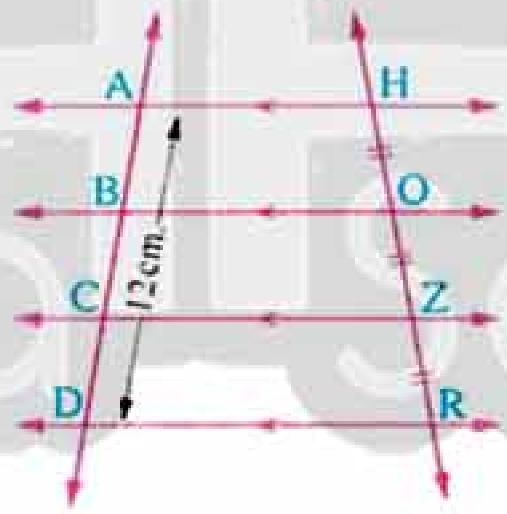


Fig. (2)

AC = ... cm.

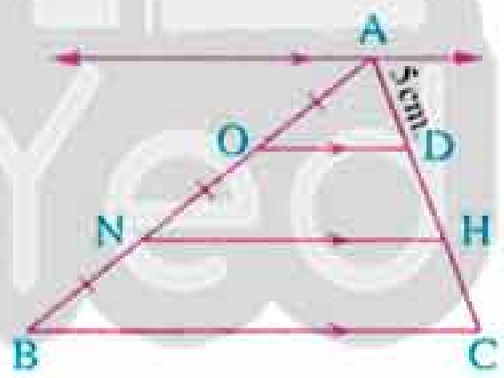


Fig. (3)

AC = ... cm.

5 In each of the following figures, if $\overleftrightarrow{MN}$ intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at E and F respectively, prove that : $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

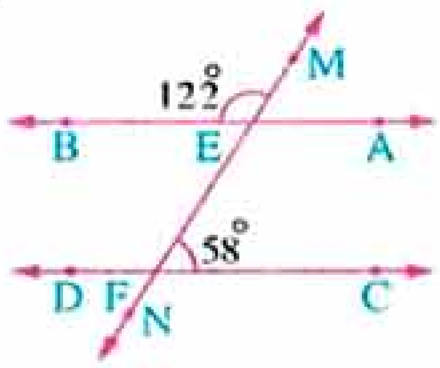


Fig. (1)

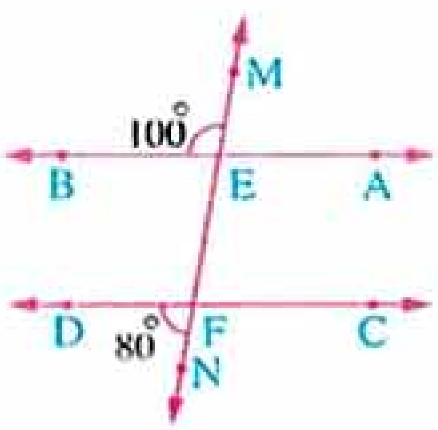


Fig. (2)

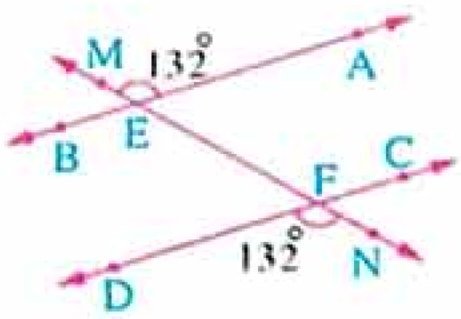


Fig. (3)

Exercise 5

6 In each of the following figures , show with reasons why is $\overline{AD} \parallel \overline{BC}$:

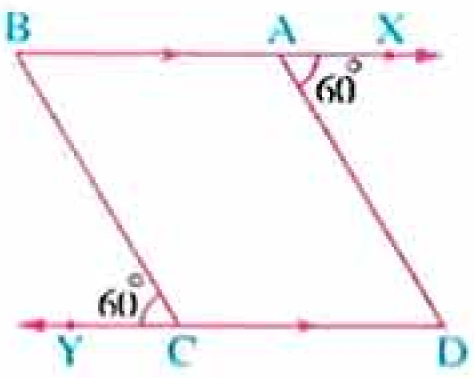


Fig. (1)

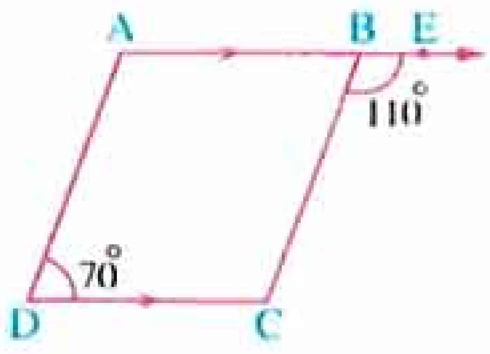


Fig. (2)

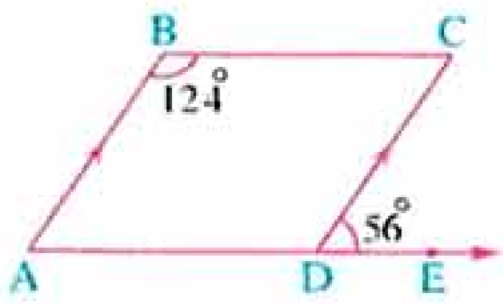
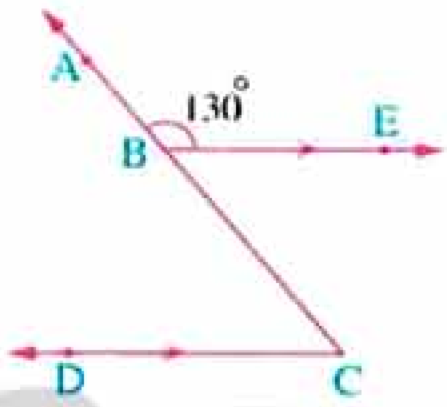


Fig. (3)

7 **Choose the correct answer from those given :**

1 **In the opposite figure :**



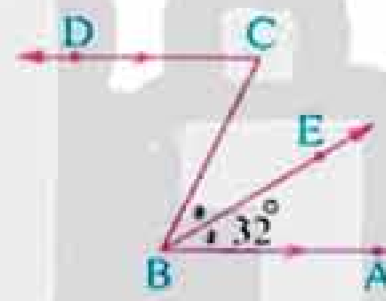
$B \in \overline{AC}$, $\overrightarrow{BE} \parallel \overrightarrow{CD}$ and m (∠ ABE) = 130°

, then m (∠ C) = ...

(a) 130° (b) 40°

(c) 50° (d) 90°

2 **In the opposite figure :**



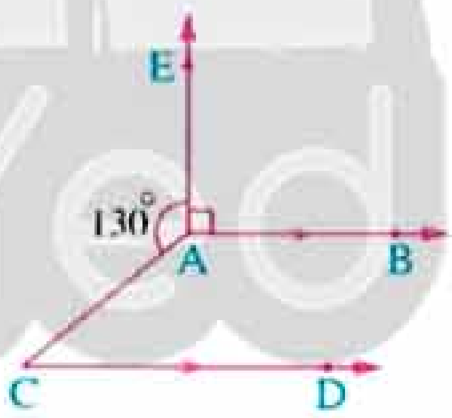
$\overrightarrow{BE}$ bisects ∠ ABC , $\overrightarrow{BA} \parallel \overrightarrow{CD}$ and

m (∠ ABE) = 32° , then m (∠ C) = ...

(a) 32° (b) 64°

(c) 60° (d) 80°

3 **In the opposite figure :**



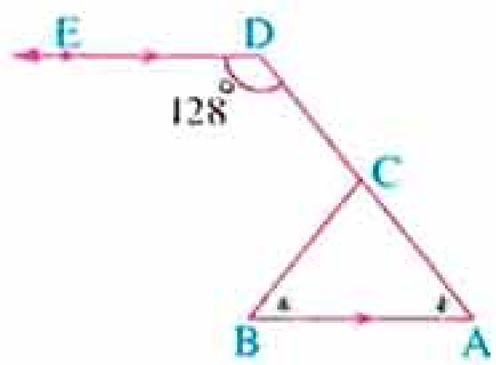
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, m (∠ EAC) = 130°

and m (∠ EAB) = 90° , then m (∠ C) = ...

(a) 90° (b) 130°

(c) 140° (d) 40°

4 **In the opposite figure :**



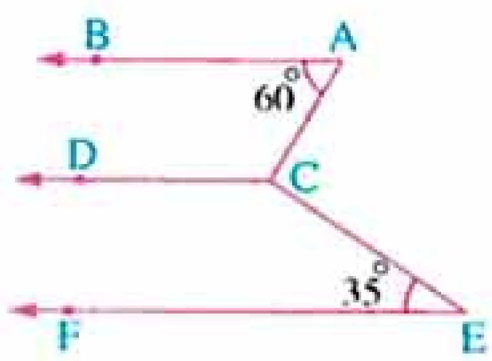
$\overline{AB} \parallel \overline{DE}$, m (∠ D) = 128° ,

m (∠ A) = m (∠ B) and $C \in \overline{AD}$, then m (∠ B) = ...

(a) 64° (b) 128°

(c) 52° (d) 26°

5 **In the opposite figure :**



$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AB} \parallel \overrightarrow{EF}$, m (∠ A) = 60° and

m (∠ E) = 35° , then m (∠ ACE) = ...

(a) 60° (b) 35°

(c) 95° (d) 85°

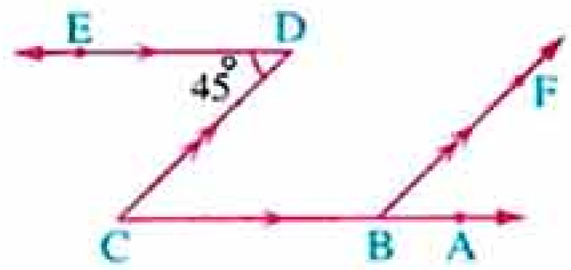
UNIT 4

6 **In the opposite figure :**

m (∠ D) = 45° , $\overrightarrow{DE}$ // $\overrightarrow{CA}$ and $\overrightarrow{CD}$ // $\overrightarrow{BF}$, then m (∠ ABF) = ...

(a) 45° (b) 90°

(c) 135° (d) 40°

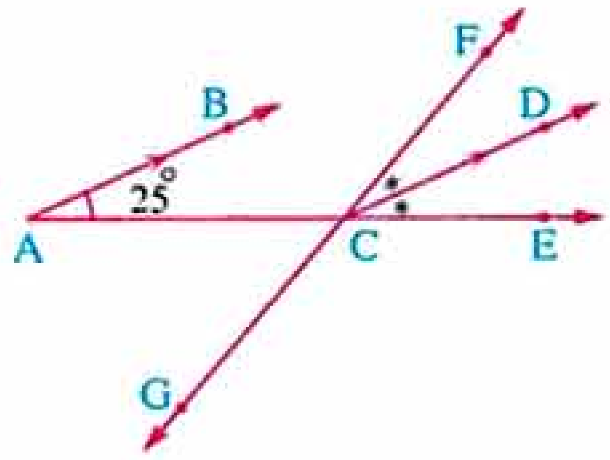


7 **In the opposite figure :**

$\overleftrightarrow{FG} \cap \overrightarrow{AE} = \{C\}$, $\overrightarrow{CD}$ bisects ∠ FCE , $\overrightarrow{CD}$ // $\overrightarrow{AB}$ and m (∠ A) = 25° , then m (∠ GCA) = ...

(a) 25° (b) 50°

(c) 130° (d) $12\frac{1}{2}°$

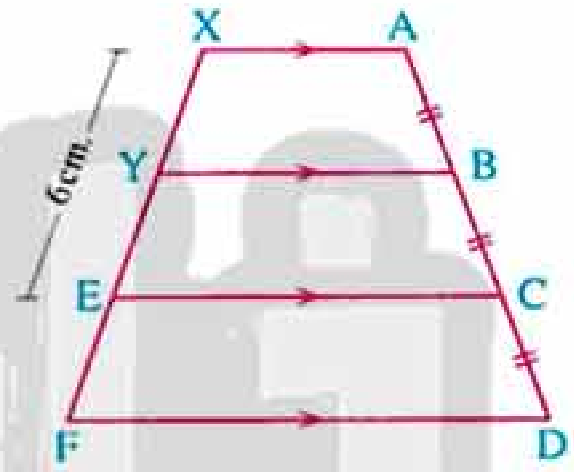


8 **In the opposite figure :**

$\overline{AX}$ // $\overline{BY}$ // $\overline{CE}$ // $\overline{DF}$, AB = BC = CD and XE = 6 cm. , then the length of $\overline{YF}$ = ...

(a) 3 cm. (b) 6 cm.

(c) 12 cm. (d) 9 cm.

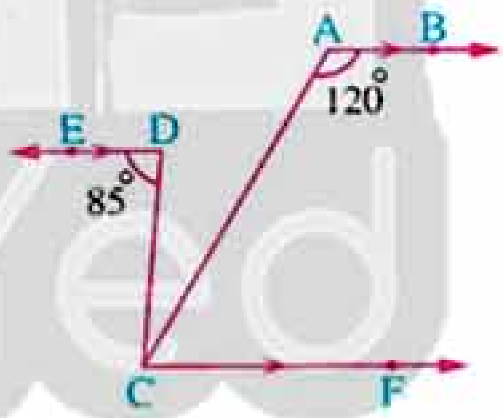


9 **In the opposite figure :**

$\overrightarrow{AB}$ // $\overrightarrow{CF}$ // $\overrightarrow{DE}$, m (∠ A) = 120° and m (∠ D) = 85° , then m (∠ ACD) = ...

(a) 60° (b) 85°

(c) 25° (d) 120°

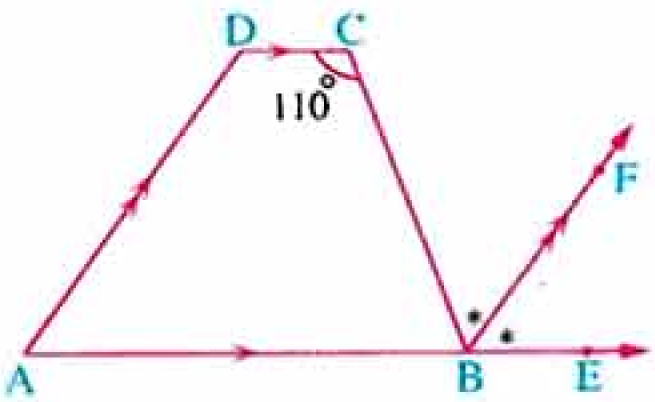


10 **In the opposite figure :**

$\overline{CD}$ // $\overrightarrow{AB}$, m (∠ C) = 110° , $\overline{AD}$ // $\overrightarrow{BF}$ and $\overrightarrow{BF}$ bisects ∠ CBE where E ∈ $\overleftrightarrow{AB}$, then m (∠ A) = ...

(a) 55° (b) 110°

(c) 70° (d) 160°

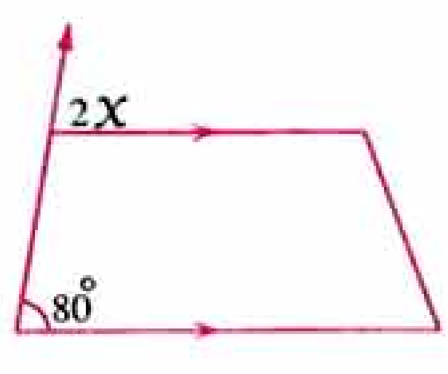


11 **In the opposite figure :**

What is the value of x ?

(a) 40° (b) 60°

(c) 80° (d) 100°



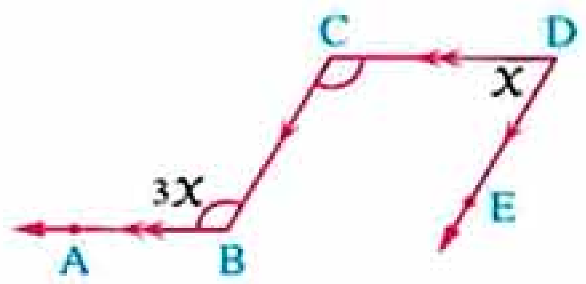
Exercise 5

12 **In the opposite figure :**

$\overline{CD} \parallel \overrightarrow{BA}$, $\overrightarrow{DE} \parallel \overline{CB}$, then $x =$ ....

(a) 60° (b) 45°

(c) 120° (d) 90°



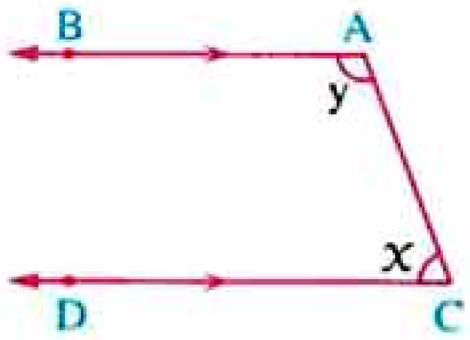
13 **In the opposite figure :**

If $\overrightarrow{AB} \parallel \overrightarrow{CD}$ and $\frac{x}{y} = \frac{7}{11}$

, then $x =$ ....

(a) 60° (b) 70°

(c) 100° (d) 110°

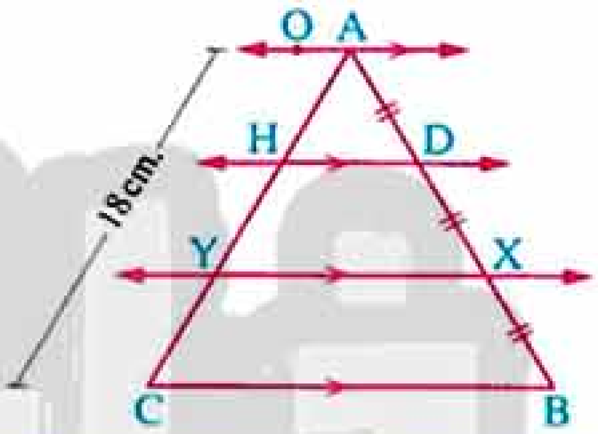


8 **In the opposite figure :**

$\overleftrightarrow{AO} \parallel \overleftrightarrow{HD} \parallel \overleftrightarrow{YX} \parallel \overline{CB}$, AD = DX = XB

and AC = 18 cm.

Find the length of $\overline{AY}$

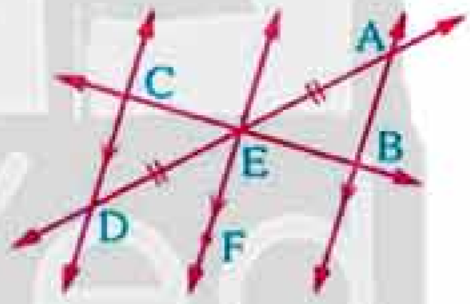


9 **In the opposite figure :**

$\overleftrightarrow{AD} \cap \overleftrightarrow{BC} = \{E\}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$, AE = DE

and BC = 8 cm.

Find the length of $\overline{BE}$

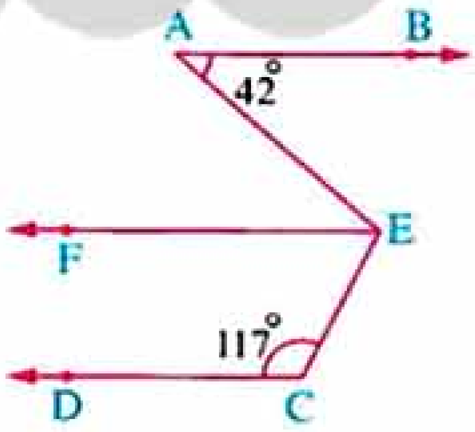


10 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{EF} \parallel \overrightarrow{CD}$

, m (∠ A) = 42° and m (∠ C) = 117°

Find : m (∠ AEC)

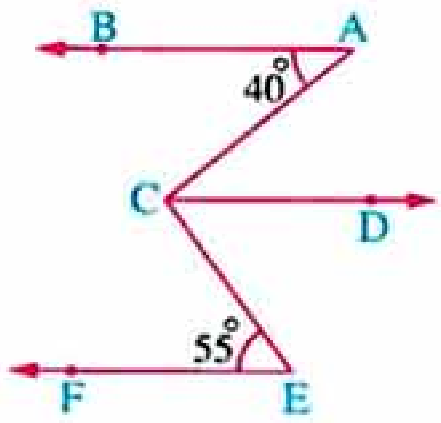


11 **In the opposite figure :**

m (∠ A) = 40° , m (∠ E) = 55°

, $\overrightarrow{AB} \parallel \overrightarrow{EF}$ and $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Find : m (∠ ACE)



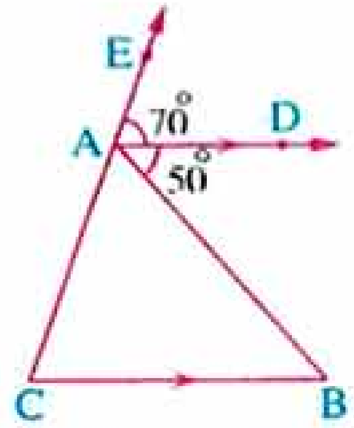
UNIT 4

12 **In the opposite figure :**

$\overrightarrow{AD} \parallel \overrightarrow{BC}$, $E \in \overrightarrow{CA}$,

$m(\angle DAE) = 70°$ and $m(\angle DAB) = 50°$

Find the measures of the angles of the triangle ABC

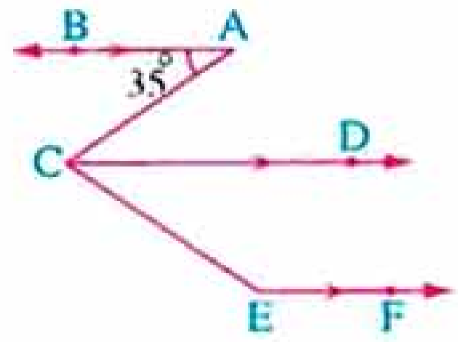


13 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{EF}$, $m(\angle A) = 35°$ and

$\overrightarrow{CD}$ bisects $\angle ACE$

Find : 1 $m(\angle DCE)$ 2 $m(\angle CEF)$

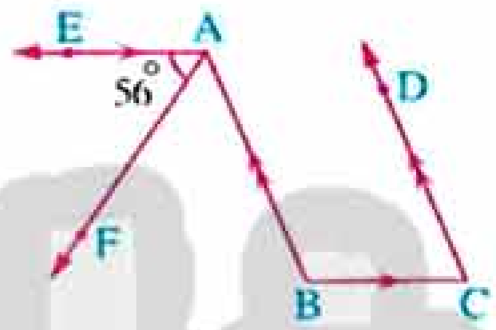


14 **In the opposite figure :**

$\overrightarrow{AE} \parallel \overrightarrow{CB}$, $\overrightarrow{BA} \parallel \overrightarrow{CD}$,

$\overrightarrow{AF}$ bisects $\angle BAE$ and $m(\angle EAF) = 56°$

Find : $m(\angle C)$

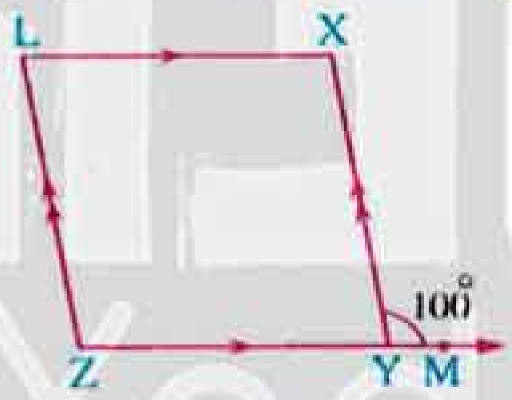


15 **In the opposite figure :**

$\overline{XL} \parallel \overline{YZ}$, $\overline{XY} \parallel \overline{LZ}$ and $m(\angle XYM) = 100°$

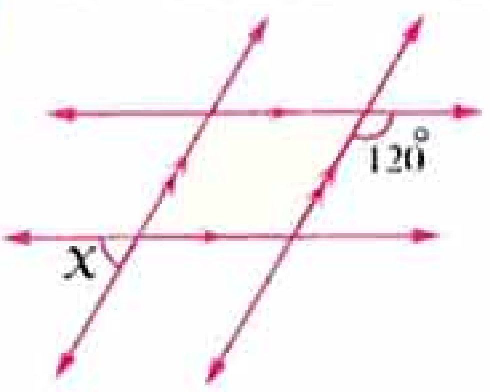
, where $M \in \overrightarrow{ZY}$

Find : 1 $m(\angle X)$ 2 $m(\angle Z)$ 3 $m(\angle L)$



16 **Find the value of x in each figure :**

1



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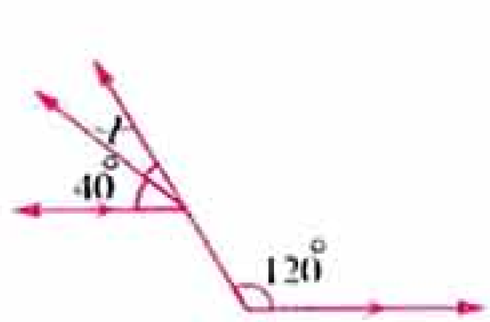

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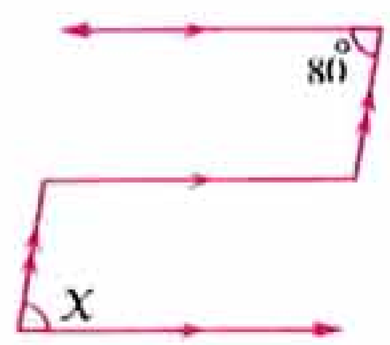

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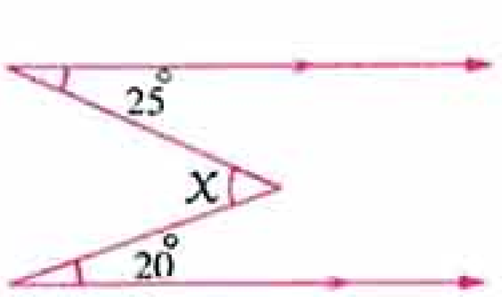

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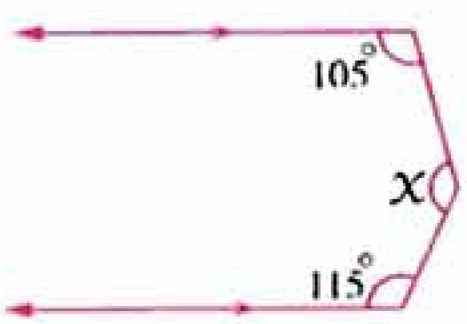

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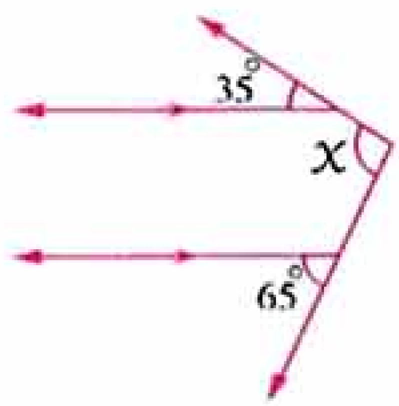
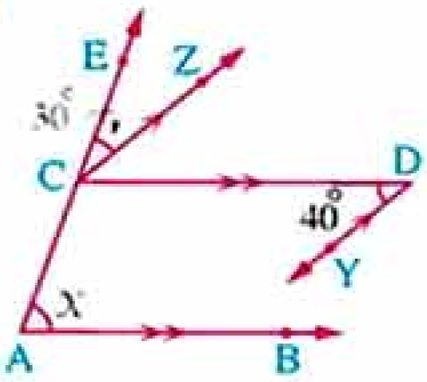


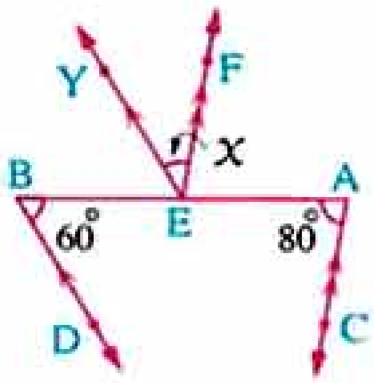
Exercise 5

17 **Find the value of x in each of the following figures :**

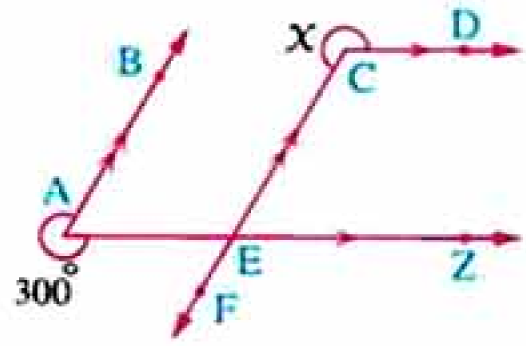
1



2



3



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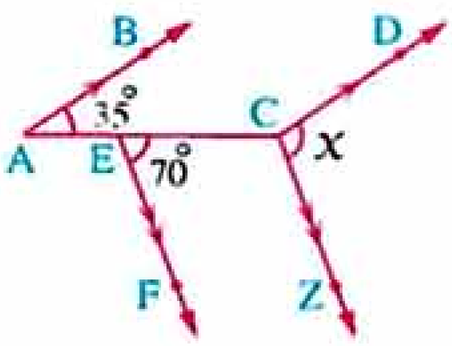

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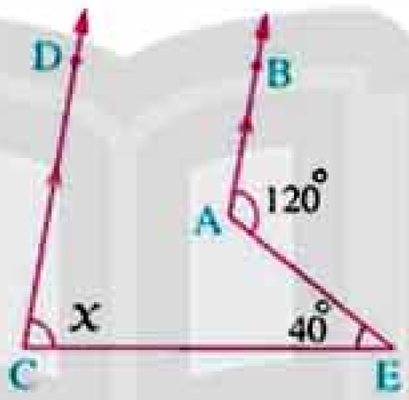

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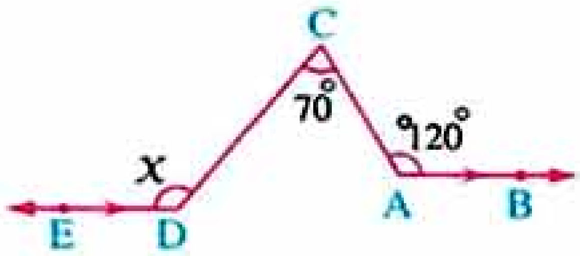
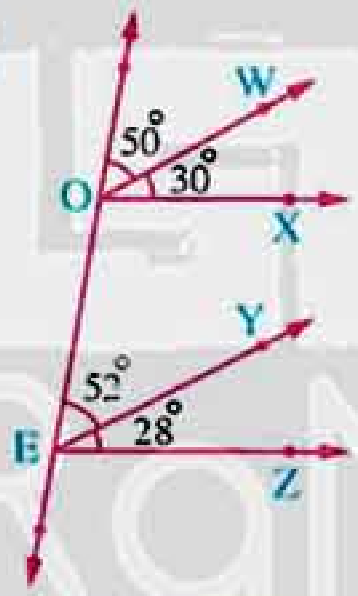
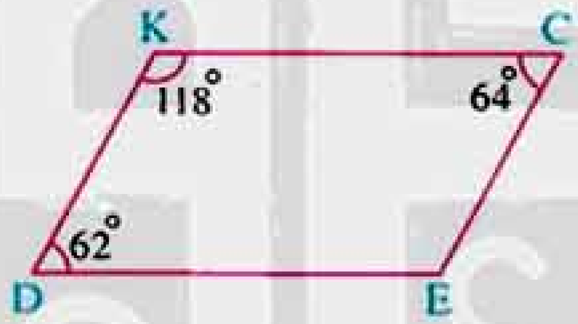


18 **Find the pairs of parallel lines in each figure :**

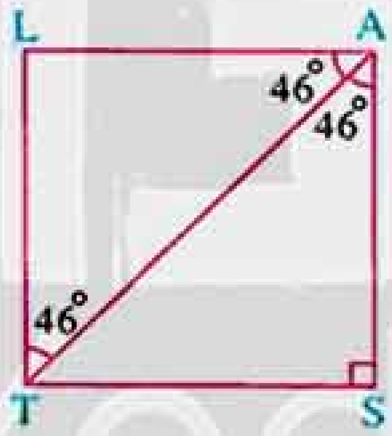
1



2



3

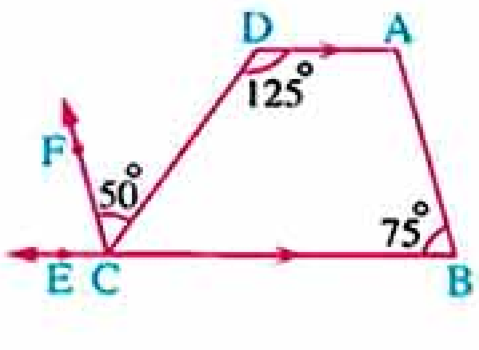


19 **In the opposite figure :**

$\overline{AD} \parallel \overrightarrow{BC}$, $E \in \overrightarrow{BC}$,

m ($\angle$ B) = 75° , m ($\angle$ D) = 125° and

m ($\angle$ DCF) = 50° Is $\overleftrightarrow{AB} \parallel \overrightarrow{CF}$? Why ?

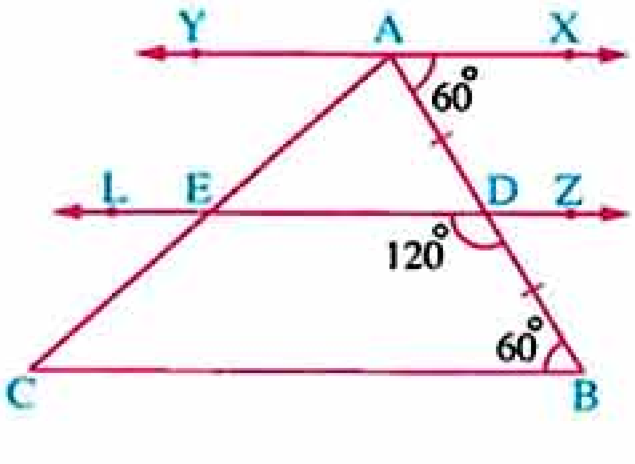


20 **In the opposite figure :**

m ($\angle$ XAD) = m ($\angle$ B) = 60°

, m ($\angle$ EDB) = 120° , AD = DB and AC = 18 cm.

Find the length of $\overline{AE}$ giving the reason.



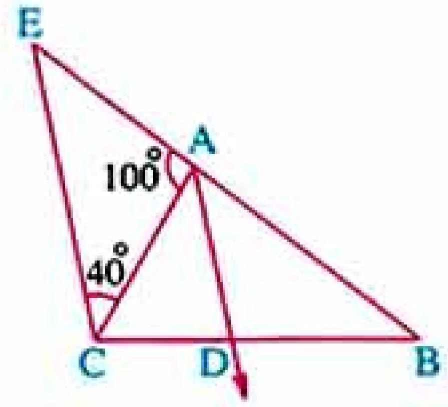
UNIT 4

21 **In the opposite figure :**

$A \in \overline{BE}$, $\overrightarrow{AD}$ bisects $\angle BAC$

, $m(\angle EAC) = 100°$ and $m(\angle ACE) = 40°$

Is $\overrightarrow{AD} \,//\, \overline{CE}$? Why ?

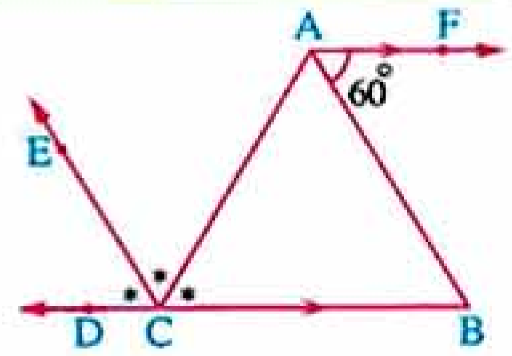


22 **In the opposite figure :**

$m(\angle FAB) = 60°$, $\overrightarrow{AF} \,//\, \overrightarrow{BD}$,

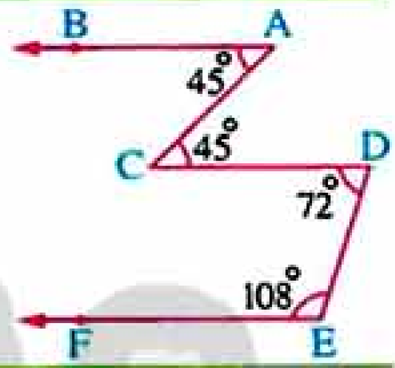
$C \in \overrightarrow{BD}$ and $m(\angle ACB) = m(\angle ACE) = m(\angle ECD)$

Is $\overline{AB} \,//\, \overrightarrow{CE}$? Why ?



23 **In the opposite figure :**

Is $\overrightarrow{AB} \,//\, \overrightarrow{DC} \,//\, \overrightarrow{EF}$? Why ?

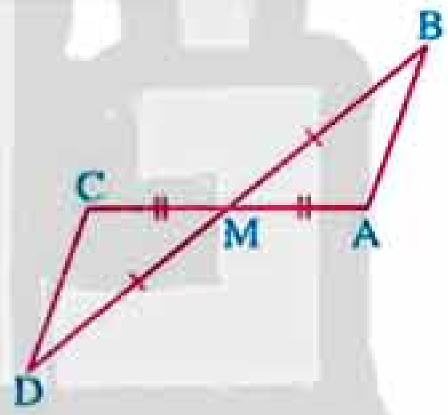


24 **In the opposite figure :**

$\overline{BD} \cap \overline{AC} = \{M\}$

, $MB = MD$ and $MA = MC$

1. Is $\Delta AMB \equiv \Delta CMD$? Why ?
2. Is $\overline{AB} \,//\, \overline{CD}$? Why ?

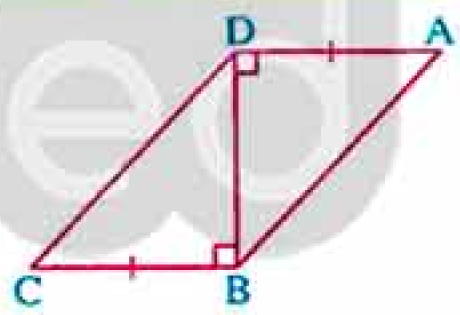


25 **In the opposite figure :**

ABCD is a quadrilateral in which

$AD = CB$ and $m(\angle ADB) = m(\angle CBD) = 90°$

Is $\overline{AB} \,//\, \overline{CD}$? Why ?

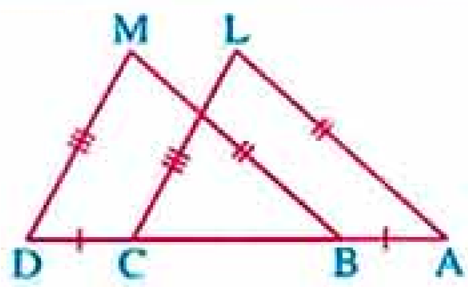


26 **In the opposite figure :**

$B \in \overline{AD}$ and $C \in \overline{AD}$

such that : $AB = CD$, $AL = BM$ and $LC = MD$

Is $\overline{AL} \,//\, \overline{BM}$, $\overline{CL} \,//\, \overline{DM}$? Why ?

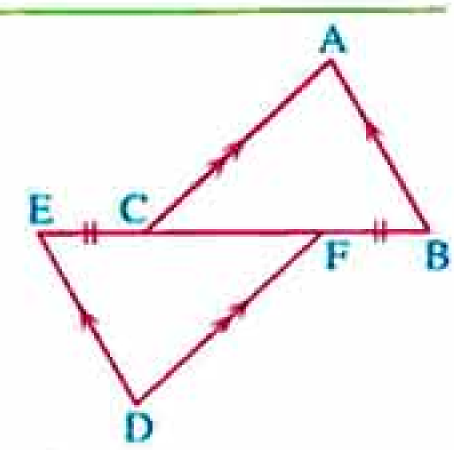


27 **In the opposite figure :**

$\overline{AB} \,//\, \overline{ED}$, $\overline{AC} \,//\, \overline{FD}$

and $\overline{BF} \equiv \overline{CE}$

Is $\overline{AB} \equiv \overline{DE}$? Why ?



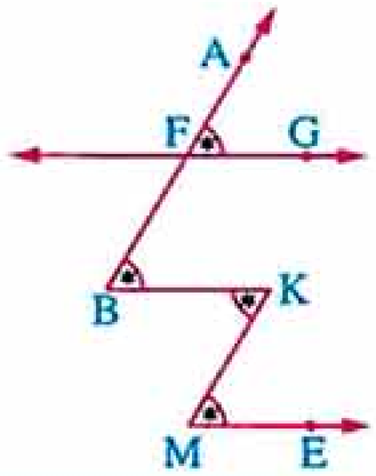
Exercise 5

28 **In the opposite figure :**

m (∠ AFG) = m (∠ B) = m (∠ K) = m (∠ M)

Write the four pairs of parallel lines.

Give your reasons.



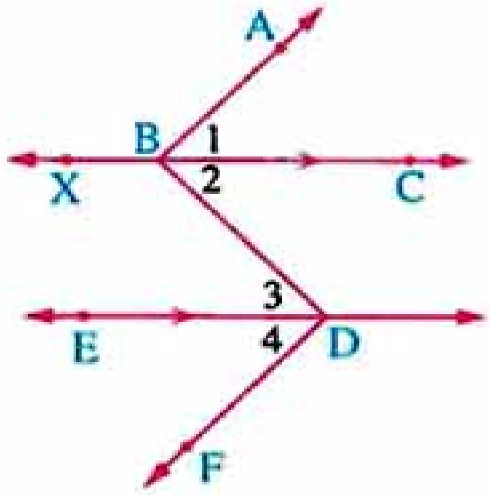
29 **In the opposite figure :**

m (∠ 1) = m (∠ 4)

and $\overleftrightarrow{BC} // \overleftrightarrow{ED}$

Does $\overrightarrow{BA} // \overrightarrow{DF}$?

Give reason.

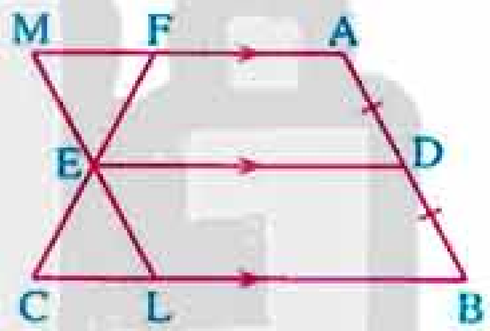


30 **In the opposite figure :**

$\overleftrightarrow{AM} // \overleftrightarrow{DE} // \overleftrightarrow{BC}$, AD = DB , F ∈ $\overline{AM}$

, L ∈ $\overline{BC}$, $\overline{ML} \cap \overline{FC} = \{E\}$

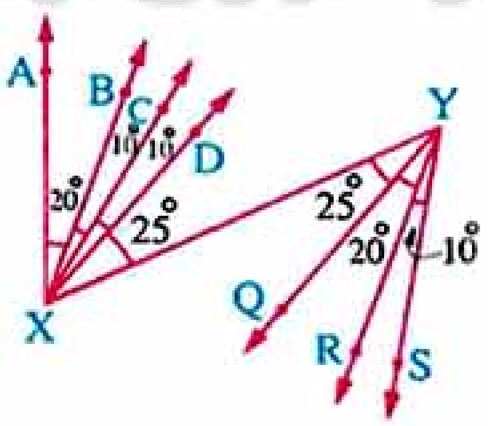
Is FM = LC ? Why ?



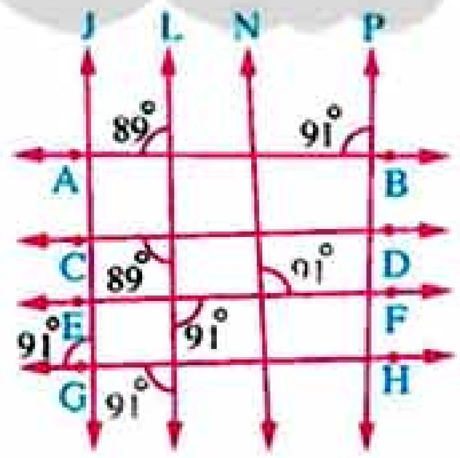
For excellent pupils

31 **In each of the following figures , name the pairs of parallel lines :**

1



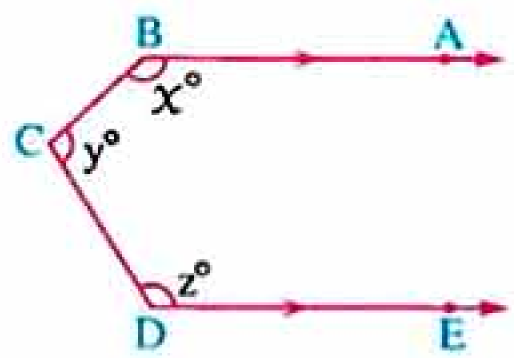
2



32 **In the opposite figure :**

$\overrightarrow{BA} // \overrightarrow{DE}$

Find the value of the expression : $x + y + z$



EXERCISE 6

Geometric Constructions

From the school book

First Constructing a perpendicular from a given point to a straight line

1 Using the ruler and the compasses, draw Δ ABC in which AB = AC = 5 cm. , BC = 6 cm. , then draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$
Then find by measuring the length of $\overline{AD}$ (Don't remove the arcs). «4 cm.»

2 Using the geometric tools, draw the equilateral triangle ABC of side length 5 cm. , then draw $\overline{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$ (Don't remove the arcs).

3 Draw Δ ABC in which AB = 6 cm. , m ($\angle$ A) = 50° , m ($\angle$ B) = 70° , then draw $\overline{CD} \perp \overline{AB}$ to cut it at D , then find the length of $\overline{CD}$ by measuring and calculate the area of Δ ABC (Don't remove the arcs). «5 cm. , 15 cm^2.»

4 Draw the equilateral triangle ABC in which the length of each side is 4 cm. , then draw $\overrightarrow{CD} \perp \overleftrightarrow{CB}$ that intersects $\overrightarrow{BA}$ at D , find by measuring the length of $\overline{DA}$ « 4 cm. »

5 **Using the geometric instruments , draw a triangle , then draw its altitudes if the triangle is :**

1. an acute-angled triangle.
2. a right-angled triangle.
3. an obtuse-angled triangle.

Are the straight lines that contain the altitudes concurrent.
Where is the position of the intersection point with respect to the triangle ?
Is it inside or outside the triangle or belongs to one of its sides ?

Exercise 6

Second — Bisecting a given line segment "Constructing the axis of symmetry of a line segment"

6 Using the ruler and the compasses , draw the line segment $\overline{BC}$ of length 7 cm. , then draw the straight line L as an axis of symmetry of it. (Don't remove the arcs)

7 Draw $\overline{AB}$ of length 6 cm. , using compasses and a ruler , draw the straight line L that is the axis of symmetry of $\overline{AB}$, where $\overline{AB} \cap L = \{C\}$

Label the point $D \in L$ such that CD = 4 cm. Measure the lengths of $\overline{DA}$ and $\overline{DB}$

(Don't remove the arcs) « 5 cm. »

8 Draw $\overline{BC}$ with a suitable length. Using compasses and the unscaled ruler, bisect $\overline{BC}$ at D and from D, draw $\overline{DA}$ perpendicular to $\overline{BC}$, then draw $\overline{AB}$ and $\overline{AC}$

Compare the lengths of $\overline{AB}$ and $\overline{AC}$ using the compasses. What do you observe ?

9 Draw the isosceles triangle ABC in which AB = AC Using the compasses , bisect $\overline{BC}$ at D Draw $\overline{AD}$, is $\overline{AD} \perp \overline{BC}$?

10 Using the geometric instruments, draw Δ XYZ in which m ($\angle$ Y) = 90° ,

XY = YZ = 4 cm., then bisect $\overline{XZ}$ at L, then draw $\overline{YL}$

Find by measuring m ($\angle$ XLY) (Don't remove the arcs) « 90° »

11 Draw the triangle ABC in which AB = AC = 4 cm. , BC = 6 cm. Bisect $\overline{AB}$ at D and $\overline{AC}$ at E , then draw $\overline{DE}$, and find its length. (Don't remove the arcs) « 3 cm. »

12 Draw the triangle ABC in which m ($\angle$ B) = 90° , AB = 8 cm. , BC = 6 cm. Bisect $\overline{AC}$ at D , is BD = $\frac{1}{2}$ AC ?

13 Draw the triangle ABC in which AB = 4 cm. , BC = 5 cm. and AC = 6 cm.

Construct the bisector altitudes of the sides of the triangle. What do you notice ?

14 **Using the geometric instruments, draw a triangle, then draw the axes of symmetry of its sides if the triangle is :**

1 an acute-angled triangle. 2 a right-angled triangle. 3 an obtuse-angled triangle.

Are the symmetry axes of the sides of the triangle concurrent ?

UNIT 4

15 Draw Δ ABC using an unscaled ruler and compasses , bisect $\overline{AB}$ and $\overline{AC}$ at D and E respectively. Draw $\overline{DE}$

1 Using the compasses , measure the length of $\overline{DE}$ and check that BC = 2 DE

2 Does $\angle$ ABC $\equiv$ $\angle$ ADE ? Does $\overline{DE}$ // $\overline{BC}$?

16 Draw the right-angled triangle XYZ at Y using the compasses and the ruler only. Bisect $\overline{XZ}$ at M. Draw $\overline{YM}$ Are MX = MY = MZ ?
Draw other right-angled triangles and repeat the same construction.
Are MX = MY = MZ ?

Third Constructing the bisector of a given angle

17 Using the geometric instruments, draw an angle of measure 120° and bisect it
(Don't remove the arcs)

18 Using the geometric tools, draw an angle of measure 75° and bisect it
(Don't remove the arcs)

19 Draw an angle whose vertex is A and its measure is 130° , use a ruler and a compasses to divide the angle A into 4 equal angles in measure. (Don't remove the arcs)

20 Using the ruler and the compasses, draw Δ ABC in which AB = AC = 3 cm. , BC = 5 cm. , then bisect $\angle$ A by the bisector $\overrightarrow{AD}$ where D $\in$ $\overline{BC}$ (Don't remove the arcs)

21 Using a ruler and the compasses , draw a triangle ABC in which AB = AC = 7 cm. , BC = 6 cm. Bisect $\angle$ B and $\angle$ C by two bisectors which intersect at M
Is MB = MC ? (Don't remove the arcs)

22 Using the geometric tools, draw Δ ABC in which AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then bisect $\angle$ B by the bisector $\overrightarrow{BD}$ to cut $\overline{AC}$ at D Find the length of $\overline{BD}$ by measuring.
(Don't remove the arcs) «2.4 cm.»

23 Draw the equilateral triangle ABC of side length 4 cm. using the compasses and the ruler bisect each of $\angle$ ABC and $\angle$ ACB , If the two bisectors intersect at M , find by measuring m ($\angle$ BMC) (Don't remove the arcs) «120°»

24 **Using the geometric instruments , draw a triangle and bisect each of its angles if the triangle is :**

1 an acute-angled triangle. 2 a right-angled triangle. 3 an obtuse-angled triangle.

What do you notice about the three bisectors of the angles of the triangle ?

Exercise 6

Fourth — Constructing an angle to be congruent to a given angle and drawing a straight line from a given point parallel to a given straight line

25 Draw $\angle$ A of measure 100° , then using the ruler and the compasses , draw another angle B such that angle B equals in measure angle A , and bisect it.

26 Using the protractor , draw $\angle$ ABC of measure 70° and on the other side of $\overline{BA}$, draw using the ruler and the compasses $\overrightarrow{AE} \parallel \overrightarrow{BC}$ (Don't remove the arcs)

27 Draw Δ ABC in which AB = 6 cm. , m ($\angle$ A) = 50° , m ($\angle$ B) = 70° using the compasses and ruler draw $\overleftrightarrow{XY}$ passing through A and parallel to $\overleftrightarrow{BC}$ (Don't remove the arcs)

28 **Using the compasses and the ruler , draw the triangle ABC in which AB = 5 cm. , BC = 6 cm. and CA = 7 cm. , D $\in \overrightarrow{CB}$ and D $\notin \overline{CB}$:**

1 Draw $\angle$ DBE congruent to $\angle$ A such that the ray $\overrightarrow{BE}$ is lying between the two rays $\overrightarrow{BA}$ and $\overrightarrow{BD}$

2 **Complete :** m ($\angle$ ABE) = m ($\angle$ ....)

29 Draw Δ ABC in which AB = 6 cm. , BC = 5 cm. and AC = 4 cm. , then bisect $\overline{BC}$ at D , then draw $\overrightarrow{DE} \parallel \overline{AB}$ to cut $\overline{AC}$ at E , then draw $\overrightarrow{EF} \parallel \overline{CB}$ to cut $\overline{AB}$ at F
Find by measuring the length of each of $\overline{ED}$ and $\overline{EF}$, then write the name of the figure DEFB and find its perimeter. « ED = 3 cm. , EF = 2.5 cm. , The perimeter = 11 cm.»

For excellent pupils

30 Without using the protractor, draw an angle of measure $22\frac{1}{2}^\circ$

31 Draw $\angle$ ABC with measure 60° using the ruler and the compasses , bisect $\angle$ ABC , from C , draw $\overrightarrow{CE} \parallel \overrightarrow{BA}$ to meet the bisector of the angle at E , from E , draw $\overrightarrow{EF} \perp \overrightarrow{BA}$ where $\overleftrightarrow{EF} \cap \overleftrightarrow{BA} = \{F\}$ Does m ($\angle$ ABC) = m ($\angle$ FEB) ? Why ? (Don't remove the arcs)

Summary of the second part of unit 4
"From lesson 4 to lesson 6"

• Cases of congruence of two triangles :

- **First case (Two sides and the included angle) :**

  Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

- **Second case (Two angles and one side) :**

  Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

- **Third case (Three sides) :**

  Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.

- **Fourth case (Hypotenuse and one side in the right-angled triangle) :**

  Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

• Parallelism :

- **If a straight line intersects two parallel straight lines , then :**

  1. Each two alternate angles are equal in measure.
  2. Each two corresponding angles are equal in measure.
  3. Each two interior angles in the same side of the transversal are supplementary.

- **Two straight lines are parallel if a third straight line intersects them and one of the following cases is satisfied :**

  1 Two alternate angles have the same measure.

  or 2 Two corresponding angles have the same measure.

  or 3 Two interior angles in the same side of the transversal are supplementary.

- The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other.
- If two straight lines are parallel to a third straight line , then these two straight lines are parallel.
- If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.
- The axis of symmetry of a line segment is the straight line perpendicular to it from its midpoint.

Exams on the second part of unit four from lesson (4) to lesson (6)

Model 1

Answer the following questions :

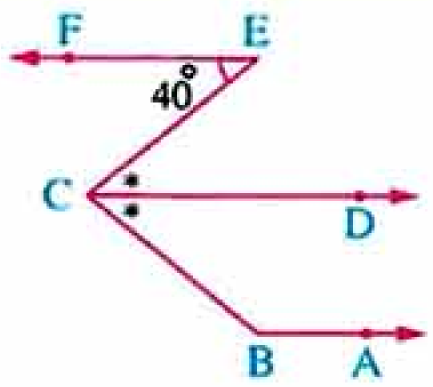
1 Choose the correct answer from the given ones :

1. If two straight lines are parallel to a third straight line , then they are ....
   (a) perpendicular. (b) coincedent. (c) parallel. (d) intersecting.
2. If $\Delta ABC \equiv \Delta XYZ$, then AB = ...
   (a) BC (b) YZ (c) XZ (d) XY
3. If L_1 and L_2 are two straight lines and $L_1 \cap L_2 = \varnothing$, then the two straight lines L_1 and L_2 are ....
   (a) intersecting. (b) perpendicular. (c) parallel. (d) coincedent.
4. If $\Delta ABC \equiv \Delta XYZ$, $m(\angle B) = 30°$, $m(\angle Z) = 60°$, then $m(\angle X) =$ ....
   (a) 30° (b) 60° (c) 90° (d) 180°
5. If L_1 , L_2 and L_3 are straight lines , $L_1 \perp L_2$ and $L_1 // L_3$, then L_2 .... L_3
   (a) $\perp$ (b) // (c) is coincedent to (d) bisects
6. In the parallelogram ABCD , m (∠ DAC) = m (∠ ....).
   (a) ACD (b) ACB (c) DBC (d) ADC

2 Complete the following :

1. Two triangles are congruent if each .... is congruent to the corresponding side of the other triangle.
2. If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are ....
3. The axis of symmetry of a line segment is ....
4. If two straight lines are perpendicular to a third straight line , then these two straight lines are ....
5. Two right-angled triangles are congruent if ....

3 [a] In the opposite figure :



$\overrightarrow{BA} // \overrightarrow{CD}$, $\overrightarrow{CD} // \overrightarrow{EF}$

, $\overrightarrow{CD}$ bisects ∠ BCE and m (∠ CEF) = 40°

Find : m (∠ B)

UNIT 4

[b] In the opposite figure :

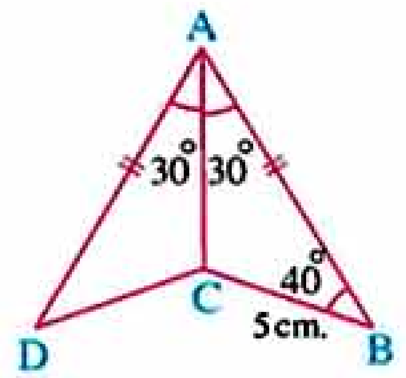
$AB = AD$, $BC = 5$ cm. , $m(\angle B) = 40°$

, $m(\angle BAC) = m(\angle DAC) = 30°$

1 Are $\Delta BAC \equiv \Delta DAC$? Give reason.

2 **Find :** $m(\angle D)$

3 **Find :** The length of $\overline{CD}$



4 **[a]** Draw $\angle A$ of measure 120° , then bisect it by using the ruler and the compasses. (Don't remove the arcs)

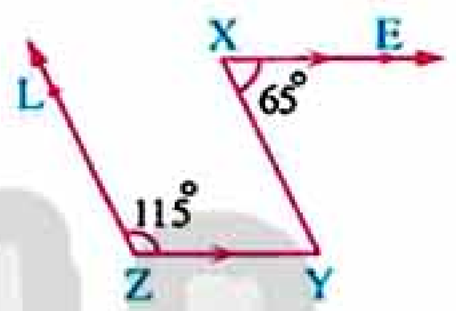
[b] In the opposite figure :

$\overrightarrow{XE} \parallel \overrightarrow{ZY}$, $m(\angle X) = 65°$

and $m(\angle Z) = 115°$

1 **Find :** $m(\angle Y)$

2 Is $\overline{YX} \parallel \overrightarrow{ZL}$? Why ?



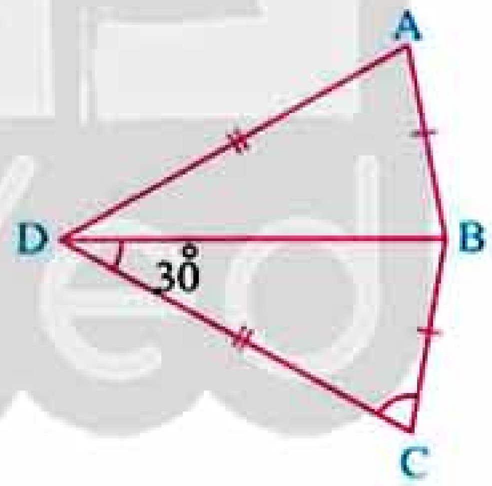
5 **[a] In the opposite figure :**

$AB = CB$, $AD = CD$

and $m(\angle CDB) = 30°$

1 Write the conditions of congruence of ΔABD and ΔCBD

2 **Find :** $m(\angle ADC)$



[b] Draw ΔABC in which $AB = AC = 5$ cm. and $BC = 6$ cm.

, then draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

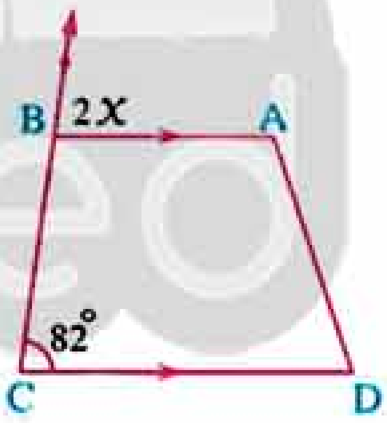
, then find by measuring the length of $\overline{AD}$ (Don't remove the arcs)

Model 2

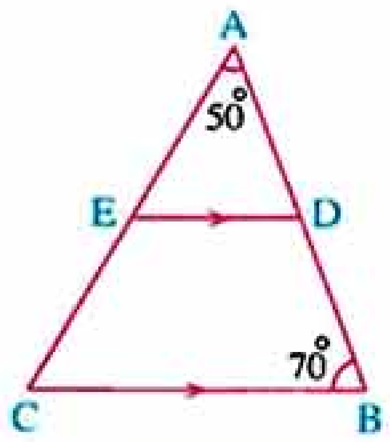
Answer the following questions :

1 **Choose the correct answer from the given ones :**

1. If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) + m(\angle B) = 140°$, then $m(\angle Z) =$ ....
   (a) 100° (b) 40° (c) 80° (d) 140°
2. If a straight line intersects two parallel straight lines , then each two alternate angles are ....
   (a) complementary. (b) equal in measure.
   (c) supplementary. (d) corresponding.
3. If $\Delta ABC \equiv \Delta DEF$, the perimeter of $\Delta ABC = 18$ cm. and $BC = 6$ cm.
   , then $DE + DF =$ ....
   (a) 12 cm. (b) 6 cm. (c) 3 cm. (d) 24 cm.
4. If L_1 , L_2 and L_3 are straight lines , $L_1 \perp L_3$, $L_2 \perp L_3$, then L_1 .... L_2
   (a) // (b) ⊥ (c) coincides (d) intersects
5. **In the opposite figure :**
   If $\overline{AB} \mathbin{//} \overline{DC}$, $m(\angle C) = 82°$, then $x =$ ....
   (a) 82° (b) 98°
   (c) 41° (d) 49°



6. **In the opposite figure :**
   ABC is a triangle in which $m(\angle A) = 50°$
   , $m(\angle B) = 70°$ and $\overline{DE} \mathbin{//} \overline{BC}$, then $m(\angle AED) =$ ....
   (a) 120° (b) 70°
   (c) 50° (d) 60°



2 **Complete the following :**

1. Two triangles are congruent if two angles and ....
2. If a straight line intersects two straight lines and there are two corresponding angles having the same measure , then the two straight lines are ....

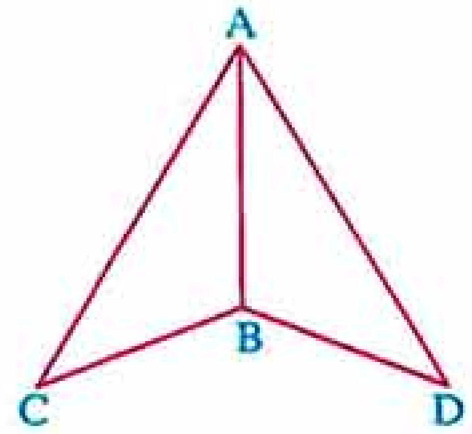
UNIT 4

3 The perpendicular to a line segment from its midpoint is called ....

4 The straight line which is perpendicular to one of two parallel straight lines is .... to the other straight line.

5 **In the opposite figure :**

$\Delta ABC \equiv \Delta ABD$, the perimeter of the figure ACBD equals 20 cm. and AB = 5 cm. , then the perimeter of ΔABC = ... cm.

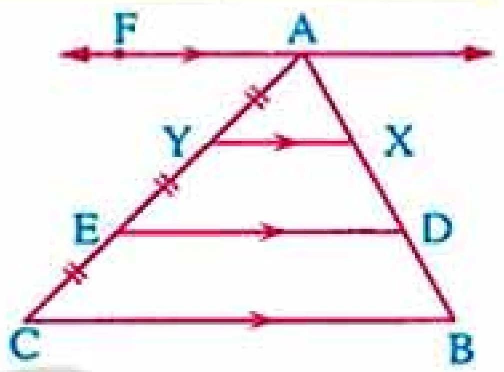


3 **[a] In the opposite figure :**

$\overleftrightarrow{AF} // \overline{XY} // \overline{DE} // \overline{BC}$

, AY = YE = EC where

AB = 18 cm. **Find :** The length of $\overline{AD}$



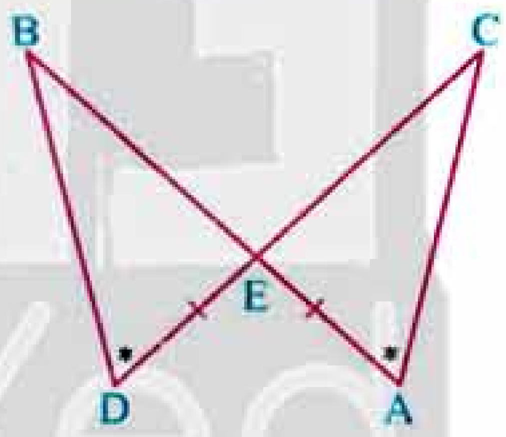
[b] Draw the line segment $\overline{AC}$ of length 6 cm. , then bisect it by using the geometric tools (Don't remove the arcs).

4 **[a] In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$, AE = ED

and $\angle A \equiv \angle D$

Is $\Delta ACE \equiv \Delta DBE$? Why ?



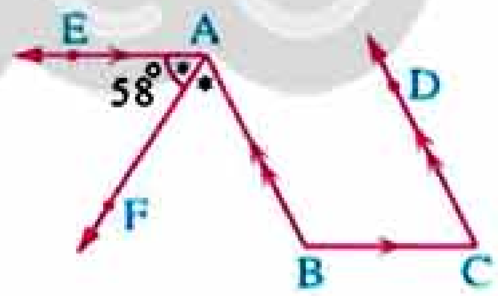
[b] In the opposite figure :

$\overrightarrow{CD} // \overrightarrow{BA}$, $\overrightarrow{CB} // \overrightarrow{AE}$

, $\overrightarrow{AF}$ bisects $\angle$ BAE and

m ($\angle$ FAE) = 58°

Find : m ($\angle$ C)



5 **[a]** Draw $\angle$ A of measure 120° , then bisect it into four equal angles in measure by using the ruler and the compasses. (Don't remove the arcs)

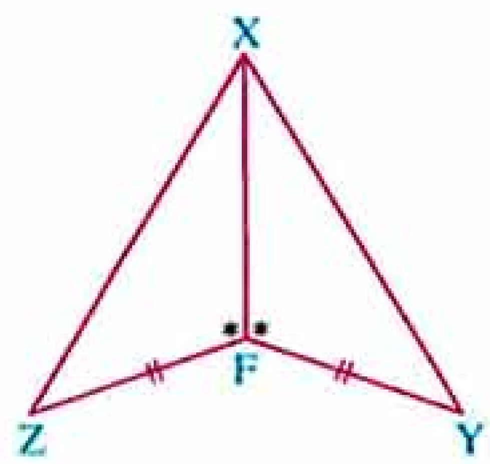
[b] In the opposite figure :

YF = ZF and

m ($\angle$ XFY) = m ($\angle$ XFZ)

Is $\Delta XFY \equiv \Delta XFZ$?

, then show why $\overrightarrow{XF}$ bisects $\angle$ X

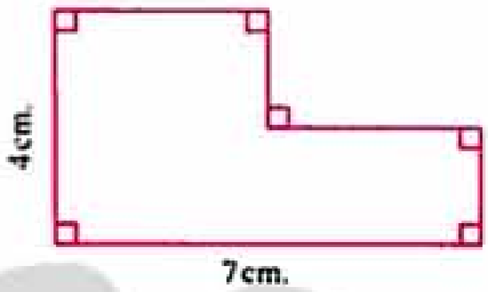


TIMSS Problems

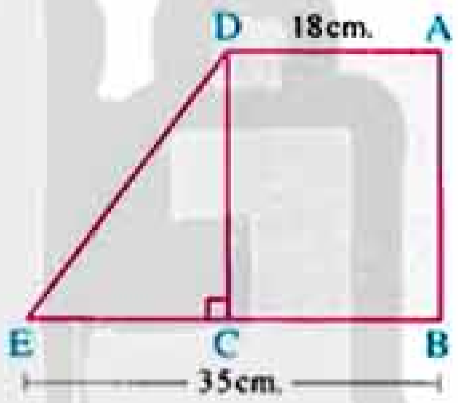
Accumulative basic skills

1 Complete the following :

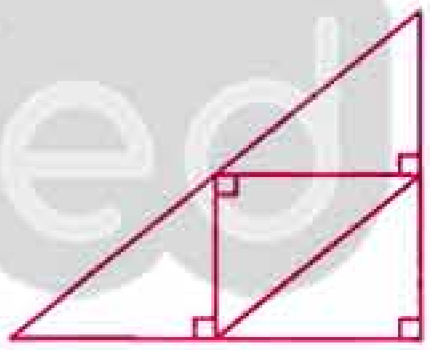
1. The perimeter of the opposite figure is .... cm.



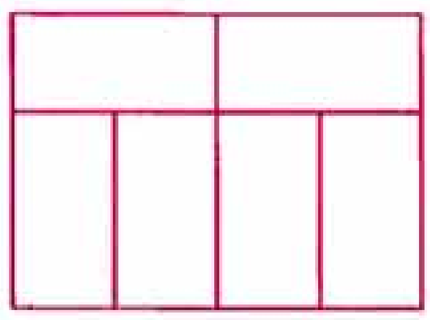
2. **In the opposite figure :**
   ABCD is a rectangle whose area is 360 cm^2.
   , AD = 18 cm. and BE = 35 cm.
   , then the area of Δ DCE = ... cm^2.



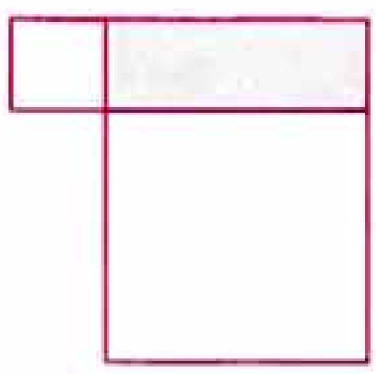
3. The number of right-angled triangles in the opposite figure equals ....



4. If A = (3 , – 4) , B = (– 2 , – 4) , then AB = ...
5. The number of axes of symmetry of the circle is ....
6. If the sum of measures of two angles in a triangle is $\frac{5}{6}$ the sum of measures of all its angles , then the measure of the third angle is ....°
7. **In the opposite figure :**
   A rectangle of area 48 cm^2. is divided into 6 congruent rectangles , then its perimeter equals .... cm.



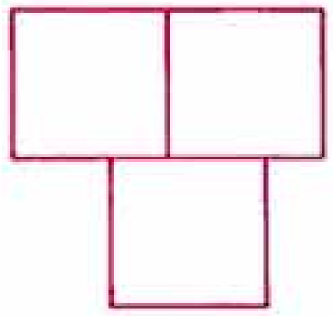
8. The point which belongs to all diameters of a circle is ....
9. **In the opposite figure :**
   If the sum of perimeters of the two squares is 28 cm.
   , then the perimeter of the coloured rectangle is .... cm.



Basic Skills

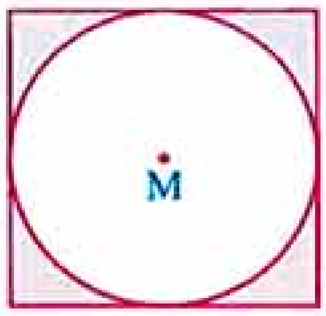
10 In the opposite figure :

Three squares equal in area , the sum of their areas is 12 cm^2 , then the perimeter of the figure is .... cm.



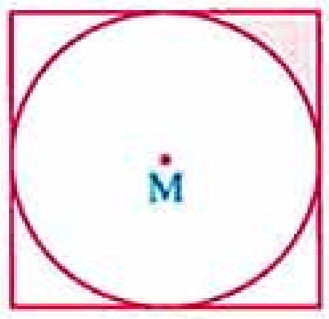
11 In the opposite figure :

A circle is inscribed in a square whose side length is 14 cm. , then the area of the coloured region is .... cm^2 ($\pi = \frac{22}{7}$)



12 In the opposite figure :

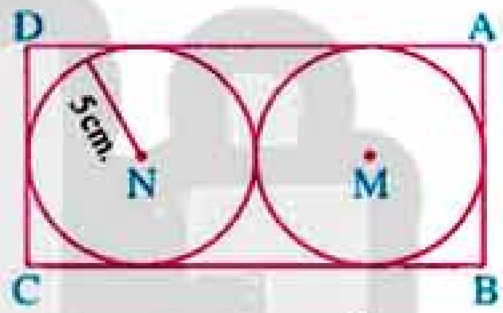
A circle is inscribed in a square whose side length is 10 cm. , then the perimeter of the coloured region is .... cm. ($\pi = 3.14$)



2 Choose the correct answer from the given ones :

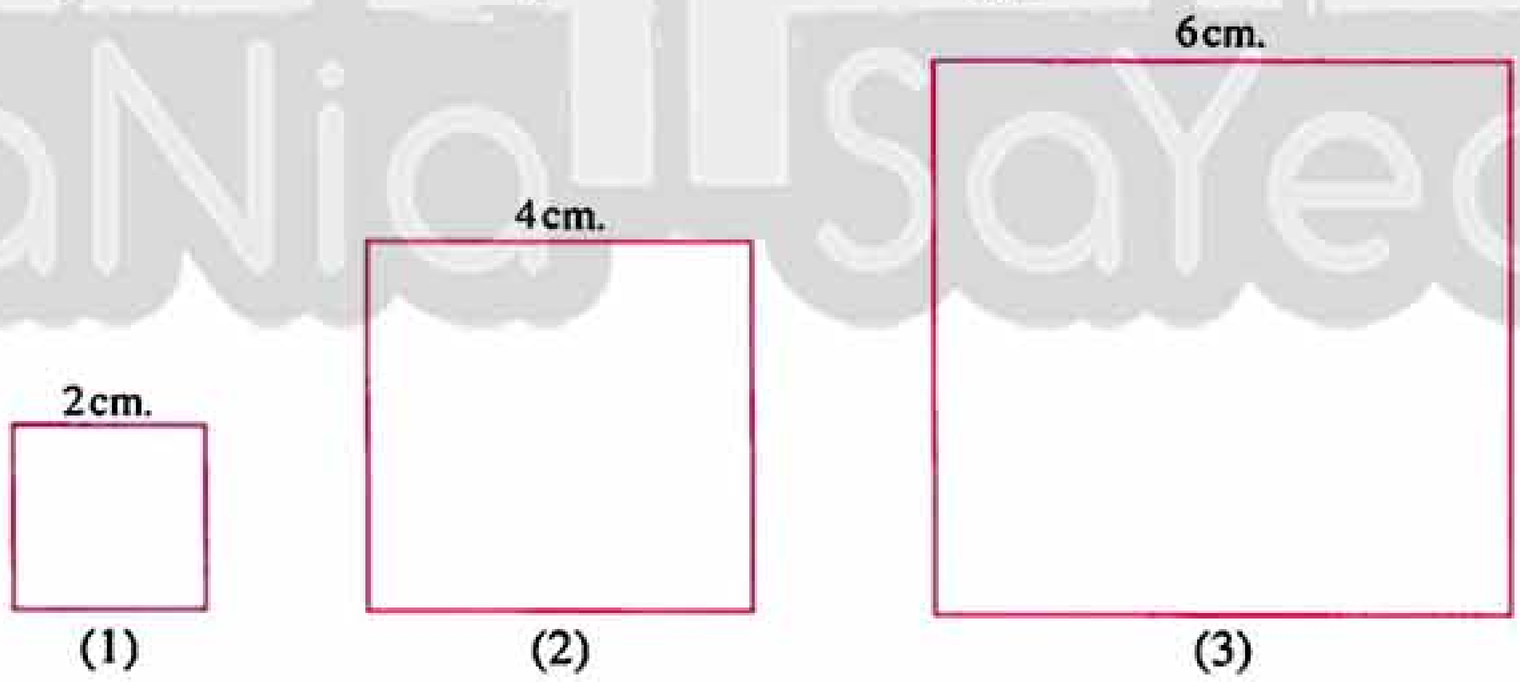
1 In the opposite figure :

Two circles in a rectangle , the radius length of each one is 5 cm. What is the area of the rectangle ?



(a) 200 cm^2 (b) 100 cm^2 (c) 60 cm^2 (d) 50 cm^2

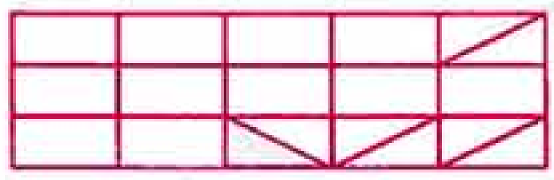
2 What is the perimeter of the 6th square in the following pattern ?



(a) 32 cm. (b) 40 cm. (c) 48 cm. (d) 56 cm.

3 In the opposite figure :

The area of the coloured part = ... the area of the whole figure.



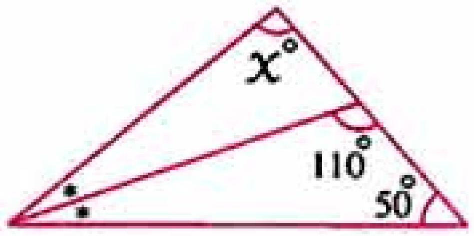
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{7}{15}$

4 The best unit to measure the area of a room is ....

(a) mm^2 (b) cm^2 (c) m^2 (d) km^2

5 **In the opposite figure :**

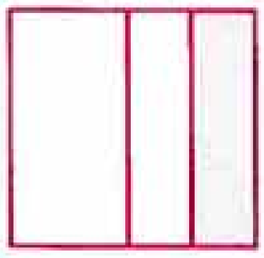
$x = ...$

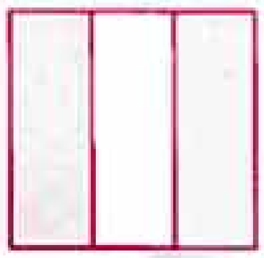


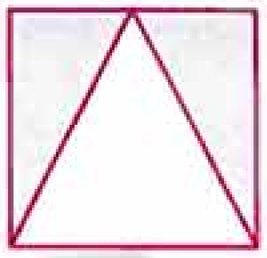
(a) 50°　　(b) 80°

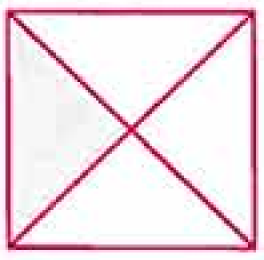
(c) 90°　　(d) 100°

6 Which of the following figures shows that $\frac{2}{3}$ the square is coloured ?


(a)

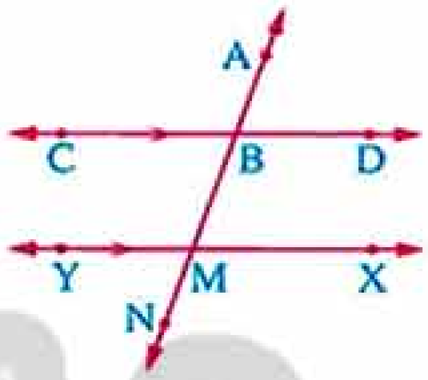

(b)


(c)


(d)

7 **In the opposite figure :**

If $\overleftrightarrow{DC} \parallel \overleftrightarrow{XY}$, then $\angle$ ABC and $\angle$ XMN are ....



(a) complementary.　　(b) supplementary.

(c) congruent.　　(d) adjacent.

8 Which of the following sentences is wrong for all rectangles ?

(a) Opposite sides are parallel.　　(b) Opposite sides are equal in length.

(c) All angles are right.　　(d) The diagonals are perpendicular.

9 The small squares in the figures (a) and (b) are congruent. If the perimeter of fig. (a) is 48 cm. , then the perimeter of fig. (b) is .... cm.

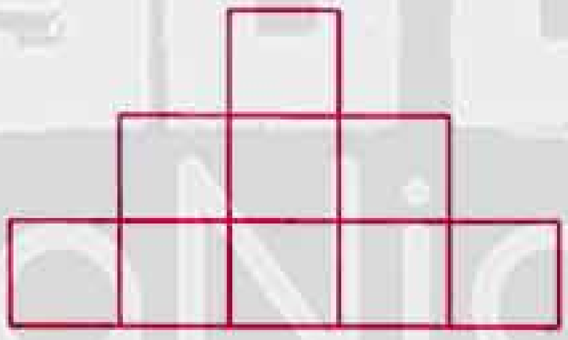


Fig. (a)

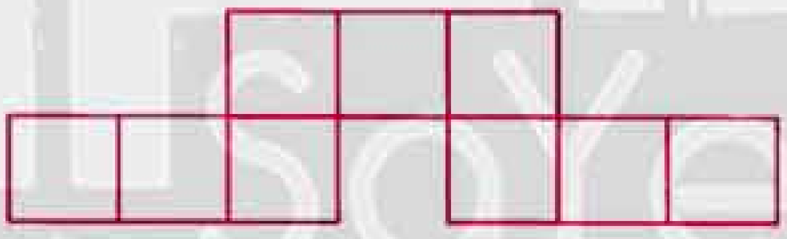


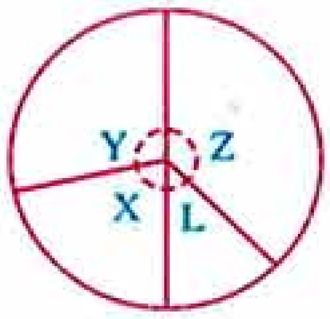
Fig. (b)

(a) 48　　(b) 57　　(c) 60　　(d) 63

10 The image of the point (– 3 , 5) by translation of 3 units in the negative direction of the y-axis is ....

(a) (– 3 , 8)　　(b) (– 3 , 2)　　(c) (– 6 , 5)　　(d) (0 , 5)

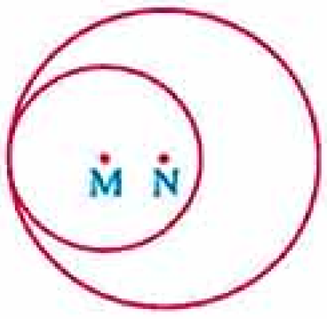
11 Which of the angles in the opposite figure has the closest measure to 45° ?



(a) X　　(b) Y

(c) Z　　(d) L

12 The number of axes of symmetry of the opposite figure is ....



(a) 1　　(b) 2

(c) 3　　(d) an infinite number

Maths

By a group of supervisors

NOTEBOOK

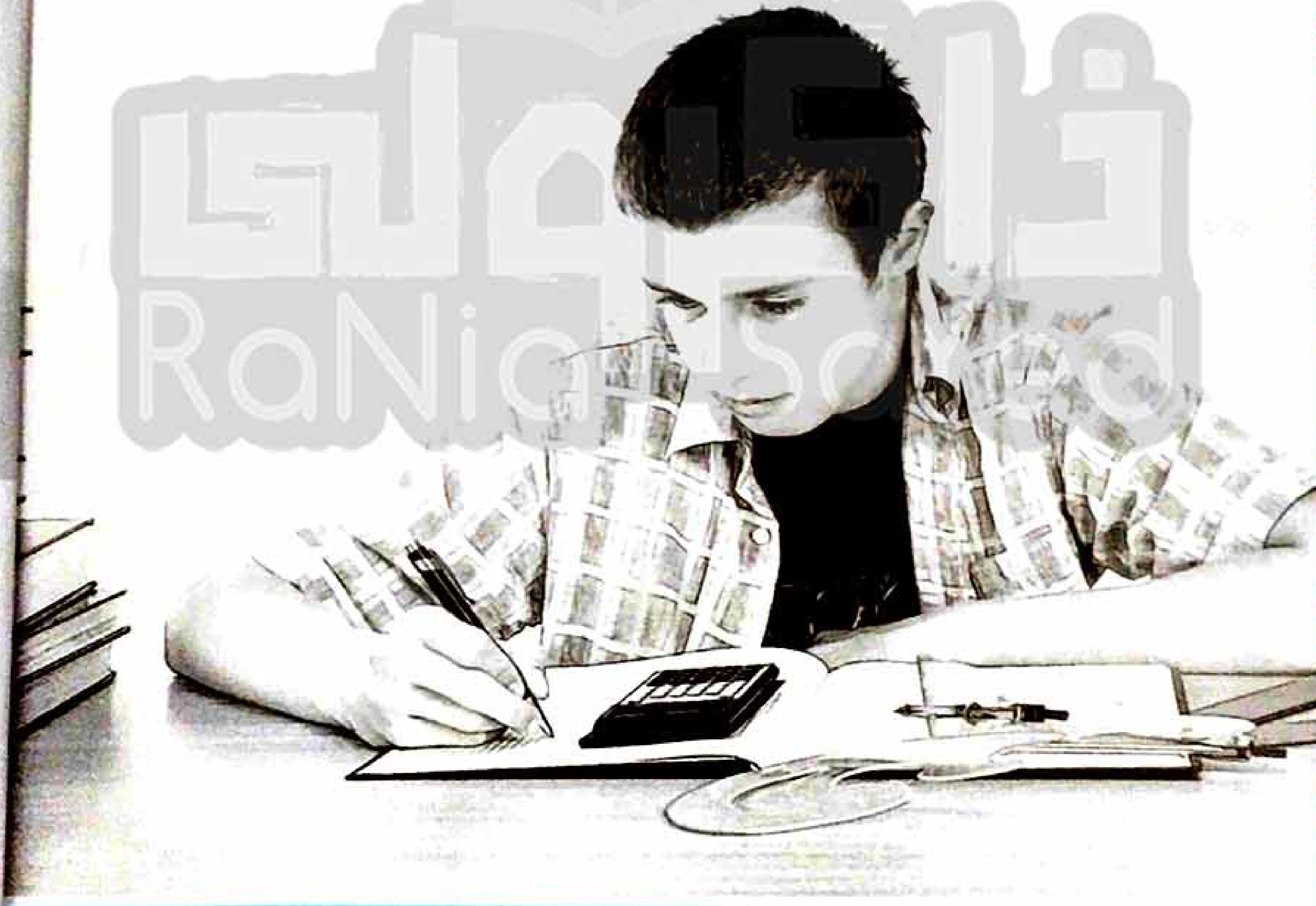
- Quizzes
- Final Revision
- Final Examinations
- Research Projects

1st PREP. 2021
FIRST TERM

EL-MOASSER

Quizzes

on Algebra and Statistics



Quiz 1 on lesson 1 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 If $\frac{7}{X+5}$ is a rational number , then $X \neq$ ....

(a) – 5 (b) 0 (c) 5 (d) 10

2 $\frac{5-X}{X-3}$ = zero if $X =$ ....

(a) 3 (b) 5 (c) zero (d) – 3

3 $\frac{5}{11}$ = ... on the form of a recurring decimal.

(a) 0. 45 (b) $0.\dot{4}5\dot{4}$ (c) $0.\dot{4}\dot{5}$ (d) $0.0\dot{4}\dot{5}$

2 [a] Write each of the following on the form of $\frac{a}{b}$ in the simplest form :

1 $|-2.25|$ 2 35 %

[b] Write three rational numbers expressing each of the following rational numbers :

1 $\frac{5}{7}$ 2 $\frac{2}{9}$

Quiz 2 till lesson 2 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 Which of the following fractions is closest to 0. 35 numerically ?

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2 $\frac{7}{X-3} \notin \mathbb{Q}$ if $X =$ ....

(a) zero (b) 3 (c) 4 (d) 7

3 The rational number which lies between $\frac{1}{4}$ and $\frac{1}{3}$ is ....

(a) 0. 25 (b) $\frac{31}{120}$ (c) $\frac{41}{120}$ (d) $0.\dot{3}$

2 [a] Find three rational numbers lying between : $\frac{1}{2}$ and $\frac{1}{3}$

[b] If a = 3 and b = 5 , which of the following numbers is rational and which is not ?

1 $\frac{b}{3-a}$ 2 $\frac{b-5}{a}$

Quiz 3 till lesson 3 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 The additive identity in $\mathbb{Q}$ is ....

(a) 1 (b) -1 (c) $\frac{5}{8-8}$ (d) $\frac{7-7}{15+1}$

2 The remainder of subtracting $\frac{3}{7}$ from $\frac{9}{21}$ is ....

(a) zero (b) $\frac{6}{21}$ (c) $\frac{6}{14}$ (d) $-\frac{6}{21}$

3 $-\frac{3}{4}$ .... $-\frac{6}{5}$

(a) $>$ (b) $<$ (c) $=$ (d) $\leq$

2 [a] Using the properties of addition operation in $\mathbb{Q}$, find the result of each of the following :

1 $\frac{3}{4}+\frac{5}{9}+\frac{1}{4}+\frac{4}{9}$

2 $\frac{5}{8}+\frac{1}{3}+\frac{3}{8}+\left(-\frac{1}{3}\right)$

[b] Find four rational numbers between : $\frac{4}{3}$, $\frac{4}{5}$ in which one of them is an integer.

Quiz 4 till lesson 4 – unit 1

time 20 min.

1 Complete the following :

1 The multiplicative neutral element in $\mathbb{Q}$ is ....

2 The additive inverse of the number $-\frac{4}{9}$ is ....

3 If $\frac{a}{b}=\frac{2}{3}$, then $\frac{3a}{2b}=$ ....

2 [a] Using the distribution property , find the value of each of the following :

1 $\frac{7}{12}\times\frac{23}{45}+\frac{17}{12}\times\frac{23}{45}-2\times\frac{23}{45}$

2 $\frac{22}{25}\times\frac{7}{11}+\frac{5}{11}\times\frac{22}{25}-\frac{22}{25}$

[b] If $x=\frac{2}{3}$, $y=-\frac{1}{2}$, $z=\frac{1}{6}$ **, find the value of :** $(x-y)-z$

Quiz 5 till lesson 5 – unit 1

time 20 min.

1 Choose the correct answer from those given :

1 If $a \times \frac{b}{2} = \frac{a}{2}$, then b = ……………

(a) $\frac{a}{2}$ (b) zero (c) 1 (d) – a

2 The rational number which lies at the midpoint of the distance between $\frac{1}{2}$ and $\frac{7}{8}$ is ……………

(a) $\frac{11}{16}$ (b) $\frac{11}{8}$ (c) $\frac{11}{4}$ (d) $\frac{11}{32}$

3 The number $\frac{x-3}{x-5} \in \mathbb{Q}$, if $x \neq$ ……………

(a) 3 (b) – 3 (c) 5 (d) – 5

2 [a] Find a rational number lying at the quarter of the distance between $-\frac{3}{5}$ and $-\frac{4}{5}$ from the side of the smaller number.

[b] Find the value of : $\left(\frac{3}{4} - \frac{2}{3}\right) \div \left(\frac{1}{4} \times \frac{1}{3}\right)$

Quiz 6 till lesson 1 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The degree of the algebraic expression $3x^2 + 2xy^2 + 3z^4$ equals the degree of the algebraic expression ……………

(a) $5xy + 3y^2z$ (b) $2x^2y^2 + 3x^2y$ (c) $2xy + 3x^4z$ (d) $5a^2b + 4ab^2$

2 If a = zero , b = 5 , c = 2 , then the numerical value of the expression $a^2b + ac$ = ……………

(a) zero (b) 2 (c) 6 (d) 8

3 If the degree of the algebraic term $2a^3b^n$ is ninth , then n = ……………

(a) 8 (b) 6 (c) 2 (d) 9

2 [a] Arrange the algebraic expression descendingly according to the powers of a :

$7ab + 5a^5b^3 - 3a^2b^5$

[b] Use the distribution property to find the value of : $\frac{3}{7} \times 2 + \frac{3}{7} \times 6 - \frac{3}{7}$

Quiz 7 — till lesson 2 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The remainder of subtracting $-5x$ from $3x$ is ....

(a) $-2x$ (b) $2x$ (c) $-8x$ (d) $8x$

2 The multiplicative inverse of the rational number $\frac{3}{2}$ is ....

(a) $-\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) -0.6 (d) $0.\dot{6}$

3 $-2x$ exceeds x by ....

(a) $3x$ (b) $-3x$ (c) $3x^2$ (d) $2x$

2 [a] Simplify to the simplest form : $3x^2y + 4xy^2 - 2y^3 + 3 + x^2y + 3y^3 - z$

[b] If $x = -\frac{1}{9}$, $y = \frac{3}{4}$, $z = -3$ **, find the value of each of :**

1 $x \div (y \times z)$ 2 $(x + z) \times y$

Quiz 8 — till lesson 3 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The coefficient of the algebraic term $(-5xy^2)$ is ....

(a) 5 (b) -5 (c) 3 (d) 2

2 The used property in the operation performing of $\frac{6}{7} \times 1 = \frac{6}{7}$ is the property of ....

(a) associative.

(b) commutative.

(c) the existence of the multiplicative neutral.

(d) the existence of the multiplicative inverse.

3 The additive inverse of the number $|-\frac{2}{5}|$ is ....

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) $-\frac{2}{5}$ (d) $-\frac{5}{2}$

2 [a] Subtract : $x + x^2 - 5$ **from** $2x^2 + x - 3$, then find the numerical value of the result when $x = 6$

[b] Use the properties of addition operation in $\mathbb{Q}$ to find the value of the expression :

$\frac{5}{4} + \left(-\frac{13}{5}\right) + \left(-\frac{25}{4}\right) + \frac{28}{5}$

Quiz 9 till lesson 4 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 If the rational number $\frac{x-3}{x-5}=0$, then $x=$ ....

(a) 3 (b) 5 (c) -3 (d) -5

2 $\frac{y^5}{y^2}+y^3=$ .... , where $y \neq 0$

(a) y^6 (b) y^5 (c) $2y^3$ (d) y^3

3 $98\,a^7\,b^4=$ .... $\times 14\,a^7\,b$

(a) $7\,b^3$ (b) $7\,a\,b$ (c) $7\,b^4$ (d) $1372\,b^3$

2 [a] Simplify to the simplest form :

$$\frac{6x^4y^2}{7}\times\frac{28xy^3}{3}$$

[b] Add : $2x-7y+z$ **and** $5z+6y-2x$

Quiz 10 till lesson 5 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 $2x\times 3x=$ ....

(a) $6x$ (b) $5x$ (c) $6x^2$ (d) $5x^2$

2 The number of all rational numbers that exist between $\frac{2}{5}$ and $\frac{4}{5}$ is ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

3 The expression $2a+5ab$ is of .... degree.

(a) the first (b) the second (c) the third (d) zero

2 [a] Simplify to the simplest form : $4n(n+5)+n(6-n)$

, then find the numerical value of the expression when $n=-1$

[b] Subtract : x^2-8x-3 **from the sum of :** $2x^2+5x-1$ **and** $x^2-13x-2$

Quiz 11 till lesson 6 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 The middle term in the expansion of $(2x - 5y)^2$ is ....

(a) $-10x^2y^2$ (b) $10x^2y^2$ (c) $20xy$ (d) $-20xy$

2 A rectangle , its length = $4x$ cm. and its width = $3x$ cm. , then its area = ... cm^2.

(a) $7x$ (b) $12x$ (c) $12x^2$ (d) $14x$

3 $(4x - 3)(x - 4) =$ ....

(a) $4x^2 - 19x - 12$ (b) $4x^2 - 7$ (c) $4x^2 - 12$ (d) $4x^2 - 19x + 12$

2 [a] Simplify to the simplest form :

$(2a - 3)(2a + 3) + 7$, then find the numerical value of the result when $a = -1$

[b] Use the distribution property to find the value of : $6 \times \frac{5}{7} + 2 \times \frac{5}{7} - \frac{5}{7}$

Quiz 12 till lesson 7 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1 $(x^2 + x) \div x =$ .... where $x \neq$ zero

(a) zero (b) x (c) $2x + 1$ (d) $x + 1$

2 If $(x - 3)(x + 3) = x^2 + k$, then k = ...

(a) -9 (b) 3 (c) 6 (d) 9

3 The multiplicative inverse of the number 0.5 is ....

(a) 1 (b) 5 (c) -2 (d) 2

2 [a] Simplify to the simplest form : $(x - 5)(5 + x) - x^2$

[b] The area of a rectangle is $(12a^4b^2 + 8a^3b^4 - 16a^2b^2)$ cm^2.

and its width is $(2a^2b^2)$ cm.

Find the length of the rectangle.

Quiz 13 till lesson 8 – unit 2

time 20 min.

1 Complete the following :

1. $2x^3 \times 3xy =$ ....
2. If $x - y = 3$ and $x + y = 5$, then $x^2 - y^2 =$ ....
3. The rational number that lies at the midpoint of the distance between $\frac{1}{4}$ and $\frac{1}{3}$ is ....

2 [a] Find the quotient : $x^4 + 3x^2 + 2$ **by** $x^2 + 1$

[b] Simplify to the simplest form : $(x + 5)^2 - (x + 3)(x - 3)$

, then find the numerical value of the result when $x = -3$

Quiz 14 till lesson 9 – unit 2

time 20 min.

1 Choose the correct answer from those given :

1. If $\frac{x}{y} = 1$, then $3x - 3y =$ ....
   (a) zero (b) 1 (c) 3 (d) 6
2. The highest common factor of the expression $12x^3y^4 + 8x^2y^4$ is ....
   (a) $2x^2y^2$ (b) $4x^2y^4$ (c) $4x^3y$ (d) $12x^3y^4$
3. The rational number $\frac{x}{-5}$ is negative if x ....
   (a) > zero (b) < zero (c) ≤ zero (d) = zero

2 [a] Factorize each of the following by identifying the highest common factor :

1. $10a^4x^2 + 15a^5x^4 - 30a^3x^3$
2. $3a(a - 2b) - 6b(a - 2b)$, then find the numerical value of the result when $a - 2b = |-\frac{1}{3}|$

[b] Find the quotient : $2x^2 + 13x + 15$ **by** $2x + 3$ where $(2x + 3) \neq 0$

Quiz 15 till lesson 1 – unit 3

time 20 min.

1 Choose the correct answer from those given :

1 The arithmetic mean of the values 34 , 23 , 25 , 40 , 22 , 6 is ....

(a) 22 (b) 23 (c) 24 (d) 25

2 If the arithmetic mean of the numbers 3 , 3 and x is 4 , then x = ...

(a) 3 (b) 4 (c) 12 (d) 6

3 The expression $3x^3 + 7x^3y$ is of the .... degree.

(a) first (b) second (c) third (d) fourth

2 [a] Find the mean mark of the marks of a student in mathematics exam in 5 months :

Months	September	October	November	December	January
Marks	13	15	14	18	20

[b] Subtract : $-a^2 - 5ab + 4b^2$ from $3a^2 - 2ab - 2b^2$

Quiz 16 till lesson 2 – unit 3

time 20 min.

1 Choose the correct answer from those given :

1 If the median of the values 27 , 45 , 19 , 24 and 28 is x , then x = ...

(a) 24 (b) 27 (c) 28 (d) 45

2 If the arithmetic mean of the side lengths of a triangle equals 8 cm. , then the perimeter of the triangle equals ....

(a) 8 cm. (b) 18 cm. (c) 24 cm. (d) 15 cm.

3 The additive inverse of $\left(\frac{-1}{5}\right)^0$ is ....

(a) 1 (b) – 1 (c) 5 (d) $\frac{1}{5}$

2 [a] The following table shows the number of training hours of basketball for each of Sarah and Gamal during 5 days :

Sarah	6	5	3	2	5
Gamal	7	6	4	7	3

Find the median number of training hours for each of them.

[b] Find three rational numbers between : $\frac{2}{3}$ and $\frac{1}{5}$

Quiz 17 till lesson 3 – unit 3

time 20 min.

1 Choose the correct answer from those given :

1 If the order of the median of a set of values is the fourth , then the number of these values equals ....

(a) 3 (b) 5 (c) 7 (d) 9

2 If the mode of the values 7 , 5 , y + 3 , 5 and 7 is 7 , then y = ...

(a) 3 (b) 4 (c) 5 (d) 7

3 If $(x+1)^2 = x^2 + kx + 1$, then k = ...

(a) 1 (b) 2 (c) 3 (d) 4

2 [a] Find the quotient :

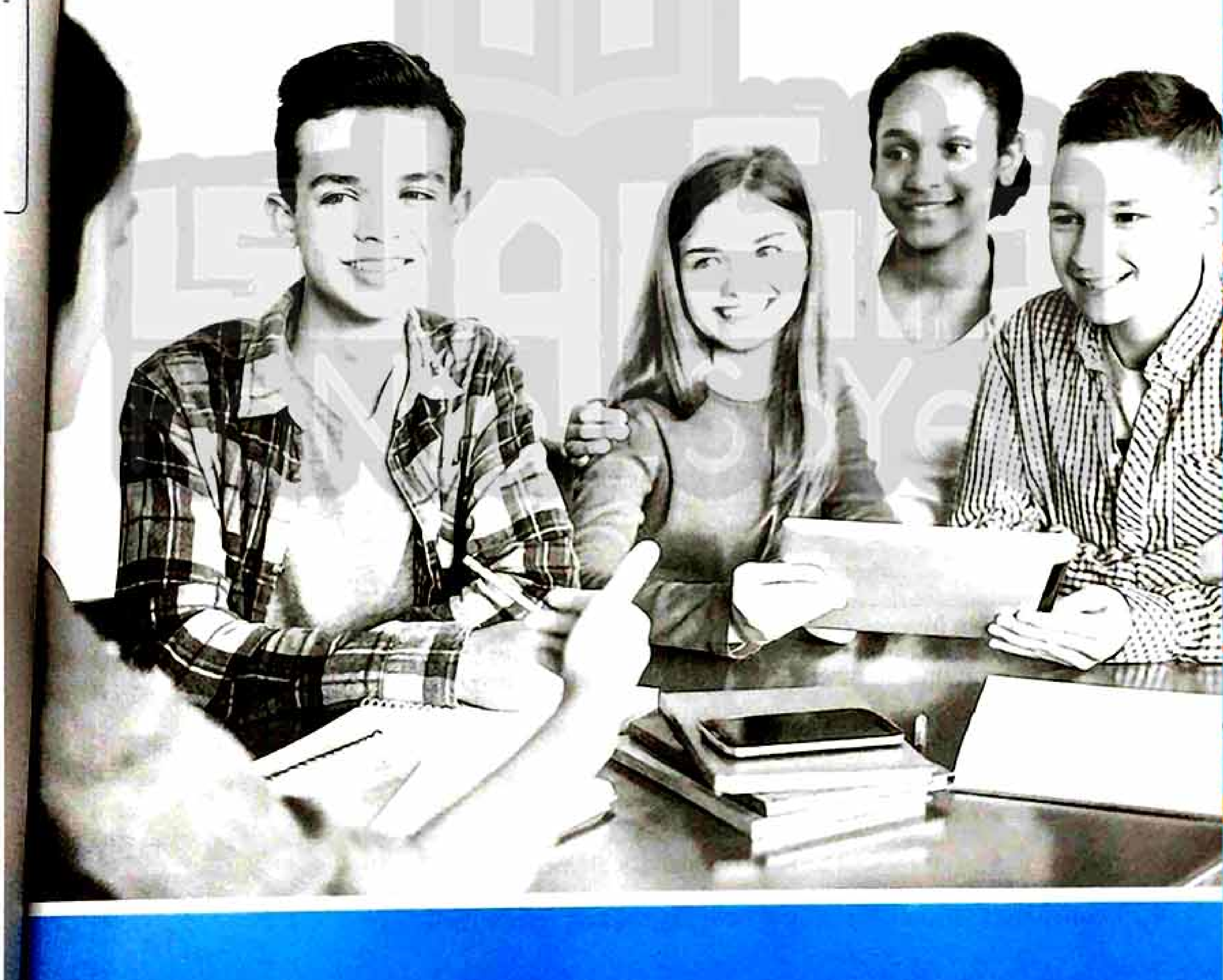
$8x^3 - 125$ by $4x^2 + 10x + 25$ where the divisor is not equal to zero.

[b] Find the arithmetic mean , the median and the mode of the following values :

12 , 15 , 8 , 20 , 10 , 9 , 7 , 15

Final Revision

of Algebra and Statistics



Revision for the important rules of algebra and statistics

Remember — The rational numbers

The rational number is the number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Examples for rational numbers : $\frac{5}{7}$, $-\frac{3}{8}$, 7 , zero , $4\frac{3}{7}$, 0.35 , 43 % , $0.\dot{3}$

Each integer is a rational number with denominator = 1 ***i.e.*** $\mathbb{Z} \subset \mathbb{Q}$

For example : $7 = \frac{7}{1}$, $0 = \frac{0}{1}$

If $\frac{a}{b}$ is a rational number , then $b \neq 0$

For example : If $\frac{3}{x-5} \in \mathbb{Q}$, then $x - 5 \neq 0$ ***i.e. :*** $x \neq 5$

If the rational number $\frac{a}{b} = 0$, then $a = 0$

For example : If the rational number $\frac{3x}{x+3} = 0$, then $3x = 0$ ***i.e. :*** $x = 0$

Zero is the additive identity element (additive neutral element) in $\mathbb{Q}$

For example : $0 + \frac{1}{2} = \frac{1}{2} + 0 = \frac{1}{2}$

The number 1 is the multiplicative identity element (multiplicative neutral element) in $\mathbb{Q}$

For example : $1 \times \frac{-2}{3} = \frac{-2}{3} \times 1 = \frac{-2}{3}$

The additive inverse :

For every rational number $\frac{a}{b}$ there exist an additive inverse to it that is $(-\frac{a}{b})$

where $\frac{a}{b} + (-\frac{a}{b}) = 0$ (The additive identity element)

For example : The additive inverse of the number $\frac{3}{7}$ is $(-\frac{3}{7})$

where $\frac{3}{7} + (-\frac{3}{7}) = 0$ (The additive identity element)

Notice that : The additive inverse of the number zero is itself.

The multiplicative inverse :

For every rational number $\frac{a}{b}$ except zero there is a multiplicative inverse that is the rational number $\frac{b}{a}$ where $\frac{a}{b} \times \frac{b}{a} = 1$ (The multiplicative identity element)

For example :

The multiplicative inverse of the number $\frac{3}{7}$ is $\frac{7}{3}$

where $\frac{3}{7} \times \frac{7}{3} = 1$ (The multiplicative identity element)

Notice that

- There is no multiplicative inverse for the number 0 because $\frac{a}{0}$ is meaningless.
- The multiplicative inverse of the number 1 is itself.
- The multiplicative inverse of the number -1 is itself.

The number that lies at the middle of the way (halfway) between two numbers

= The smaller number $\oplus$ $\frac{1}{2}$ |The difference between the two numbers|

= The greater number $\ominus$ $\frac{1}{2}$ |The difference between the two numbers|

$= \frac{\text{First number + second number}}{2}$

For example :

The number that lies at the middle of the way between $\frac{3}{4}$ and $\frac{1}{8} = \frac{\frac{3}{4}+\frac{1}{8}}{2} = \frac{7}{16}$

The number that lies at one third of the way between two numbers :

1. From the side of the smaller number

   = the smaller number $\oplus$ $\frac{1}{3}$ |The difference between the two numbers|

2. From the side of the greater number

   = the greater number $\ominus$ $\frac{1}{3}$ |The difference between the two numbers|

For example :

The number that lies at one third of the way between $\frac{3}{8}$ and $-\frac{5}{6}$:

1. From the side of the smaller number $= -\frac{5}{6} + \frac{1}{3}\left|\frac{3}{8} - \left(-\frac{5}{6}\right)\right| = -\frac{31}{72}$
2. From the side of the greater number $= \frac{3}{8} - \frac{1}{3}\left|\frac{3}{8} - \left(-\frac{5}{6}\right)\right| = -\frac{1}{36}$

Remember **The algebraic term and the algebraic expression and their degree**

The algebraic term is formed from the product of two or more factors.

For example : $5\,x$ is an algebraic term is formed from :

- 5 numerical factor (Coefficient of the term)
- x algebraic factor (Symbolic factor)

The degree of the algebraic term is the sum of the indices of the algebraic factors (symbolic factors) in this term.

For example :
- The term $5\,x$ is of the 1^{st} degree.
- The term $-9\,z^2$ is of the 2^{nd} degree.
- The term $7\,x^2\,y$ is of the 3^{rd} degree.
- The term 7 is of zero degree.

The algebraic expression is formed from an algebraic term or more.

For example :
- $7\,a + 5\,b$ is an algebraic expression consisting of two terms.
- $3\,x^2 + 4\,x^2 y - 7\,x$ is an algebraic expression consisting of three terms.

The degree of the algebraic expression is the highest degree of the terms forming it.

For example :
- $3\,y + 7$ is of the 1^{st} degree.
- $4\,x^2 + 5\,x + 3$ is of the 2^{nd} degree.

Remember **Like algebraic terms**

The algebraic terms are said to be like if the algebraic symbols forming their factors are like and the indices of these symbols are equal.

For example :
- $3\,x$, $-7\,x$ and x are like terms.
- $3\,x\,y^2$ and $-5\,y^2\,x$ are like terms.

but :
- $6\,x^2$, $7\,x\,y$ and $-y^2$ are unlike terms because their symbols are different.
- $7\,x$, $-5\,x^2$ and $3\,x^3$ are unlike terms because their indices are different.

Remember — Adding and subtracting algebraic expressions

We can add the two expressions : $3x - 5y + 7$ and $2y - 7x + 3$ by

The horizontal method

In this method, we use the commutative and associative properties.

$\therefore (3x - 5y + 7) + (2y - 7x + 3)$

$= (3x - 7x) + (-5y + 2y) + (7 + 3)$

$= -4x - 3y + 10$

The vertical method

In this method, we arrange the two expressions vertically such that the like terms lie under each other using the commutative property.

$$\begin{array}{r} \therefore\ 3x - 5y + 7 \\ -7x + 2y + 3 \\ \hline -4x - 3y + 10 \end{array}$$

We can subtract $3x - 5y + 7$ from $2y - 7x + 3$ by

The horizontal method

In this method, we put the subtraction operation in the form :

The remainder

= The minuend − the subtrahend

$\therefore$ The remainder

$= (2y - 7x + 3) - (3x - 5y + 7)$

$= 2y - 7x + 3 - 3x + 5y - 7$

$= (-7x - 3x) + (2y + 5y) + (3 - 7)$

$= -10x + 7y - 4$

The vertical method

In this method, we arrange the terms of the subtrahend down its like terms of the minuend, then we add the minuend to the additive inverse of the subtrahend.

The minuend : $2y - 7x + 3$

$\oplus \quad \ominus \quad \ominus$

The subtrahend : $-5y + 3x + 7$

The remainder = $7y - 10x - 4$

Remember — Multiplying algebraic terms and expressions

Multiplying a monomial by a monomial

When multiplying a monomial by another monomial :

1. Multiply the coefficients.
2. Multiply the symbols regarding adding the indices of the factors which have like bases.

For example :

- $5x \times (-3y) = -15xy$
- $-3x^2 \times 4xy = -12x^3y$

Multiplying a monomial by an algebraic expression

When we multiply a monomial by an algebraic expression , we multiply this monomial by each term of the algebraic expression.

For example : $3x(4y - x) = 12xy - 3x^2$

Multiplying two binomials

First — Multiplying by inspection

$6a^2$; $-2b^2$; $-3ab + 4ab = ab$

$$(2a - b)(3a + 2b) = \underset{\substack{\text{First}\\ \times \\ \text{First}}}{6a^2} + \underset{\substack{\text{Product of means}\\ + \\ \text{Product of extremes}}}{ab} \underset{\substack{\text{Second}\\ \times \\ \text{Second}}}{- 2b^2}$$

Second — The product of the sum of two terms and the difference between them :

$(a + b)(a - b) = a^2 - b^2$

i.e. Sum of two terms × difference between them = square of the first – square of the second

For example : $(a + 3b)(a - 3b) = a^2 - 9b^2$

Third — The square of an expression containing two terms : $(a \pm b)^2 = a^2 \pm 2ab + b^2$

1. The square of an expression consisting of the sum of two terms =
   The square of the first ⊕ 2 × the first × the second + the square of the second.

For example : $(2a + 3)^2 = (2a)^2 + (2 \times 2a \times 3) + 3^2 = 4a^2 + 12a + 9$

2. The square of an expression consisting of the difference between two terms =
   The square of the first ⊖ 2 × the first × the second + the square of the second.

For example : $(2x - 5y)^2 = (2x)^2 - (2 \times 2x \times 5y) + (5y)^2 = 4x^2 - 20xy + 25y^2$

Remember | Dividing algebraic terms and expressions

Dividing a monomial by a monomial

When dividing a monomial by another monomial :

1. Divide the coefficients.
2. Divide the symbols regarding that the indices of the factors of like bases should be subtracted.

For example : • $-12a^3 \div 3a = -4a^2$

• $-28a^5b^3c^3 \div (-4a^3b) = 7a^2b^2c^3$

Dividing an algebraic expression by a monomial

When we divide an algebraic expression by a monomial, we divide each term of the expression by this monomial.

For example : $(16x^3y + 24x^2y^4 - 8x^2y) \div (-4x^2y)$

$$= \frac{16x^3y}{-4x^2y} + \frac{24x^2y^4}{-4x^2y} - \frac{8x^2y}{-4x^2y} = -4x - 6y^3 + 2$$

Dividing an algebraic expression by another one

To perform the division operation of an algebraic expression by another one, we will remember the steps of division :

For example : To find the quotient of dividing $5a - 10a^2 + 6a^3 + 3$ **by** $3 + 2a^2 - 4a$

1. Arrange the terms of each of the dividend and the divisor descendingly according to the power of a before performing of the division operation.
2. Divide $6a^3$ **by** $2a^2$, then the result $3a$
3. Multiply $3a$ **by** $(2a^2 - 4a + 3)$
4. Subtract $(6a^3 - 12a^2 + 9a)$ from $(6a^3 - 10a^2 + 5a + 3)$
5. Repeat the previous steps in the same order till the difference will be equal to zero, then the operation of division will be finished and the quotient will be : $3a + 1$

$$\begin{array}{r|l} 2a^2 - 4a + 3 & 6a^3 - 10a^2 + 5a + 3 \\ \hline 3a + 1 & \overset{\ominus}{}6a^3 \overset{\oplus}{-} 12a^2 \overset{\ominus}{+} 9a \\ & \qquad\quad 2a^2 - 4a + 3 \\ & \qquad\quad \overset{\ominus}{}2a^2 \overset{\oplus}{-} 4a \overset{\ominus}{+} 3 \\ & \qquad\quad 0 \quad\ \ 0 \quad\ \ 0 \end{array}$$

Remember The steps of factorization by identifying the highest common factor (H.C.F.)

1. Find H.C.F. of the algebraic terms of the expression.
2. Put H.C.F. outside two brackets.
3. Divide each term of the algebraic expression by the H.C.F. and write the quotients inside the two brackets.

For example : $21\,a^3 b^2 - 7\,a^2 b^2 - 35\,a^2 b^3 = 7\,a^2 b^2 (3\,a - 1 - 5\,b)$

Remember Statistics

The arithmetic mean of a set of values $= \dfrac{\text{Sum of these values}}{\text{Number of these values}}$

For example : The arithmetic mean of the values : 2 , 5 , 7 , 5 , 4 , 9 , 3

is $\dfrac{2+5+7+5+4+9+3}{7} = \dfrac{35}{7} = 5$

The median of a set of values is the value which divides this set such that the number of values which are greater than it is equal to the number of values which are smaller than it.

To get the median, we arrange the values ascendingly or descendingly

If the number of values is odd , then :

The median is the value which is in the middle exactly.

For example :

- **If the values are :**

42 , 23 , 17 , 30 , 20

Then its ascending order is :

17 , 20 , 23 , 30 , 42

⇩

∴ The median = 23

If the number of values is even , then :

The median $= \dfrac{\text{The sum of the two middle values}}{2}$

For example :

- **If the values are :**

27 , 13 , 23 , 24 , 13 , 21

Then its ascending order is :

13 , 13 , 21 , 23 , 24 , 27

⇩

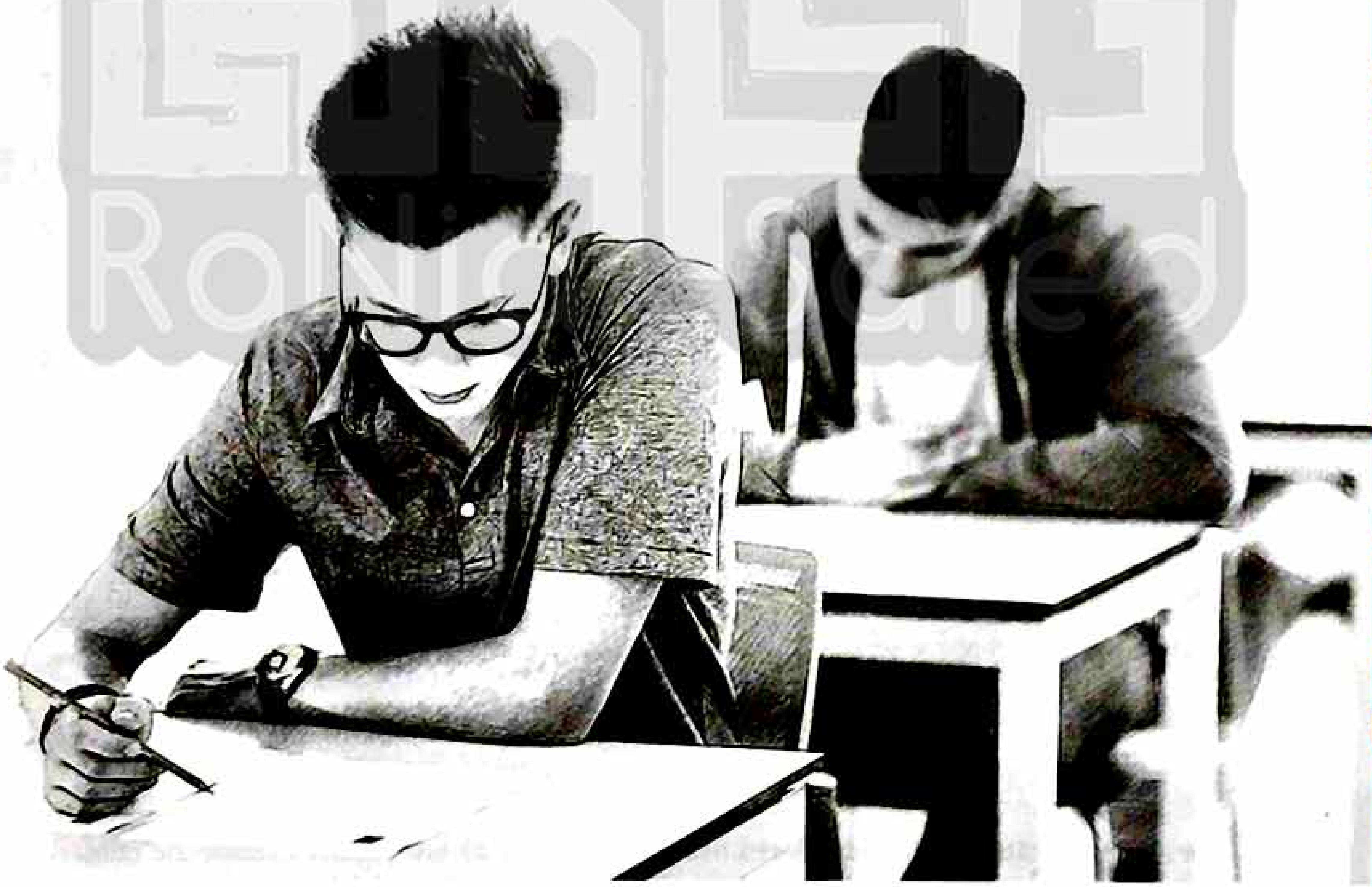
∴ The median $= \dfrac{21+23}{2} = 22$

The mode of a set of values is the most common value.

For example : The mode of the values : 6 , 2 , 3 , 11 , 6 , 8 , 6 , 3 is 6

Final Examinations

on Algebra and Statistics



Model Examinations of the School Book on Algebra and Statistics

Model 1

Answer the following questions :

1 Complete each of the following :

1. $2\frac{1}{5} \times$ .... $= 1$
2. If the order of the median of a set of values is the fourteenth , then the number of these values equals ....
3. $0.18 - 30\ \% =$ ....
4. $7x^3y^2 \times$ .... $= 21x^3y^5$
5. $(2x - 3)(x + 5) = 2x^2 +$ .... $- 15$

2 Choose the correct answer from those given :

1. The rational number that lies one third of the way between 8 and 12 from the smaller is ....
   (a) $8\frac{1}{3}$ (b) 10 (c) $9\frac{1}{3}$ (d) $10\frac{2}{3}$
2. If the mode of the values 7 , 5 , $x + 4$, 5 , 7 is 5 , then $x =$ ....
   (a) 1 (b) 4 (c) 5 (d) 7
3. If $\Delta + \square = 20$, $\Delta + \Delta + \square = 35$, then $\Delta =$ ....
   (a) 15 (b) 20 (c) 5 (d) 10
4. The arithmetic mean of the values 1 , 6 , 4 , 8 , 6 is ....
   (a) 25 (b) 5 (c) 6 (d) 8
5. If $\frac{2}{5}x = 10$, then $\frac{3}{5}x =$ ....
   (a) 25 (b) 15 (c) 20 (d) 5
6. $0.7 + 0.\dot{3} =$ ....
   (a) 1 (b) 3.7 (c) $0.\dot{3}7$ (d) $1\frac{1}{30}$

3 [a] Subtract : $5x^2 + y^2 - 3xy + 1$ **from** $6x^2 - 2xy + 3y^2$

[b] Use the distribution property to find the value of :

$$\frac{27}{16} \times \frac{11}{7} + \frac{27}{16} \times \frac{11}{7} - \frac{27}{16} \times \frac{6}{7}$$

4 [a] Simplify to the simplest form : $(2x - 3)(2x + 3) + 7$, then calculate the numerical value of the result when $x = -1$

[b] Find three rational numbers that lie between : $\frac{1}{2}$ and $\frac{1}{3}$

5 **[a]** **Divide :** $2x^3+3x^2-4x-6$ **by** $2x+3$ (where $x \neq -\frac{3}{2}$)

[b] **The following table shows Gehad's marks in mathematics exam in 6 months :**

Month	October	November	December	February	March	April
Mark	30	35	42	37	44	50

Find the arithmetic mean of the marks.

Model 2

Answer the following questions :

1 **Complete each of the following :**

1. $24x^4y^6 = 6x^2y^3 \times$ ....
2. The remainder of subtracting $-3x$ from $2x$ is ....
3. 1 , 1 , 2 , 3 , 5 , 8 , ... (in the same pattern)
4. If the mode of the values 7 , 5 , a + 3 , 5 , 7 is 7 , then a = ...
5. $5x^2 + 15xy = 5x$ (.............. + ...)

2 **Choose the correct answer from those given :**

1. The algebraic term $6x^3y^2$ is of the .... degree.
(a) third (b) fourth (c) fifth (d) sixth
2. The rational number that lies in half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$
3. The multiplicative inverse of the number $\left(\frac{1}{2}\right)^0$ is ....
(a) 2 (b) – 2 (c) 1 (d) –1
4. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....
(a) – 2 (b) 0 (c) 2 (d) 5
5. The median of the values 5 , 4 , 7 is ....
(a) 4 (b) 5 (c) 7 (d) 16
6. If the arithmetic mean of the values 3 , 5 and $x + 2$ is 4 , then the arithmetic mean of the two values $5 - x$, $5 + 2x$ is ....
(a) 6 (b) 4 (c) 3 (d) 2

3 [a] Using the distribution property , find the value of : $\frac{3}{7} \times 2 + \frac{3}{7} \times 6 - \frac{3}{7}$

[b] Find three rational numbers that lie between : $\frac{1}{2}$ and $\frac{1}{3}$

4 [a] What is the increase of : $7x + 5y + z$ than $2x + 6y + z$?

[b] Divide : $14x^2y - 35xy^2 + 7xy$ by $7xy$ where $x \neq 0$ and $y \neq 0$

5 [a] Simplify to the simplest form : $(x - 3)(x + 3) + 9$, then calculate the numerical value of the result when $x = 5$

[b] If the arithmetic mean of the numbers : 8 , 7 , 5 , 9 , 4 , 3 , k + 4 is 6 , then find the value of : k

Model examination for the merge students

Answer the following questions :

1 Complete each of the following :

1 The algebraic term $5 x y$ is of the .... degree.

2 $(x - 3)$ (............... + ...) $= x^2 - 9$

3 The rational number which hasn't a multiplicative inverse is ....

4 The median of the values 3 , 4 , 5 is ....

5 The number $\frac{4}{x}$ is a rational number if $x \neq$ ....

2 Choose the correct answer from those given :

1 If $\frac{4}{7} x = \frac{4}{7}$, then $x =$ ....

(a) 1 (b) 0 (c) 4 (d) 7

2 The arithmetic mean of the values 2 , 3 , 8 , 2 , 5 equals ....

(a) 3 (b) 2 (c) 4 (d) 8

3 The additive inverse of the number – 3 is ....

(a) – 3 (b) 3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

4 The remainder of subtracting $7 x$ from $9 x$ equals ....

(a) $2 x$ (b) $16 x$ (c) $-2 x$ (d) 0

5 The mode of the values 3 , 3 , 4 , 4 , 5 , 3 is ....

(a) 4 (b) 22 (c) 5 (d) 3

3 [a] Using the distribution property , complete to find :

$\frac{5}{7} \times 8 + \frac{5}{7} \times 5 + \frac{5}{7} = \frac{5}{7}$ (............... + ... + ...) $= \frac{5}{7}$ (...............) = ...

[b] If $a = \frac{1}{2}$, $b = -2$, complete the following :

$b \div a =$ (...............) ÷ (...............) = (...............) × (...............) = ...

4 Put true (✔) or false (✘) :

1 The quotient of $12 x^4 + 6 x$ by $6 x$ is $2 x^3 + 1$ (   )

2 The H.C.F. of : $15 x^5 + 5 x$ is $5 x^5$ (   )

3. The rational number that lies between $\frac{1}{4}$ and $\frac{3}{4}$ is $\frac{1}{2}$ ( )
4. $5x + 3x = 8x$ ( )
5. If $(x+4)^2 = x^2 + k + 16$, then $k = 4x$ ( )

5 Match from column (A) to column (B) :

Column (A)	Column (B)
1. If $\frac{x-7}{5} = 0$, then $x =$	3
2. $3x^2 + 15y =$ .... $(x^2 + 5y)$	7
3. $(3x + 5) + (4x - 5) =$	50
4. $\frac{1}{2} =$ .... %	1
5. If $\frac{a}{b} = \frac{1}{2}$, then $\frac{2a}{b} =$	$7x$

Some Schools Examinations on Algebra and Statistics

1 Cairo Governorate

El-Maadi Educational Zone

Answer the following questions :

1 Complete each of the following :

1. The median of the values : 5 , 9 , 7 , 4 , 3 , 8 is ....
2. The remainder of subtracting $-3x$ from $2x$ is ....
3. $5x^2 + 15xy = 5x$ (.............. + ...)
4. If the mode of the values : 8 , 5 , y + 3 , 5 , 8 is 8 , then y = ...
5. The rational number that hasn't a multiplicative inverse is ....

2 Choose the correct answer :

1. The mean of the values : 4 , 7 , 3 , 9 , 2 is ....
   (a) 2 (b) 3 (c) 5 (d) 7
2. The additive inverse of the number $\left(\frac{1}{2}\right)^{zero}$ is ....
   (a) 2 (b) -2 (c) 1 (d) -1
3. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....
   (a) -2 (b) 0 (c) 2 (d) 5
4. The number that lies at half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....
   (a) $\frac{2}{3}$ (b) $\frac{5}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$
5. The algebraic term : $6x^3y$ is of the .... degree.
   (a) first (b) fourth (c) sixth (d) fifth
6. If $\frac{a}{b} = 1$, then $5a - 5b$ = ...
   (a) zero (b) 1 (c) 3 (d) 5

3 [a] Use the distribution property to find the value of : $\frac{3}{7} \times \frac{5}{6} + \frac{3}{7} \times \frac{7}{6} - \frac{3}{7}$

[b] Simplify : $(x-3)(x+3)+7$

4 [a] Write three rational numbers between : $\frac{1}{3}$ and $\frac{5}{6}$

[b] Factorize by identifying the H.C.F. : $3a(4a+5b) - 2b(4a+5b)$

5 [a] Add : $3a^2 + 2a + 5$ **and** $2a^2 - 5a + 3$

[b] Divide : $x^2 + 6x + 5$ **by** $x + 5$ (where $x \neq -5$)

2 Cairo Governorate

El-Waily Educational Zone
Modern Future Language School

Answer the following questions :

1 Choose the correct answer :

1. The algebraic term : $6x^3y$ is of the .... degree.
   (a) first (b) fourth (c) sixth (d) fifth
2. If the mode of the values : 7 , 5 , $x + 4$, 5 , 7 is 5 , then $x =$ ....
   (a) 1 (b) 4 (c) 5 (d) 7
3. If the rational number : $\frac{x-2}{x+3} = 0$, then the value of $x =$ ....
   (a) 1 (b) 2 (c) -2 (d) -3
4. The multiplicative inverse of the number $\left(\frac{1}{2}\right)^{zero}$ is ....
   (a) -2 (b) 2 (c) 1 (d) -1
5. Subtracting $-2x$ from $3x$ equals ....
   (a) x (b) $-5x$ (c) $5x$ (d) $-6x^2$
6. If the arithmetic mean for the numbers : 3 , 5 , x is 4 , then $x =$ ....
   (a) 3 (b) 4 (c) 5 (d) 6

2 Complete the following :

1. The median for the values : 4 , 8 , 3 , 5 , 7 is ....
2. If $\frac{x}{y} = 1$, then $x - y =$ ....
3. $(x - 5)(x + 5) =$ ....
4. $6x^3 = 2x \times$ ....
5. The number that lies at half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....

3 [a] Add : $3x - 2y + 5$ **and** $x + 2y - 2$

[b] Find three rational numbers that lie between : $\frac{1}{3}$ and $\frac{1}{2}$

4 [a] Use the distribution property to find the value of : $\frac{3}{7} \times 2 + \frac{3}{7} \times 6 - \frac{3}{7}$

[b] Divide : $21x^2y - 7xy + 35xy^3$ **by** $7xy$ (where $xy \neq 0$)

5 **[a] Simplify to the simplest form :** $(x-3)(x+3)+9$

[b] Subtract : $4x^2-5x+3$ **from** $5x^2+4x-3$

[c] This table shows a pupil's marks of mathematics in five months :

Month	Oct.	Nov.	Dec.	Feb.	March
Marks	40	30	55	45	35

Find the arithmetic mean of the marks.

3 Cairo Governorate

Western Cairo Educational Zone
Mathematics Inspection

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 The number $\frac{x-3}{x-5} \in \mathbb{Q}$, if $x \neq$ ....

(a) 3 (b) -3 (c) 5 (d) -5

2 The multiplicative inverse of the rational number $\frac{3}{2}$ is ....

(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) -0.6 (d) 0.6

3 $9a^7b^4 =$ .... $\times a^7 b$

(a) $3b^3$ (b) $9b^3$ (c) $-3ab$ (d) $9ab$

4 If the degree of the algebraic term : $2a^3b^n$ is ninth , then n = ...

(a) 8 (b) 6 (c) 2 (d) 9

5 The median of the values : 4 , 5 , 7 is ....

(a) 4 (b) 7 (c) 16 (d) 5

6 The mode of the values : 5 , 6 , 5 , 4 , 3 is ....

(a) 3 (b) 4 (c) 5 (d) 6

2 **Complete each of the following :**

1 $5x^2+15xy=5x($.............. + ...$)$

2 If $x-y=3$, $x+y=5$, then $x^2-y^2=$ ....

3 The arithmetic mean of the values : 5 , 4 , 8 , 3 , 10 is ....

4 The rational number in half way between $\frac{1}{7}$ and $\frac{5}{7}$ is ....

5 $|-5|-|2|=$ ....

3 **[a]** **Use the property of distribution to find the value of :** $\frac{6}{14} \times 10 + \frac{6}{14} \times 5 - \frac{6}{14}$

[b] **Find three rational numbers that lie between :** $\frac{1}{7}$ and $\frac{1}{3}$

[c] **Find the result of :** $\frac{-3}{5} + \frac{2}{3}$

4 **[a]** **Divide :** $12x^3 + 8x^2 - 4x$ **by** $4x\ (x \neq 0)$

[b] **Add :** $4x^2 - 5x - 1$ **and** $5x + 3x^2 - 7$

5 **[a]** **Simplify to the simplest form :** $(2x - 3)(2x + 3) + 9$, then find the value of the result if $x = -2$

[b] **The following table shows the distribution of marks of 20 students in an exam :**

Marks	7	8	9	10	Total
No. of students	5	9	4	2	20

Find the mode of these marks.

4 Giza Governorate

Al-Agoza Directorate
Supervision of Math

Answer the following questions :

1 **Choose the correct answer :**

1 If $\square + \square + \square + \triangle + \triangle + \triangle = 60$, then $\square + \triangle = \ldots\ldots\ldots$

(a) 30 (b) 40 (c) 20 (d) 50

2 If the mode of the set of values : 5 , $x + 2$, 4 , 6 , 9 is 6 , then $x = \ldots\ldots\ldots$

(a) 2 (b) 4 (c) 6 (d) 5

3 The algebraic term : $3x^2y^3$ is of the .... degree.

(a) fifth (b) third (c) sixth (d) fourth

4 The rational number $\frac{3 - x}{7 + x}$ = zero , when $x = \ldots\ldots\ldots$

(a) 7 (b) -7 (c) 3 (d) -3

5 If half of a number is 30 , then $\frac{3}{4}$ of this number is ....

(a) 48 (b) 42 (c) 40 (d) 45

6 The remainder of subtracting -3 a from 2 a is ....

(a) 5 a (b) -5 a (c) a (d) $-$ a

2 **Complete :**

1. If the ratio $x : 25$ equals $2 : 5$, then $x =$ ....
2. The number $\frac{-4}{7}$ has an additive inverse = ...
3. The arithmetic mean of the values : 2 , 3 , 2 , 6 , 7 equals ....
4. $(3x + 2)(x - 4) = 3x^2$ .... $- 8$
5. The median of the values : 2 , 5 , 4 , 6 , 3 is ....

3 **[a] Use the distribution property to get the result of :** $\frac{5}{19} \times 11 + \frac{5}{19} \times 9 - \frac{5}{19}$

[b] Add : $3x - 5y + 2$, $2x + 5y - 2$

4 **[a] Find the quotient of dividing :** $x^2 + 7x + 12$ **by** $x + 4$ (where $x \neq -4$)

[b] Simplify to its simplest form : $(2a - 3)(2a + 3) + 7$

, then find the numerical value of the result at $a = -1$

5 **[a] Find two numbers lying between :** $\frac{1}{2}$, $\frac{4}{3}$ one of them is rational , the other is an integer.

[b] If the median of the values : $x + 5$, $x + 3$, $x + 8$ is 9 , then find the value of x

5 Giza Governorate

Omrania Directorate
Math Inspection

Answer the following questions :

1 **Choose the correct answer :**

1. The mode of the values : 6 , 8 , 6 , 1 , 1 , 9 , 8 , 2 , 8 is ....
   (a) 1 (b) 6 (c) 8 (d) 9
2. $x^3 y \times x y^2 =$ ....
   (a) $x^3 y^2$ (b) $3 x^3 y^4$ (c) $x^4 y^3$ (d) $x^3 y^3$
3. The multiplicative inverse of $\left|\frac{-7}{8}\right|$ is ....
   (a) $\frac{-7}{8}$ (b) $\frac{8}{7}$ (c) $\frac{7}{8}$ (d) $\frac{-8}{7}$
4. The degree of the expression : $x^3 + 2xy + 3y^2x^2$ is the .... degree.
   (a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}
5. $(-5x) + (-3x) - x =$ ....
   (a) $-9x$ (b) $9x$ (c) $8x$ (d) $-8x$
6. $(3a + 2b)^2 = 9a^2 +$ .... $+ 4b^2$
   (a) $6ab$ (b) $12ab$ (c) $24ab$ (d) $36ab$

2 **Complete the following :**

1. The arithmetic mean of the values : 22 , 18 , 15 , 25 and 30 is ....
2. $-\frac{1}{4} +$ .... $= 0$
3. $(x+4)(x-4) = x^2$ ....
4. The median of the values : 23 , 16 , 12 , 28 , 21 , 32 , 9 is ....
5. $7x(x+5y) = 7x^2 +$ ....

3 **[a] By using the distribution property find :** $\frac{5}{9} \times \frac{2}{7} + \frac{5}{9} \times \frac{1}{7} + \frac{5}{9} \times \frac{4}{7}$

[b] Subtract : $5x^2 + 2x - 1$ **from** $8x^2 - 3x + 7$

4 **[a]** If $a = \frac{1}{2}$, $b = -\frac{2}{3}$, $c = 3$, **find the value of :** $c^2 - 6ab$

[b] Simplify to the simplest form : $(5x-6)^2 + 60x - 36$

5 **[a] Divide :** $x^2 + 12x + 35$ **by** $x + 5$ (where $x \neq -5$)

[b] The following table shows the marks of 50 students :

Marks	4	6	9	12	15	18
Frequency	6	13	16	7	5	3

Find the mode of these marks.

6 Alexandria Governorate

Middle Educational Zone
Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. $8.46 \simeq$ .... to the nearest tenth.
   (a) 8.4 (b) 9 (c) 8 (d) 8.5
2. If $\frac{x}{8} = \frac{3}{6}$, then $x =$ ....
   (a) 16 (b) 48 (c) 4 (d) 12
3. .... is a terminating decimal.
   (a) $\frac{7}{20}$ (b) $\frac{2}{11}$ (c) $\frac{7}{11}$ (d) $\frac{1}{3}$
4. The median for the values : 4 , 8 , 3 , 5 , 7 is ....
   (a) 3 (b) 4 (c) 5 (d) 7

5 $\frac{4}{7}$ .... $\frac{3}{5}$

(a) = (b) < (c) > (d) ≥

6 If the mode of the values : 7 , 5 , a + 3 , 5 , 7 is 7 , then a = ...

(a) 2 (b) 4 (c) 7 (d) 5

2 Complete each of the following :

1 The multiplicative inverse of $\left(\frac{1}{2}\right)^{zero}$ is ....

2 $100\,\% - \frac{1}{4} =$ ....

3 $(x + 5)(x +$ ....$) = x^2 +$ .... $+ 15$

4 $5\,x^2 + 3$ is an algebraic expression of the .... degree.

5 The arithmetic mean of the set of values : 1 , 6 , 4 , 8 , 6 is ....

3 [a] Factorize by identifying the H.C.F. : $9\,m^4\,n^2 - 6\,m^3\,n^3 + 12\,m^2\,n^4$

[b] Use the distribution property to find : $\left(-\frac{3}{7}\right) \times 8 + 5 \times \left(-\frac{3}{7}\right) + \left(-\frac{3}{7}\right)$

4 [a] Multiply : $(6\,x - 2\,y)(6\,x + 2\,y)$

[b] If $x = \frac{-1}{3}$, $y = \frac{3}{4}$, $z = -3$, find in the simplest form the numerical value of $x\,y\,z$

5 [a] What is the increase of : $7\,x + 5\,y + z$ **than** $2\,x + 6\,y + z$?

[b] The following table shows the number of hours at studying of Mona during 5 days :

Day	Saturday	Sunday	Monday	Tuesday	Wednesday
Hour	$3\frac{1}{2}$	3	$2\frac{1}{2}$	3	4

Find : 1 The mean of the studying hours. 2 The mode of the studying hours.

7 Alexandria Governorate

El-Gomrok Educational Zone
Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1 If the degree of the algebraic term : $x^3\,y^m$ is 5 , then m = ...

(a) 1 (b) 2 (c) 3 (d) 5

2 $\mathbb{Z}_+ \cap \mathbb{Z}_- =$ ....

(a) $\mathbb{Z}$ (b) $\mathbb{Z}_+$ (c) $\mathbb{Z}_-$ (d) $\varnothing$

3 If the rational number $\frac{x+4}{x-7} = 0$, then $x =$ ....

(a) 4 (b) 7 (c) – 4 (d) – 7

4 The mean of the values : 2 , 3 , 7 and 8 is ....

(a) 2 (b) 3 (c) 5 (d) 7

5 $\frac{1}{8} - \frac{3}{8} =$ ....

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{-1}{4}$ (d) $\frac{1}{4}$

6 If the order of the median for a set of ordered values is the fifth , then the number of these values is ....

(a) 3 (b) 5 (c) 7 (d) 9

2 Complete each of the following :

1 The additive inverse of the number $\left(\frac{2}{5}\right)^{\text{zero}}$ is ....

2 The greatest negative integer is ....

3 $(3x - y)(2x + 5y) = 6x^2 + 13xy -$ ....

4 The mode for the values : 3 , 9 , 12 , 3 , 7 , 8 and 3 is ....

5 If $\{1, 2, x\} = \{2, 5, 1\}$, then $x =$ ....

3 [a] Use the distribution property to find the following in the simplest form :

$\frac{7}{16} \times \frac{6}{7} - \frac{7}{16} \times \frac{4}{7}$

[b] Factorize by identifying the H.C.F. : $4x^3 - 6x^2 - 8x$

4 [a] Find three rational numbers lying between : $\frac{1}{5}$ and $\frac{1}{2}$

[b] Add : $2a^2 + 4b^2 + 5c$ and $3a^2 - 2b^2 + c$

5 [a] Find the quotient of : $16x^4y^2 - 32x^3y^3 + 24x^2y^4$ **by** $8x^2y$ where $x \neq 0$, $y \neq 0$

[b] Find the mean and the median for the following values : 7 , 8 , 2 , 4 and 9

8 El-Kalyoubia Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 $ab \times 2a^2b =$ ....

(a) $2a^3b^2$ (b) $-2a^2b$ (c) ab^4 (d) $-3ab$

2 If the mode for the set of values : 7 , 5 , y + 3 , 5 and 7 is 7 , then y = ...

(a) 3 (b) 4 (c) 5 (d) 7

3 The rational number that lies in half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$

4 If the order of the median of a set of values is the fourth , then the number of these values equals ....

(a) 3 (b) 5 (c) 7 (d) 9

5 If $2x = 10$, then $\frac{3}{5}x =$ ....

(a) 25 (b) 15 (c) 5 (d) 3

6 The algebraic term : $7xy$ is of the .... degree.

(a) first (b) second (c) third (d) fourth

2 Complete each of the following :

1 $3xy + 6x =$ .... $(y + 2)$

2 $25\% - \left|\frac{-1}{5}\right| =$ ....

3 $\frac{-4}{11} \times$ .... $= 1$

4 If the sum of 5 numbers is 30 , then the arithmetic mean for these numbers is ....

5 The number $\frac{4}{x}$ is a rational number if $x \neq$ ....

3 **[a] Subtract :** $2x + 6y - 7$ **from** $2x - 5y + 2$

[b] Divide : $14x^3 - 28x^2 + 7x$ **by** $7x$ where $x \neq$ zero

4 **[a] Use the distribution property to find the value of :** $\frac{2}{7} \times 9 + \frac{2}{7} \times 6 - \frac{2}{7}$

[b] The length of a rectangle is $(2x + 5)$ cm. and its width is $(3x + 2)$ cm. Calculate its area.

5 **[a] Find the median for the values :** 3 , 5 , 12 , 11 , 8 , 10

[b] If $x = \frac{-1}{3}$, $y = \frac{3}{4}$, $z = -3$, **find in the simplest form the numerical value of each of the following :**

1 $yz + \frac{1}{4}$ **2** $xy + yz$

9 El-Sharkia Governorate

Belbeis Education Directorate
El-Fath G.L.S

Answer the following questions :

1 Choose the correct answer :

1 The median of the values : 9 , 18 , 5 , 7 , 11 is ....

(a) 5 (b) 7 (c) 9 (d) 11

2 If $(x+y)^2 = 15$, $x^2 + y^2 = 7$, then $xy =$ ....

(a) 8 (b) 22 (c) 6 (d) 4

3 The mean of : 3 , 0 , 4 , 6 , 7 is ....

(a) 4 (b) 5 (c) 6 (d) 7

4 The rational number in half way between $\frac{2}{7}$ and $\frac{4}{7}$ is ....

(a) $\frac{5}{14}$ (b) $\frac{3}{7}$ (c) $\frac{5}{7}$ (d) $\frac{4}{14}$

5 The additive identity element in $\mathbb{Q}$ is ....

(a) zero (b) -1 (c) 1 (d) $\frac{1}{2}$

6 The multiplicative inverse of $\frac{5}{8}$ is ....

(a) $-\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{8}{3}$ (d) $\frac{8}{5}$

2 Complete each of the following :

1 $3x^2y \times$ .... $= 15x^2y^3$

2 The algebraic term : $-5x^2y^2$ is of the .... degree.

3 The mode of the values : 7 , 5 , 4 , 5 is ....

4 The number $\frac{x+3}{x-7} \in \mathbb{Q}$ if $x \neq$ ....

5 $(x+3)(3x-2) = 3x^2 +$ .... -6

3 [a] Use the distribution property to find the value of :

$\frac{8}{13} \times 11 + \frac{8}{13} \times 9 - \frac{8}{13} \times 7$

[b] Add : $2a + 3b - c$ **and** $3a - 2b - 2c$

4 [a] Find the quotient of : $x^2 - 9x + 20$ **by** $x - 4$ (where $x \neq 4$)

[b] Factorize by identifying the H.C.F. : $12xy^2 + 18x^2y - 6x^2y^2$

5 [a] Simplify : $(2x+5)(2x-5) + 25$, then find the numerical value of the result when $x = -1$

[b] The following table shows the scores of a class in maths exam :

Marks	5	6	7	8	9	10
Frequency	6	5	13	7	4	2

Find the mode mark.

10 El-Monofia Governorate

Shibon Elkom Directorate
Supervisor of Math

Answer the following questions :

1 Choose the correct answer :

1 The additive identity in the set of integers is ....

(a) zero (b) 1 (c) -1 (d) 2

2 If the mean of : 4 , 5 , x is 6 , then $x =$ ....

(a) 4 (b) 5 (c) 6 (d) 9

3 The number $\frac{\text{zero}}{-2}$ .... $\mathbb{N}$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

4 The additive inverse for the expression : $2x - 3y$ is ....

(a) $-2x - 3y$ (b) $2x + 3y$ (c) $3y - 2x$ (d) $-3y + 2x$

5 The smallest prime number is ....

(a) zero (b) 1 (c) 2 (d) 3

6 If $\frac{x+4}{x-3}$ is a rational number , then $x \neq$ ....

(a) 3 (b) -3 (c) 4 (d) -4

2 Complete each of the following :

1 The number that lies at half the distance between $\frac{1}{2}$, $\frac{3}{4}$ is ....

2 The order of the median for the values : 4 , 12 , 9 , 8 , 2 is ....

3 If the number $y + 5$ hasn't a multiplicative inverse , then $y =$ ....

4 The remainder of subtraction $2x - 1$ from .... equals $2x$

5 If the mode for the values : 2 , 4 , $k - 3$, is 4 , then $k =$ ....

3

[a] Factorize by identifying the H.C.F. : $10x^3 - 5x^2$

[b] Simplify : $(a-4)^2 + 8(a-2)$

[c] Add : $2x^2 - 5x + 3$, $4x - x^2 - 2$

4

[a] Find three rational numbers between : $\frac{3}{5}$, $\frac{1}{4}$

[b] Use the distribution property to find : $\frac{-5}{2} \times 4 + \frac{-5}{2} \times 3 + \frac{-5}{2}$

[c] If $x = \frac{3}{2}$, $y = \frac{-5}{4}$, **find in the simplest form the value of :** $x^2 - 2xy$ (Show steps)

5 **[a] Divide :** $x^2 - 5x + 6$ **by** $x - 2$ where $x \neq 2$

[b] Find the mean and the median of : 4 , 6 , 12 , 3 , 9 , 8 (Show steps)

11 El-Dakahlia Governorate

Maths Supervision

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....
   (a) – 2 (b) zero (c) 2 (d) 5
2. $(-3x) \times (-5y) =$ ....
   (a) $-15xy$ (b) $-8xy$ (c) $8xy$ (d) $15xy$
3. The mode of the values : 4 , 5 , 4 , 3 , 7 , 5 , 4 is ....
   (a) 3 (b) 4 (c) 5 (d) 7
4. The algebraic term : $6x^3y^2$ is of the .... degree.
   (a) third (b) fourth (c) fifth (d) sixth
5. The arithmetic mean for the values : 3 , $5 - x$, $7 + x$ is ....
   (a) 2 (b) 3 (c) 4 (d) 5
6. If $\frac{2}{5}x = 10$, then $\frac{3}{5}x =$ ....
   (a) 25 (b) 20 (c) 15 (d) 5

2 **Complete each of the following :**

1. The multiplicative inverse of the number $\left(\frac{-9}{8}\right)^{zero}$ is ....
2. The number that lies at half way between $\frac{1}{2}$ and $\frac{5}{8}$ is ....
3. If $\triangle + \square = 20$, $\triangle + \triangle + \square = 35$, then $\square =$ ....
4. If the order of the median of a set of values is the fifth , then the number of these values is ....
5. 1 , 1 , 2 , 3 , 5 , 8 , ... (in the same pattern)

3 **[a] Simplify :** $(x - 3)(x + 3) + 9$

, then calculate its numerical value when $x = 5$

[b] If $x = \frac{1}{2}$, $y = \frac{-2}{3}$, $z = 2$, **find the value of :** $\frac{y - z}{x}$

4 **[a]** Use the distribution property to find the value of : $\frac{7}{12} \times \frac{23}{45} + \frac{17}{12} \times \frac{23}{45} - 2 \times \frac{23}{45}$

[b] **1** Add : $5x + 2y - 1$ and $2x - 5y + 3$

2 Factorize by identifying the H.C.F : $3a(a - 2b) - 6b(a - 2b)$

, then find the value of the result when $(a - 2b) = \left|-\frac{1}{3}\right|$

5 **[a]** Divide : $2x^2 + 5xy + 2y^2$ by $2x + y$ where $2x + y \neq 0$

[b] The following table shows Omar's marks in 6 mathematics examinations :

Month	Oct.	Nov.	Dec.	Feb.	Mar.	Apr.
Mark	41	35	47	37	44	48

Find each of the median mark and the mean mark.

12 Port Said Governorate

Educational Directorate
Math Department

Answer the following questions :

1 **Choose the correct answer :**

1 The additive inverse of the number $\left(\frac{-1}{5}\right)^0$ is ....

(a) 1 (b) – 1 (c) 5 (d) $\frac{1}{5}$

2 The degree of the algebraic expression : $3x^2 + 5xy^2 + 6y^2$ is ....

(a) zero. (b) second. (c) third. (d) fourth.

3 If $\frac{x}{y} = 1$, then $3x - 3y =$ ....

(a) zero (b) 1 (c) 3 (d) 6

4 If the arithmetic mean of six values is 12 , then the sum of these values equals ....

(a) 2 (b) 6 (c) 18 (d) 72

5 The rational number that lies at the midpoint of the distance between $\frac{1}{4}$ and $\frac{1}{3}$ is ....

(a) $\frac{1}{12}$ (b) $\frac{7}{12}$ (c) $\frac{3}{4}$ (d) $\frac{7}{24}$

6 The length of a rectangle is $2x$ cm. , and its width is y cm. , then its perimeter is .... cm.

(a) $2xy$ (b) $3xy$ (c) $2x + y$ (d) $4x + 2y$

2 **Complete :**

1 The multiplicative inverse of the number $\frac{3}{4}$ is ....

2 The mode of the values : 3 , 3 , 5 , 4 , 4 , 3 is ....

3 $(2x - 3)(4x + 5) = 8x^2 +$ .... $- 15$

4 1 , 4 , 9 , 16 , ... , ... (in the same pattern)

5 The number $\frac{5}{x-4}$ is rational if $x \neq$ ....

3 **[a] Use the distribution property to find the value of :** $\frac{5}{17} \times 10 + \frac{5}{17} \times 23 + \frac{5}{17}$

[b] Add : $2a - 3b + 5c$ **and** $3a + b - 5c$

4 **[a] Divide :** $6x^2y^2 + 9x^2y^3$ **by** $6x^2y^2$ $(x \neq 0, y \neq 0)$

[b] Simplify to the simplest form : $(x+5)^2 + (x+2)(x-2)$

5 **[a] Factorize by identifying the H.C.F. :** $12a^2b + 18a^3b^2$

[b] If the set of ages of pupils in one school is : 7 , 9 , 13 , 6 , 8 , 12 , 10 , 14 , 11 , find the median age of this set.

13 Kafr El-Sheikh Governorate

General Maths Supervision

Answer the following questions :

1 **Complete :**

1 The degree of the algebraic term : $-4xy^2$ is ....

2 $(2x-3)(3x+5) = 6x^2 +$ .... $-$ ....

3 The arithmetic mean of the values : 2 , 3 , 2 , 6 , 7 is ....

4 The number that lies at half way between $\frac{1}{2}$ and $\frac{5}{8}$ is ....

5 If the mode of the values : 5 , 7 , 4 , a + 1 , 6 and 10 is 4 , then a = ...

2 **Choose the correct answer :**

1 If $\frac{x}{y} = 1$, then $3x - 3y =$ ....

(a) 6 (b) 3 (c) 1 (d) 0

2 The order of the median of the values : 6 , 2 , 5 , 4 , 1 is ....

(a) first. (b) second. (c) third. (d) fourth.

3 The number $\frac{2}{9a}$ is a rational number if $a \neq$ ....

(a) 2 (b) 0 (c) -9 (d) 9

4 The remainder of subtracting $-2x$ from $2x$ equals ....

(a) $-4x$ (b) $4x$ (c) 0 (d) -4

5 If the rational number $\frac{2-x}{x-3} = 0$, then $x =$ ....

(a) 2 (b) -2 (c) 3 (d) -3

6 If $(x+3)(x-3) = x^2 + k$, then $k =$ ....

(a) 3 (b) -3 (c) 9 (d) -9

3 **[a] Subtract :** $-a^2 - 5ab + 4b^2$ **from** $3a^2 - 2ab + 5b^2$

[b] Use the distribution property to find : $\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11)$

[c] Add : $5x + 4xy - 7y$ **and** $3x - 2xy + 5y$

4 **[a] Divide :** $6x^2 + 13x + 6$ **by** $2x + 3$ $\left(\text{where } x \neq -\frac{3}{2}\right)$ (Show steps)

[b] Simplify : $(x+2)^2 - 4x$, then find the numerical value of the result when $x = 1$

5 **[a] Factorize by taking out the H.C.F :** $3x^2y - 6xy^2 + 9xy$

[b] If $x = \frac{5}{9}$, $y = \frac{4}{3}$, $z = \frac{1}{9}$, find in the simplest form the value of $(x + z) \div y$ (Show the steps)

[c] If the arithmetic mean of the values : 8 , 7 , 5 , 6 , 4 , k + 5 is 6 , then find the value of k

14 El-Menia Governorate

N.T.S.

Answer the following questions :

1 **Choose the correct answer :**

1 The multiplicative inverse of the number $3\frac{2}{5}$ is ....

(a) $-3\frac{2}{5}$ (b) $3\frac{2}{5}$ (c) $\frac{17}{5}$ (d) $\frac{5}{17}$

2 The quotient of dividing : $2.25 \div 1.5 =$ ....

(a) 1.5 (b) 15 (c) 0.15 (d) 500

3 $(3x+5)(x+2) = 3x^2 +$ .... $+ 10$

(a) -7 (b) $11x$ (c) $5x$ (d) $7x$

4 The number $\frac{x-3}{x+5}$ is a rational number if $x \neq$ ....

(a) 3 (b) -5 (c) 5 (d) -3

5 The mode of the values : 3 , 3 , 4 , 4 , 5 , 3 is ....

(a) 4 (b) 22 (c) 5 (d) 3

6 If $\frac{15}{x} = \frac{-3}{4}$, then $x =$ ....

(a) -20 (b) -5 (c) 5 (d) 20

2 **Complete each of the following :**

1 $\frac{3}{4} + 50\% = \frac{\dots\dots}{\dots\dots}$

2 The median of the values : 4 , 8 , 3 , 5 , 7 is ....

3 $6 b^3 = 2 b \times$ ....

4 The rational number that hasn't a multiplicative inverse is ....

5 The arithmetic mean of the numbers : 10 , 4 , 7 , 3 , 1 is ....

3 **[a] Use the distribution property to find the value of :** $\frac{5}{17} \times 10 + \frac{5}{17} \times 23 + \frac{5}{17}$

[b] Divide : $6 x^2 y^2 + 9 x^2 y^3$ **by** $3 x^2 y^2$ $(x \neq 0 , y \neq 0)$

4 **[a] Factorize by taking out the H.C.F. :** $12 a^2 b + 18 a^3 b^2$

[b] If $a^2 = 25$, $b^2 = 9$ and $a b = 15$ **, then find the value of :** $(a - b)^2$

5 **[a] Find three rational numbers between :** $\frac{4}{5}$ and $\frac{2}{3}$

[b] Find the mean of the values : 2 , 5 , 3 , 6 , 9

15 Qena Governorate

Qena Directorate of Education
Directing Mathematics

Answer the following questions :

1 **Choose the correct answer :**

1 The expression : $3 x^2 y - 6 x$ its degree is ....

(a) second. (b) first. (c) third. (d) fourth.

2 The arithmetic mean of the numbers : 7 , 13 , 5 and 15 is ....

(a) 12 (b) 10 (c) 20 (d) 7

3 The median of : 4 , 7 , 8 , 6 , 5 is ....

(a) 3 (b) 4 (c) 5 (d) 6

4 The multiplicative inverse of the number 2^0 equals ....

(a) 2 (b) – 1 (c) – 2 (d) 1

5 $(x^2 + x) \div x =$ .... (where $x \neq 0$)

(a) 0 (b) x (c) $2 x + 1$ (d) $x + 1$

2 **Complete :**

1 The additive identity element in $\mathbb{Q}$ is ....

2 The sum of : $- 3 x^2 y + 4 x y^2 - 5$ and $- 3 x y^2 + x^2 y + 5$ is ....

3 The coefficient of the algebraic term : $4\,x\,y^2\,z$ is ....

4 The highest common factor of the expression : $5\,y^2\,x + 25\,y\,x^2$ is ....

5 If $\frac{x+1}{x-5} \in \mathbb{Q}$, then x ....

3 **[a] Find three rational numbers between :** $\frac{1}{2}$ and $\frac{1}{3}$

[b] Divide : $x^2 - 5\,x + 6$ **by** $x - 3$ (where $x - 3 \neq 0$)

[c] Use the distribution property to find the value of : $\frac{5}{17} \times 8 + \frac{5}{17} \times 10 - \frac{5}{17}$

4 **[a] Find :** $(2\,x - y)\,(2\,x + y)$

[b] Complete : The mode of the values 5 , 7 , 4 , 5 , 3 , 5 is ....

5 **[a] Subtract :** $-2\,x$ **from** $4\,x$

[b] Factorize the following expression by identifying the H.C.F : $3\,x^2 + 15\,x\,y$

Final Examinations 2020

on Algebra and Statistics

Some Schools Examinations on Algebra and Statistics

1 Cairo Governorate

Nozha Directorate of Education
Nozha Language Schools

Answer the following questions :

1 Choose the correct answer :

1. The degree of the algebraic term $5 x y^2$ is ……………
   (a) zero (b) 2 (c) 3 (d) 5
2. The number $\frac{x+3}{x-5}$ equals zero if $x =$ ……………
   (a) -3 (b) 3 (c) 5 (d) -5
3. The multiplicative inverse of $\left(\frac{2}{5}\right)^0$ is ……………
   (a) 1 (b) -1 (c) $-\frac{2}{5}$ (d) $\frac{-5}{2}$
4. The mode of the numbers : 5 , 8 , 4 , 9 and 8 is ……………
   (a) 9 (b) 4 (c) 8 (d) 5
5. The H.C.F. of $12 x^3 + 6 x^2$ is ……………
   (a) 6 (b) $6 x^2$ (c) x^2 (d) $3 x^2$

2 Complete :

1. $(x - y)(x + y) =$ ……………
2. $(3 x + 5)^2 =$ …………… $+ 30 x +$ ……………
3. The arithmetic mean of the values : 5 , 4 , 8 , 3 , 10 is ……………
4. $(3 x -$ ……………$)^2 =$ …………… $- 12 x + 4$
5. The number that lies half way between $\frac{2}{7}$ and $\frac{6}{7}$ is ……………

3 [a] 1 Add : $5 a - 2 b + 4 c$ and $4 b - 3 a + c$

2 Subtract : $2 x^2 + 5 x y - y^2$ from $(2 x + y)^2$

[b] Factorize by using the H.C.F : $4 x^2 y^3 - 2 x y^2 + 6 x^3 y$

4 [a] Divide : $x^2 - 5 x + 6$ by $x - 2$ (where $x \neq 2$)

[b] Use the distribution property to find : $\frac{5}{9} \times 4 + \frac{5}{9} \times 6 - \frac{5}{9}$

5 [a] Simplify : $(x - y)(x + y) - (x - y)^2$, then calculate the numerical value of the result when $x = 2$, $y = -1$

[b] Find the mean and the median of the values : 20 , 15 , 25 , 10 , 30 , 7

2 Cairo Governorate

Rod El-Farag Educational Zone
St. Mary's School

Answer the following questions :

1 Choose the correct answer :

1 If the arithmetic mean of the numbers : 5 , 8 , 7 , k , 9 , 3 is 6 , then k = ...

(a) 3 (b) 4 (c) 5 (d) 6

2 The multiplicative inverse of the number $\frac{3}{4}$ is ....

(a) $\frac{4}{3}$ (b) $-\frac{3}{4}$ (c) $-\frac{4}{3}$ (d) 1

3 If $(x-6)(x+6) = x^2 + k$, then k = ...

(a) – 10 (b) 36 (c) 10 (d) – 36

4 If the order of the median of a set of values is the fourth, then the number of these values equals ....

(a) 3 (b) 5 (c) 7 (d) 9

5 The rational number that lies on third of the way between 8 and 12 from the smaller is ....

(a) $8\frac{1}{3}$ (b) 10 (c) $9\frac{1}{3}$ (d) $10\frac{2}{3}$

6 $|-3| + |-5| =$ ....

(a) 2 (b) – 2 (c) 8 (d) – 8

2 Complete :

1 The algebraic term $6xy^3$ whose degree is ....

2 The mode of the values : 3 , 3 , 5 , 4 , 4 , 3 is ....

3 $(2x-3)(4x+5) =$ .... + ... – ....

4 1 , 4 , 9 , 16 , ... , ... (in the same pattern)

5 The number $\frac{5}{x-4}$ is rational if $x \neq$ ....

3

[a] Subtract : $3x^2 - 5xy + 6y^2$ **from** $2x^2 - 4xy - 2y^2$

[b] Find the quotient : $2x^3 + 11x^2 + 12x - 9$ **by** $x + 3$ **where** $x \neq -3$

4

[a] Find three rational numbers between : $\frac{1}{2}$ and $\frac{2}{3}$

[b] Simplify to the simplest form : $(2x-3)(2x+3) + 7$
, and calculate the numerical value of the result when $x = 1$

5 [a] Use the distribution property to find the value of : $\frac{7}{9} \times 14 + \frac{7}{9} \times 6 - \frac{7}{9} \times 2$

(without using the calculator)

[b] This table shows a pupil's marks of mathematics in five months :

Month	Oct.	Nov.	Dec.	Feb.	March
Marks	40	30	55	45	35

Find : [1] The arithmetic mean of the marks.

[2] The median of the marks.

3 Cairo Governorate

Maadi Zone
Degla Valley Language School

Answer the following questions :

1 **Choose the correct answer :**

[1] The arithmetic mean of the numbers : 3 , 6 , 1 , 6 is ....

(a) 4 (b) 3 (c) 6 (d) 18

[2] The mode of the values : 4 , 5 , 4 , 3 , 4 is ....

(a) 3 (b) 4 (c) 5 (d) 4.5

[3] The degree of the algebraic expression : $5x^3 + 2x^2 - 7$ is the ....

(a) fifth. (b) third. (c) first. (d) second.

[4] If $\frac{x}{y} = \frac{2}{3}$, then $\frac{3x}{2y} =$ ....

(a) $\frac{1}{5}$ (b) $\frac{3}{2}$ (c) $\frac{9}{4}$ (d) 1

[5] If $\frac{x+3}{x-7} = 0$, then the value of x is ....

(a) 3 (b) -7 (c) -3 (d) 7

[6] The median of the values : 2 , 1 , 6 , 5 , 7 is ....

(a) 2 (b) 6 (c) 5 (d) 7

2 **Complete :**

[1] $\frac{3}{4} =$ .... %

[2] $(x-5)(x+5) =$ ....

[3] $12x^2y^3 \div 4xy =$ ....

[4] The remainder of subtracting $-7x^2$ from $2x^2$ is ....

[5] The rational number that lies at half the way between : $\frac{1}{4}$ and $\frac{1}{2}$ is ....

3 **[a]** If $x = \frac{3}{4}$, $y = \frac{-5}{2}$, find in the simplest form the value of : $(x - y) \div (x + y)$

[b] Add : $3x^2 + 2x - 5$ and $2x^2 - 5x + 3$

4 **[a]** Divide : $\frac{10x^5 - 6x^3 + 4x^2}{2x^2}$

[b] Use the distribution property to find the value of : $\frac{3}{7} \times \frac{5}{6} + \frac{3}{7} \times \frac{7}{6} - \frac{3}{7}$

[c] Complete : $3x^2 - 6xy = 3x$ (...............)

5 **[a]** Simplify : $(2a - 3)(2a + 3) + 7$

[b] Write three rational numbers between : $\frac{1}{3}$ and $\frac{5}{6}$

[c] Find the mean of the values : 2 , 5 , 3 , 6 , 9

4 Giza Governorate

Al-Agoza Directorate
Supervision of Math

Answer the following questions :

1 **Choose the correct answer :**

1. If $\frac{3}{x-5}$ is a rational number , then $x \neq$ ....
   (a) zero (b) 3 (c) -5 (d) 5
2. The algebraic term $2x^2y$ is of the .... degree.
   (a) first (b) second (c) third (d) fourth
3. If $5a = 45$, $ab = 1$, then $b =$ ....
   (a) $\frac{1}{9}$ (b) 5 (c) $\frac{1}{5}$ (d) 9
4. Fifth the number $5^{10} =$ ....
   (a) 5^9 (b) 5^5 (c) 5^{11} (d) 3^9
5. The value of the digit 7 in the number 0.4753 is ....
   (a) $\frac{7}{10}$ (b) $\frac{7}{100}$ (c) $\frac{7}{1000}$ (d) 7
6. The mode of the values : 5 , 7 , 3 , 5 is ....
   (a) 5 (b) 7 (c) 3 (d) 4

2 **Complete :**

1. $(2a - 3b)(a + 5b) = 2a^2 +$ .... $-$ ....
2. If three times a number is 15 , then fifth this number is ....

3 The number which lies at half the distance between : $\frac{1}{2}$ and $\frac{3}{4}$ is ....

4 5 a^2 increases – 3 a^2 by ....

5 The median of the values : 4 , 8 , 3 , 5 , 7 is ....

3 **[a] Use the distribution property to get the result of :** $\frac{3}{5} \times 2 + \frac{3}{5} \times 6 - \frac{3}{5} \times 3$

[b] Simplify : $(2x - 3)(2x + 3) + 7$

4 **[a] Find two rational numbers between :** $\frac{1}{3}$ and $\frac{1}{2}$

[b] What is the increase of : $7x + 5y + z$ than $2x + 6y + z$?

5 **[a] Factorize by taking out the H.C.F. :** $18x^2y^3 + 6x^3y^2 - 3x^2y^2$

[b] If the arithmetic mean of the values : 8 , 7 , 5 , 9 , 4 , 3 , k + 4 is 6

, find the value of : k

5 Giza Governorate

Omrania Directorate
El-Sadat Governmental Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The algebraic term $7xy^3$ whose degree is ....

(a) 1 (b) 2 (c) 3 (d) 4

2 The remainder of subtracting $3x$ from $5x$ is ....

(a) $2x$ (b) $-2x$ (c) $8x$ (d) $2x^2$

3 The median of the values : 4 , 8 , 3 , 5 and 7 is ....

(a) 3 (b) 4 (c) 5 (d) 7

4 If $\frac{a}{b} = 1$, then $5a - 5b =$ ....

(a) zero (b) 1 (c) 3 (d) 5

5 The mode of the values : 7 , 3 , 7 , 2 and 7 is ....

(a) 3 (b) 7 (c) 2 (d) 5

6 If $\frac{15}{x} = \frac{3}{4}$, then $x =$ ....

(a) 20 (b) – 20 (c) 5 (d) – 5

2 **Complete each of the following :**

1 The multiplicative inverse of $\frac{-7}{5}$ is ....

2 The additive identity element in $\mathbb{Q}$ is ....

3 The mean of the numbers : 6 , 4 , 1 , 5 and 9 is ....

4 If $\frac{x+3}{x-2} \in \mathbb{Q}$, then $x \neq$ ....

5 The rational number in half way between : $\frac{1}{7}$ and $\frac{5}{7}$ is ....

3 [a] **Add :** $5x^2 - 7xy + 4y^2$ **and** $4x^2 + 5xy - 9y^2$

[b] **Use the distribution property to find :** $\frac{8}{13} \times 11 + \frac{8}{13} \times 9 + \frac{8}{13} \times 6$

4 [a] **Simplify :** $(x-5)(x+5) + 25$, then find the value of the result if $x = 3$

[b] **Find three rational numbers between :** $\frac{1}{3}$ and $\frac{1}{2}$

5 [a] **Factorize by taking out the H.C.F. :** $27x^3y^2 - 9x^2y^3 + 3xy$

[b] **The following table shows the distribution of marks of 20 students in an exam :**

Marks	7	8	9	10	Total
No. of students	5	9	4	2	20

Find the mode of these marks.

6 Alexandria Governorate

Middle Educational Zone
Math's Supervision

Answer the following questions :

1 **Complete each of the following :**

1 If $\frac{4}{6} = \frac{12}{x}$, then $x + 2 =$ ....

2 The multiplicative inverse of $-\frac{2}{3}$ is ....

3 $\frac{1}{2} =$ .... %

4 The rational number in half way between $\frac{3}{5}$ and $\frac{4}{5}$ is ....

5 If a + 3 b = 7 , and c = 3 , then the numerical value of : a + 3 (b + c) is ....

6 The arithmetic mean of the set of values : 2 , 3 , 8 , 2 , 5 equals ....

2 **Choose the correct answer :**

1 $0.0635 \simeq$ .... to the nearest hundredth.

(a) 0.63 (b) 0.07 (c) 0.06 (d) 0.063

2 $0.7 + 0.\dot{3} =$ ....

(a) 1 (b) 3.7 (c) $0.\dot{3}7$ (d) $1\frac{1}{30}$

3 If the order of the median of a set of values is the fourteenth, then the number of these values equals ....

(a) 27 (b) 15 (c) 7 (d) 28

4 $(4x-3)(x-4) =$ ....

(a) $4x^2 - 19x - 12$ (b) $4x^2 - 7$ (c) $4x^2 - 12$ (d) $4x^2 - 19x + 12$

5 The mode of the values : 3 , 3 , 4 , 4 , 5 , 3 is ....

(a) 4 (b) 22 (c) 5 (d) 3

3 **[a] Multiply : $(2x + y)(x + 2y)$, then find the numerical value at : $x = 2$, $y = 1$**

[b] Use the distribution property to find : $\frac{7}{12} \times \frac{23}{45} + \frac{17}{12} \times \frac{23}{45} - 2 \times \frac{23}{45}$

4 **[a] Divide : $x^3 y - 4xy^2 + 6xy + x^2 y^2$ by xy**

[b] Find three rational numbers between : $\frac{4}{5}$ and $\frac{2}{3}$

5 **[a] Subtract : $5x^2 + y^2 - 3xy$ from $x^2 - 2xy + 3y^2$**

[b] The following table shows the marks of Alaa in maths tests in 6 months :

Month	Oct.	Nov.	Dec.	Feb.	March	April
Mark	41	35	47	37	44	48

Find : 1 The median for the previous marks. 2 The mean for the previous marks.

7 Alexandria Governorate

El-Montaza Educational Zone
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The additive inverse of the number $\left(\frac{-1}{5}\right)^0$ is ....

(a) 1 (b) -1 (c) 5 (d) $\frac{1}{5}$

2 The degree of the algebraic expression : $3x^2 + 5xy^2 + 6y^2$ is ....

(a) zero (b) second (c) third (d) fourth

3 If $\frac{x}{y} = 1$, then $3x - 3y =$ ....

(a) zero (b) 1 (c) 3 (d) 6

4 If the arithmetic mean of six values is 12 , then the sum of these values equals ....

(a) 2 (b) 6 (c) 18 (d) 72

5 The rational number that lies at the midpoint of the distance between $\frac{1}{4}$ and $\frac{1}{3}$ is ....

(a) $\frac{1}{12}$ (b) $\frac{7}{12}$ (c) $\frac{3}{4}$ (d) $\frac{7}{24}$

6 The length of a rectangle is $2x$ cm. and its width is y cm. , then its perimeter = ...

(a) $2xy$ (b) $3xy$ (c) $2x + y$ (d) $4x + 2y$

2 **Complete :**

1. $2 x^3 \times 3 x y =$ ....
2. $2 \frac{1}{5} \times$ .... $= 1$
3. The remainder of subtracting $(-3x)$ from $(2x)$ is ....
4. If the mode of the values : 7 , 5 , a + 3 , 5 , 7 is 7 , then a = ...
5. The median of the values : 5 , 9 , 7 , 4 , 3 , 8 is ....

3 **[a] Use the distribution property to find the value of :** $\frac{5}{17} \times 10 + \frac{5}{17} \times 23 + \frac{5}{17}$.

[b] Add : $2a - 3b + 5c$ **and** $3a + b - 5c$

[c] Divide : $6x^2y^2 + 9x^2y^3$ **by** $6x^2y^2$ $(x \neq 0 , y \neq 0)$

4 **[a]** If $a + b = \frac{5}{4}$ and $b + c = \frac{3}{4}$ **, find the value of :** $a + 2b + c$

[b] From : $5x^2 + 4x - 3$ **subtract :** $4x^2 - 5x + 3$

[c] Simplify : $(x-1)^2 + (x+3)(x-3)$

5 **[a] Factorize :** $12a^2b + 18a^3b^2$

[b] If $a^2 = 25$, $b^2 = 9$ and $ab = 15$ **, then find the value of :** $(a-b)^2$

[c] If the arithmetic mean of the values : 3 , 5 and $x + 2$ is 4
, then find the arithmetic mean of the two values : $5 - x$, $5 + 2x$

[d] If the set of ages of pupils in one school is as follows :
$\{7 , 9 , 13 , 6 , 8 , 12 , 10 , 14 , 11\}$, find the median age of this set.

8 El-Kalyoubia Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. $|-5| - |2| =$ ....
   (a) 3 (b) −7 (c) 10 (d) −3
2. If the arithmetic mean for the numbers 3 , 5 , x is 4 , then $x =$ ....
   (a) 3 (b) 4 (c) 5 (d) 6
3. The remainder of subtracting $9x$ from $7x$ equals ....
   (a) $2x$ (b) $-2x$ (c) $16x$ (d) −2
4. If 6 , 5 , 12 and x are proportional numbers , then $x =$ ....
   (a) 8 (b) 10 (c) 5 (d) 7

5 The algebraic term $3x^2y$ is of the .... degree.

(a) third (b) fourth (c) fifth (d) sixth

6 If the mode of the values : 7 , 5 , $x + 4$, 5 , 7 is 5 , then $x =$ ....

(a) 1 (b) 4 (c) 5 (d) 7

2 **Complete each of the following :**

1 $5x^2 + 15xy = 5x$ (.............. + ...)

2 12 % of 500 kg. = ... kg.

3 The median of the values : 4 , 8 , 3 , 5 , 7 is ....

4 The rational number which hasn't a multiplicative inverse is ....

5 The rational number that lies one third of the way between 8 and 12 from the smaller number is ....

3 **[a] Find three rational numbers that lie between :** $\frac{1}{2}$ and $\frac{1}{3}$

[b] Simplify to the simplest form : $(x + 5)^2 + (x + 2)(x - 2)$

4 **[a] 1 Subtract :** $5x^2 + y^2 - 3xy - 1$ **from** $6x^2 - 2xy + 3y^2$

2 Divide : $x^2 - 5x + 6$ **by** $x - 3$ (where $x \neq 3$)

[b] If $a = \frac{3}{4}$, $b = -\frac{5}{2}$ **, find in the simplest form the numerical value of :** $\frac{a+b}{a-b}$

5 **[a]** The length of a rectangle is $4x$ cm. and its width is $3x$ cm. calculate its area.

[b] The following table shows Gehad's marks in mathematics exam in 6 months :

Month	October	November	December	February	March	April
Mark	20	25	42	27	40	50

Find the arithmetic mean of the marks.

9 El-Gharbia Governorate

East-Tanta Educational Directorate
Al-Salam Language School

Answer the following questions :

1 **Complete each of the following :**

1 $\frac{3}{4} + 50\ \% = \frac{..............}{..............}$

2 $\frac{4}{5} =$ .... %

3 The additive inverse of the number $\left|-\frac{2}{3}\right|$ is ....

4 The most repeated value of a set of values is called ....

5 The smallest natural number is ....

6 If the arithmetic mean of the values : 8 , x , 7 , 5 is 6 , then $x =$ ....

2 Choose the correct answer :

1 The number $\frac{5}{3} >$ ....

(a) $\frac{10}{3}$ (b) $\frac{25}{9}$ (c) $\frac{10}{6}$ (d) $\frac{3}{5}$

2 If 3 a = 27 and a b = 1 , then b = ...

(a) $\frac{1}{9}$ (b) $\frac{1}{5}$ (c) 5 (d) 9

3 The coefficient of the algebraic term $-5x^2y$ is ....

(a) 5 (b) -5 (c) 3 (d) -3

4 The median of the values : 11 , 18 , 7 , 10 , 21 is ....

(a) 10 (b) 11 (c) 7 (d) 21

5 The H.C.F. of : $10x^2 + 5x$ is ....

(a) $2x$ (b) $5x$ (c) 5 (d) x

3 [a] Add : $2a - 3b + 5c$ **and** $3a + b - 5c$

[b] Divide : $x^2 + 6x + 5$ **by** $x + 5$ (where $x \neq -5$)

4 [a] Use the property of distribution to find the value of :

$\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11)$

[b] Factorize by identifying the H.C.F. : $27x^4 - 18x^3$

5 [a] Add : $2x + y + 5$ **and** $3x + 2y - 1$

[b] 1 Find the mode of : 2 , 4 , 7 , 4 , 5

2 Find the median of : 4 , 8 , 3 , 5 , 7

10 El-Dakahlia Governorate

Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1 If $a \times \frac{b}{3} = \frac{a}{3}$, then b = ...

(a) $\frac{a}{3}$ (b) 0 (c) a (d) 1

2 If the mode of the values : 7 , 5 , y + 3 , 5 and 7 is 7 , then y = ...

(a) 3 (b) 4 (c) 5 (d) 7

3 The algebraic term $2^2 x^3 y^2$ is of the .... degree.

(a) third (b) fourth (c) fifth (d) seventh

4 $(15 x^4 + 5 x^3) \div 5 x^3 =$ ....

(a) $3 x^2 + x$ (b) $5 x^2 + 1$ (c) $3 x + 1$ (d) $4 x^4$

5 The rational number that lies in half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$

6 The additive inverse of the number $\left(\frac{1}{2}\right)^{\text{zero}}$ is ....

(a) 2 (b) – 1 (c) 1 (d) – 2

2 Complete each of the following :

1 The order of the median for the values : 4 , 8 , 7 , 5 , 3 is ....

2 $0.18 - 30\ \% =$ ....

3 If $(2 x + y)^2 = 4 x^2 + k x y + y^2$, then k = ...

4 If $\frac{5}{a+2}$ is a rational number , then a ≠ ....

5 The arithmetic mean for the values : 18 , 35 , 24 , 7 is ....

3 [a] Use the distribution property to find the value of :

$$\frac{7}{12} \times \frac{23}{45} + \frac{17}{12} \times \frac{23}{45} - 2 \times \frac{23}{45}$$

[b] Subtract : $(- x^2 - 4 x + 7)$ **from** $(3 x^2 - 4 x - 2)$

4 [a] Factorize by identifying the H.C.F. : 3 a (4 a + 5 b) – 2 b (4 a + 5 b)

[b] Find three rational numbers between : $\frac{4}{5}$ and $\frac{2}{3}$

5 [a] Simplify to the simplest form : (y – 3) (y + 3) + 9

[b] The following table shows a student's marks of mathematics in 6 months :

Month	Oct.	Nov.	Dec.	Feb.	March	April
Mark	41	35	47	37	44	48

Find : 1 The median for the previous marks.

2 The mean for the previous marks.

11 Suez Governorate

Directorate of Education
Mathematics Inspectorate

Answer the following questions :

1 Choose the correct answer :

1. The multiplicative inverse of $\left(\frac{1}{2}\right)^0$ is ....
   (a) 2 (b) -2 (c) 1 (d) -1
2. The degree of the algebraic term $6x^3y^2$ is .... degree.
   (a) third (b) fourth (c) fifth (d) sixth
3. $2ab^2 \div$ zero = ...
   (a) undefined. (b) zero. (c) ab (d) $2ab^2$
4. If the mode of the values : 7 , 5 , $x+4$, 5 , 7 is 5 , then $x =$ ....
   (a) 7 (b) 4 (c) 5 (d) 1
5. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....
   (a) -2 (b) 0 (c) 2 (d) 5
6. The number that lies half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....
   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$

2 Complete :

1. $2\frac{1}{5} \times$ .... $= 1$
2. If the order of the median of the values is fourteenth , then the number of these values is ....
3. The result of subtracting $-7x$ from $2x$ is ....
4. $(2x-3)(x+5) = 2x^2 +$ .... $- 15$
5. The arithmetic mean of the values : 1 , 6 , 8 , 4 , 6 is ....

3 **[a] By using the distribution property , find the value of :** $\frac{3}{7} \times 2 + \frac{3}{7} \times 6 - \frac{3}{7}$

[b] Find three rational numbers between : $\frac{1}{2}$ and $\frac{1}{3}$

4 **[a] Find the quotient :** $2x^2 + 13x + 15$ **by** $x+5$

[b] Simplify to its simplest form : $(x+3)(x-3)+9$
, then find the numerical value at $x = 5$

5 **[a] What is the increase of :** $7x + 5y + 2$ than $2x + 6y + 7$?

[b] Factorize by taking out the H.C.F : $12a^2b + 18a^3b^2$

12 Port Said Governorate

East Educational Administration
Math Orientation

Answer the following questions :

1 Complete each of the following :

1. $24\,x^4\,y^6 = 6\,x^2\,y^3 \times$ ....
2. The remainder of subtracting $-3\,x$ from $2\,x$ is ....
3. 1 , 1 , 2 , 3 , 5 , 8 , ... (in the same pattern).
4. If the mode of the values : 7 , 5 , a + 3 , 5 , 7 is 7 , then a = ...
5. $5\,x^2 + 15\,x\,y = 5\,x$ (.............. + ...)

2 Choose the correct answer from those given :

1. The algebraic term $8\,x^3\,y^2$ is of the .... degree.
   (a) third (b) fourth (c) fifth (d) sixth
2. The rational number that lies in half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....
   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$
3. The multiplicative inverse of the number $\left(\frac{1}{2}\right)^{\text{zero}}$ is ....
   (a) 2 (b) -2 (c) 1 (d) -1
4. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....
   (a) -2 (b) zero (c) 2 (d) 5
5. The median of the values : 5 , 4 , 7 is ....
   (a) 4 (b) 5 (c) 7 (d) 16
6. If the arithmetic mean for the set of values : 3 , 5 , $x + 2$ is 4 , then the arithmetic mean for the two values : $5 - x$, $5 + 2\,x$ is ....
   (a) 6 (b) 4 (c) 3 (d) 2

3 [a] Use the distribution property to find the value of : $\frac{3}{7} \times 2 + \frac{3}{7} \times 6 - \frac{3}{7}$

[b] Find three rational numbers that lie between : $\frac{1}{2}$ and $\frac{1}{3}$

4 [a] What is the increase of : $7\,x + 5\,y + z$ than $2\,x + 6\,y + z$?

[b] Divide : $14\,x^2\,y - 35\,x\,y^2 + 7\,x\,y$ **by** $7\,x\,y$, $x \neq$ zero , $y \neq$ zero

5 [a] Simplify to the simplest form : $(x - 3)\,(x + 3) + 9$

[b] **The following table shows Gehad's marks of mathematics in 6 months :**

Month	October	November	December	February	March	April
Mark	30	35	42	37	44	50

Find the arithmetic mean of the marks.

13 Kafr El-Sheikh Governorate

Mathematics Inspectorate
Language Schools

Answer the following questions :

1 Choose the correct answer :

1. The median of the values : 7 , 3 , 4 , 5 , 2 is ....
 (a) 7 (b) 5 (c) 4 (d) 3
2. The rational number $\frac{x-7}{x+3}$ = zero , when ....
 (a) $x = -3$ (b) $x = 7$ (c) $x \neq 3$ (d) $x \neq 7$
3. The quotient of dividing $2.25 \div 1.5 =$ ....
 (a) 1.5 (b) 15 (c) 0.15 (d) 500
4. The arithmetic mean of the numbers : 3 , 9 , 1 , 7 is ....
 (a) 20 (b) 5 (c) 4 (d) 3
5. $(x^2 + x) \div x =$ ....
 (a) zero (b) x (c) $2x + 1$ (d) $x + 1$
6. $\left|\frac{-5}{3}\right|$ .... zero.
 (a) < (b) = (c) > (d) ≤

2 Complete :

1. $6 b^3 = 2 b \times$ ....
2. The mode of the values : 7 , 5 , a + 4 , 5 , 7 is 7 , then a = ...
3. The additive inverse of $\left[4 \times \left(-1 \frac{1}{4}\right)\right]$ is ....
4. The degree of the algebraic term : $3^2 x^2 y^2$ is ....
5. The rational number that hasn't a multiplicative inverse is ....

3 [a] **Subtract :** $5x^2 + y^2 - 3xy$ **from** $x^2 - 2xy + 3y^2$

[b] **Use the distribution property to find :** $\frac{5}{7} \times 5 + \frac{5}{7} \times 10 - \frac{5}{7}$

[c] **Simplify :** $(2x + 3)(2x - 3) + 7$

4 **[a] If $x = \frac{3}{4}$, $y = -\frac{5}{2}$, find the numerical value of :** $(x - y) \div (x + y)$

[b] Divide : $6x^2 - xy - 15y^2$ **by** $2x + 3y$ **where** $(2x + 3y) \neq 0$

[c] Add : $3a^2 + 2a + 5$ **and** $2a^2 - 5a + 3$

5 **[a] Factorize by identifying the H.C.F. :** $12xy^3 + 18xy^2$

[b] Find four rational numbers between : zero and $\frac{1}{2}$

[c] The following table shows Gehad's marks of mathematics in 6 months :

Months	October	November	December	February	March	April
Marks	31	35	42	36	46	50

Find : 1 The arithmetic mean. 2 The median.

14 El-Menia Governorate

Maghagha Educational Directorate
St. Mark & El Tewfik Schools

Answer the following questions :

1 **Choose the correct answer :**

1 The number $\frac{x - 3}{x + 5}$ is a rational number if $x \neq$ ....

(a) 3 (b) -5 (c) 5 (d) -3

2 The mode of the values : 3 , 3 , 4 , 4 , 5 , 3 is ....

(a) 4 (b) 22 (c) 5 (d) 3

3 $\frac{3y}{5} - \frac{y}{5} =$ ....

(a) $\frac{2}{5}$ (b) $\frac{y}{5}$ (c) $\frac{2y}{5}$ (d) $2y$

4 The algebraic expression : $x^3 - 3x^2 + 4$ is of the .... degree.

(a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}

5 If $\frac{15}{x} = \frac{-3}{4}$, then $x =$ ....

(a) -20 (b) -5 (c) 5 (d) 20

6 $(x + y)(x - y) =$ ....

(a) $2x$ (b) $(x - y)^2$ (c) x^2 (d) $x^2 - y^2$

2 **Complete the following :**

1 The mean of the numbers : 10 , 4 , 7 , 3 , 1 is ....

2 If $(x - y)(3x + 2y) = 3x^2 + kxy - 2y^2$, then $k =$ ....

3 The coefficient of the algebraic term $(-5\,x\,y^2)$ is ....

4 The rational number which hasn't a multiplicative inverse is ....

5 If the order of the median of a set of values is fourth , then the number of these values is ....

3 **[a] Find three rational numbers lying between : $\frac{1}{3}$ and $\frac{1}{2}$**

[b] Simplify : $(2\,x+3)^2-12\,x$, then find the numerical value of the result at $x=-2$

4 **[a] Using the distribution property, find the value of :** $\frac{3}{7}\times 10+\frac{3}{7}\times 5-\frac{3}{7}$

[b] Divide : $(x^2+6\,x+5)$ **by** $(x+5)$ where $(x\neq -5)$

5 **[a] Factorize by taking out the H.C.F. :** $3\,m^4\,n^2-6\,m^3\,n^3+9\,m^2\,n^4$

[b] Subtract : $(-x^2-4\,x+7)$ **from** $(x^2-4\,x-2)$

[c] Find k if the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 , k is 14

15 Aswan Governorate

M.M. Yackoub English Language Government School

Answer the following questions :

1 **Choose the correct answer :**

1 The algebraic term $6\,x^3\,y$ is of the .... degree.

(a) first (b) fourth (c) sixth (d) fifth

2 The mode of the values : 7 , 5 , $x+4$, 5 , 7 is 5 , then $x=$ ....

(a) 1 (b) 4 (c) 5 (d) 7

3 If the rational number $\frac{x-2}{x+3}=0$, then the value of $x=$ ....

(a) 1 (b) 2 (c) -2 (d) -3

4 The multiplicative inverse of the number $3\,\frac{2}{5}$ is ....

(a) $-3\,\frac{2}{5}$ (b) $3\,\frac{2}{5}$ (c) $\frac{17}{5}$ (d) $\frac{5}{17}$

5 Subtracting $-2\,x$ from $3\,x$ equals ....

(a) x (b) $-5\,x$ (c) $5\,x$ (d) $-6\,x^2$

6 $(3\,x+5)(x+2)=3\,x^2+$ .... $+10$

(a) -7 (b) $11\,x$ (c) $5\,x$ (d) $7\,x$

2 **Complete :**

1. $5x^3y^3 \times$ .... $= 15x^4y^5$
2. If $\frac{x}{y} = 1$, then $5x - 5y =$ ....
3. $1\frac{2}{5} \times$ .... $= 1$
4. The number that lies at half way between $\frac{1}{4}$ and $\frac{5}{8}$ is ....
5. The median for the values : 4 , 8 , 3 , 5 , 7 is ....

3 **[a] Add :** $3x - 2y + 5$ **and** $x + 2y - 2$

[b] Find three rational numbers that lie between : $\frac{1}{4}$ and $\frac{1}{2}$

4 **[a] Use the distribution property to calculate :**

$$\frac{7}{12} \times \frac{23}{45} + \frac{17}{12} \times \frac{23}{45} - 2 \times \frac{23}{45}$$

[b] Divide : $21x^2y - 7xy + 35xy^3$ **by** $7xy$

5 **[a] What is the increase of :** $8x + 4y + 3z$ than $2x + 6y - z$?

[b] Simplify to the simplest form : $(5x - 2)^2 - (5x - 2)(5x + 2) + 7$

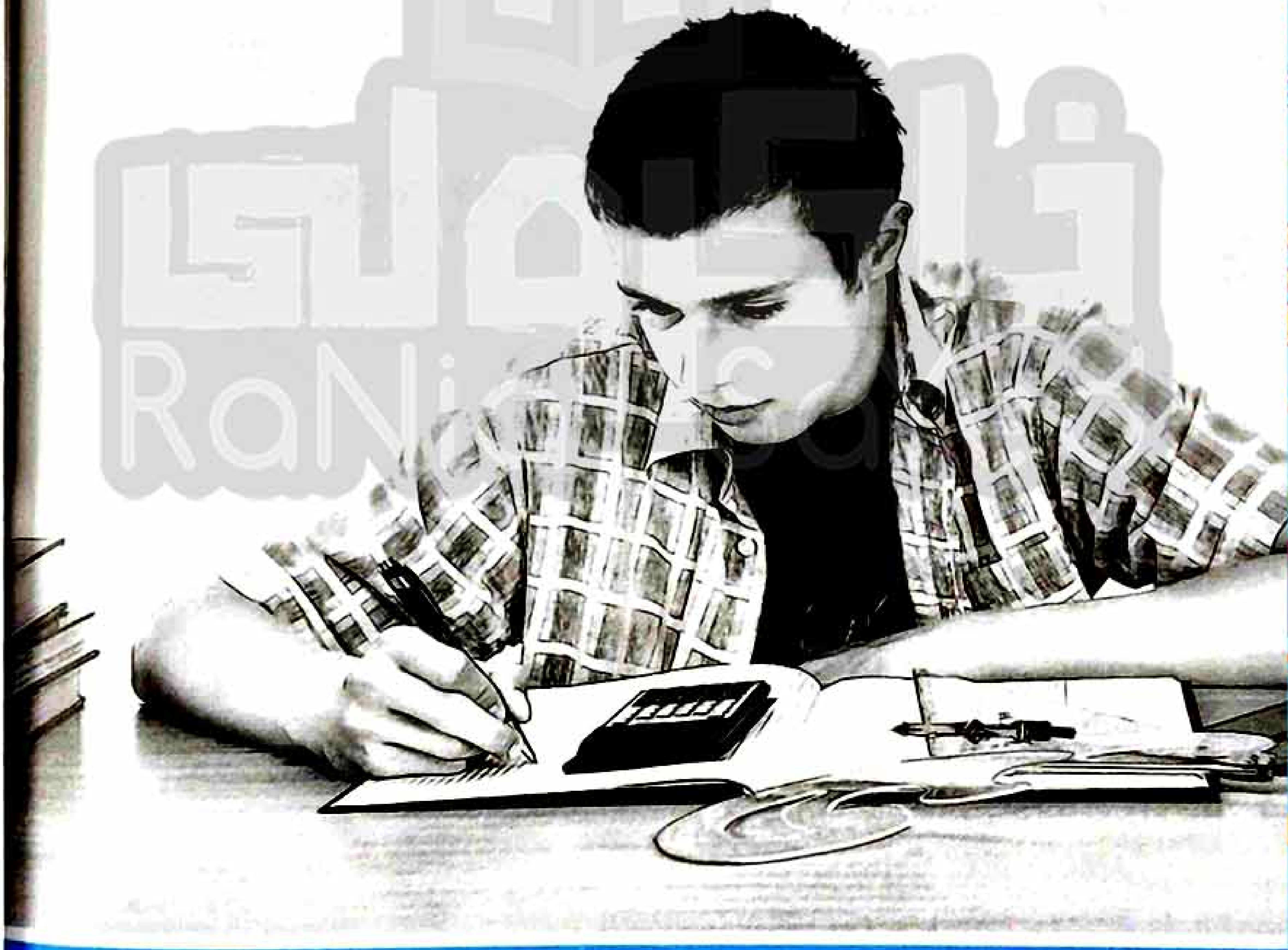
[c] The following table shows Habiba's marks of mathematics in 6 months :

The month	Oct.	Nov.	Dec.	Feb.	March.	April
The mark	41	35	47	37	44	48

Find the arithmetic mean of the marks.

Quizzes

on Geometry



Quiz 1 on lesson 1 – unit 4

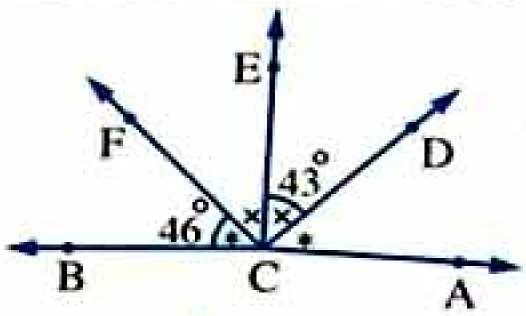
time 20 min.

1 Complete the following :

1. The acute angle supplements .... angle.
2. If m (∠ ABC) = 60° , then m (reflex ∠ ABC) = ...°
3. If the ratio between the measures of two supplementary angles is 1 : 2 , then the measure of the smaller angle equals ....°

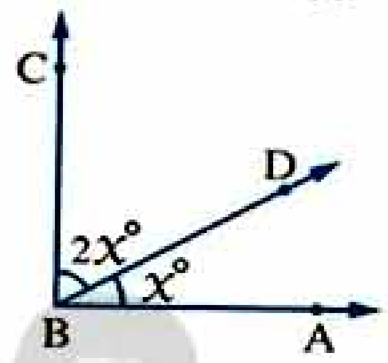
2 [a] In the opposite figure :

Are $\overrightarrow{CA}$ and $\overrightarrow{CB}$ on the same straight line ? Why ?



[b] In the opposite figure :

If $\overrightarrow{BC} \perp \overrightarrow{BA}$, then find the value of x

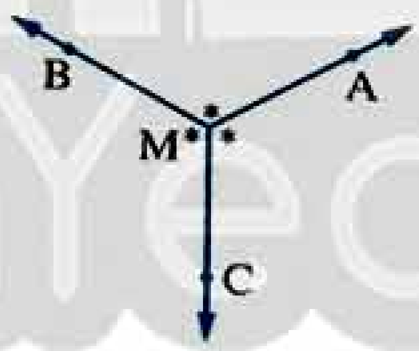


Quiz 2 till lesson 2 – unit 4

time 20 min.

1 Choose the correct answer from those given :

1. From the opposite figure :



m (∠ AMC) = ...

(a) 60° (b) 120° (c) 150° (d) 360°

2. If ∠ A complements ∠ B and m (∠ A) = 48° , then m (reflex ∠ B) = ...

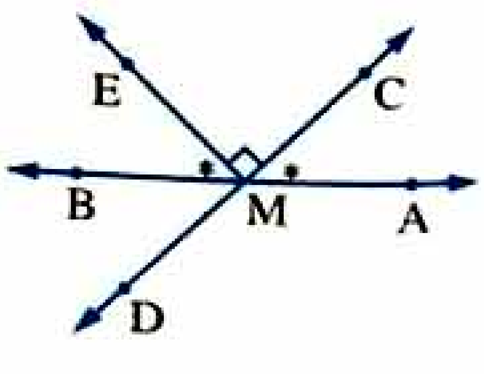
(a) 309° (b) 312° (c) 315° (d) 318°

3. The sum of measures of the accumulative angles at a point equals ....

(a) 2 right angles. (b) 3 right angles. (c) 4 right angles. (d) 5 right angles.

2 In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m (∠ CME) = 90° , m (∠ AMC) = m (∠ EMB)



Find : 1 m (∠ AMC) 2 m (∠ BMD) 3 m (∠ AMD)

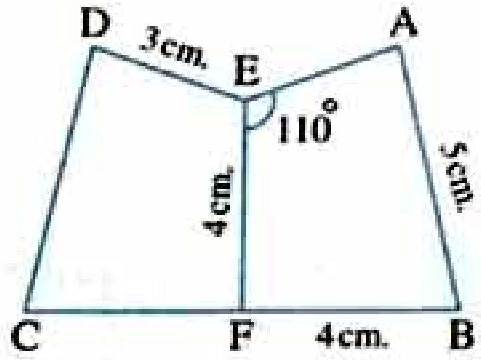
Quiz 3 till lesson 3 – unit 4

time 20 min.

1 In the opposite figure :

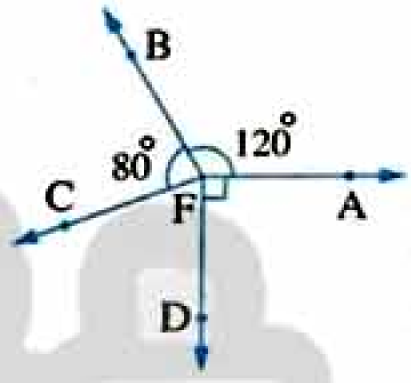
If $F \in \overline{BC}$ and the figure ABFE ≡ the figure DCFE

, complete the following :



1. The axis of symmetry of the figure is ……………
2. AE = …………… cm.
3. ∠ D ≡ ∠ ……………
4. m (∠ FED) = ……………°
5. m (∠ EFB) = ……………°
6. The perimeter of the figure ABCDE = …………… cm.

2 In the opposite figure :



m (∠ AFB) = 120° , m (∠ BFC) = 80°

, m (∠ AFD) = 90°

Find : m (∠ CFD)

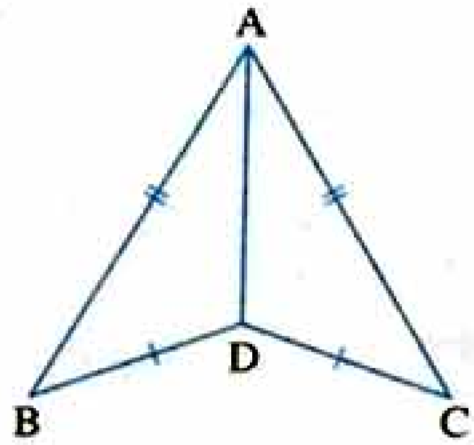
Quiz 4 till lesson 4 – unit 4

time 20 min.

1 Complete the following :

1. If Δ ABC ≡ Δ XYZ , m (∠ A) = 40° , m (∠ Y) = 60° , then m (∠ C) = ……………°
2. The two triangles are congruent if in one triangle , two sides and ……………
3. The two complementary adjacent angles , their outer sides are ……………

2 [a] In the opposite figure :

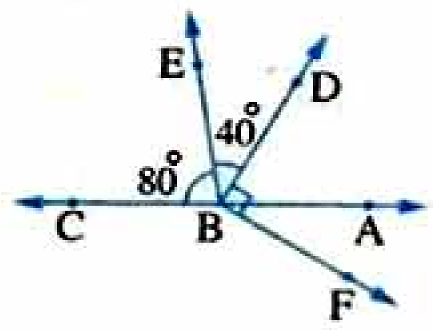


AB = AC

, BD = CD

Is Δ ABD ≡ Δ ACD ? Why ?

[b] In the opposite figure :



$B \in \overleftrightarrow{AC}$, m (∠ EBC) = 80°

, m (∠ DBE) = 40°

and m (∠ DBF) = 90°

Find : m (∠ CBF)

Quiz 5 till lesson 5 – unit 4

time 20 min.

1 Complete the following :

1 If a straight line cuts two parallel straight lines , then each two alternate angles are ....

2 The right angle complements an angle of measure ....°

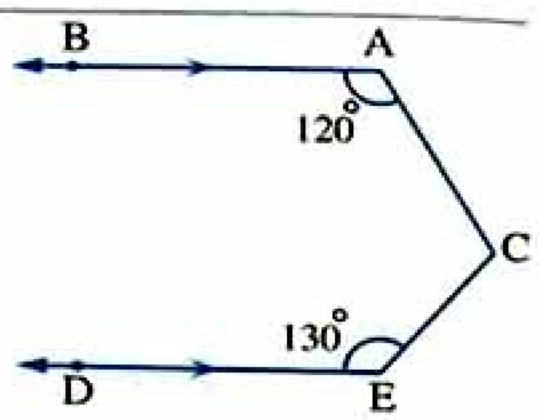
3 If two straight lines are parallel to a third straight line , then they are ....

2 [a] In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{ED}$, m (∠ A) = 120°

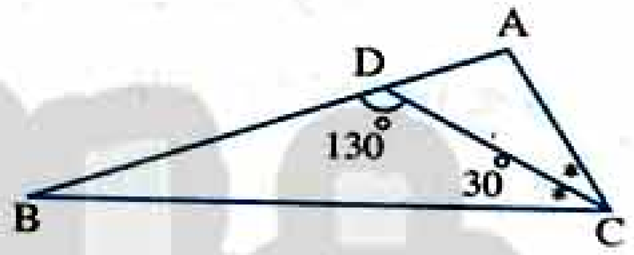
and m (∠ E) = 130°

Find : m (∠ C)



[b] In the opposite figure :

Find : m (∠ A)



Quiz 6 till lesson 6 – unit 4

time 20 min.

1 Complete the following :

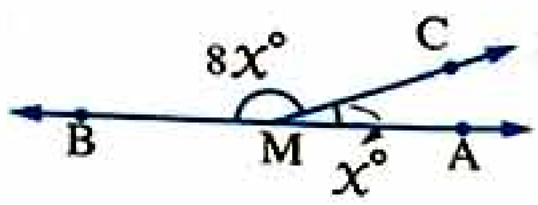
1 If the two adjacent angles are supplementary , then their outer sides are ....

2 The axis of symmetry of the line segment is ....

3 **In the opposite figure :**

If M ∈ $\overleftrightarrow{AB}$,

then the value of x = ...°

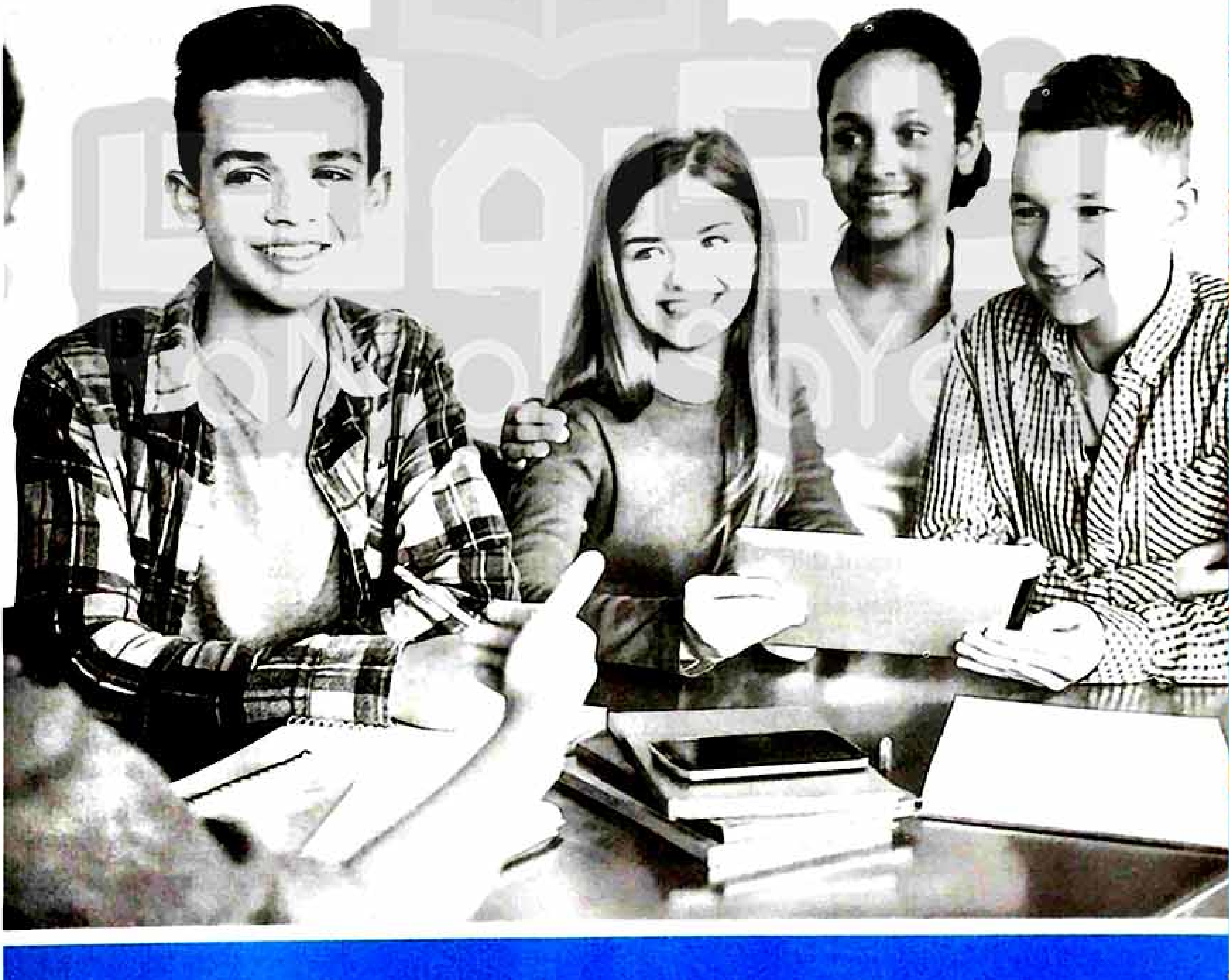


2 [a] Using the geometric instruments , draw an angle of measure 120° and bisect it into four equal angles in measure. (Don't remove the arcs)

[b] Draw Δ ABC in which AB = BC = 5 cm. and AC = 6 cm. Using the compasses , bisect $\overline{AC}$ in D , then draw $\overline{BD}$ Does $\overline{BD} \perp \overline{AC}$? (Don't remove the arcs)

Final Revision

of Geometry



Revision for the important theorems , corollaries and rules of geometry

The types of angles

Zero angle
Its measure is 0°

Acute angle
Its measure is more than 0° and less than 90°

Right angle
Its measure is 90°

Obtuse angle
Its measure is more than 90° and less than 180°

Straight angle
Its measure is 180°

Reflex angle
Its measure is more than 180° and less than 360°

The relation between the angles

The two vertically opposite angles are equal in measure.

Sum of measures of the two complementary angles is 90°

→ The complements of the same angle are equal in measure.

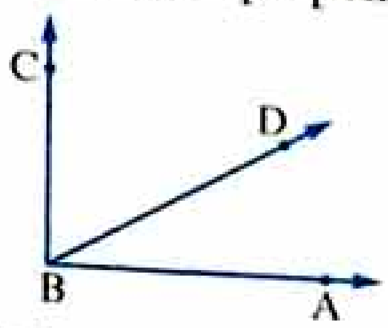
Sum of measures of the accumulative angles at a point is 360°

Sum of measures of the two supplementary angles is 180°

→ The supplements of the same angle are equal in measure.

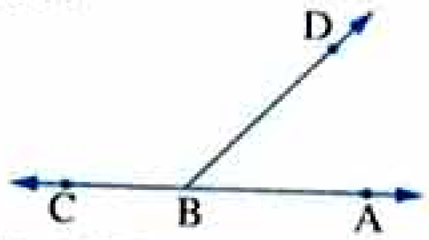
The two adjacent angles

The two adjacent complementary angles :
Their outer sides are perpendicular.



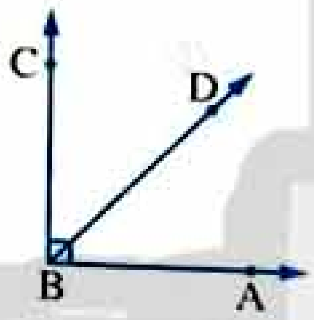
If m (∠ ABD) + m (∠ DBC) = 90°
, then $\overline{AB} \perp \overline{BC}$

The two adjacent supplementary angles :
Their outer sides are on the same straight line.



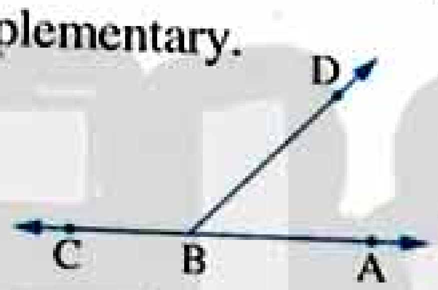
If m (∠ ABD) + m (∠ DBC) = 180°
, then $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on the same straight line.

The two adjacent angles whose outer sides are perpendicular , are complementary



If $\overrightarrow{BA} \perp \overrightarrow{BC}$
, then m (∠ ABD) + m (∠ DBC) = 90°

The two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are supplementary.

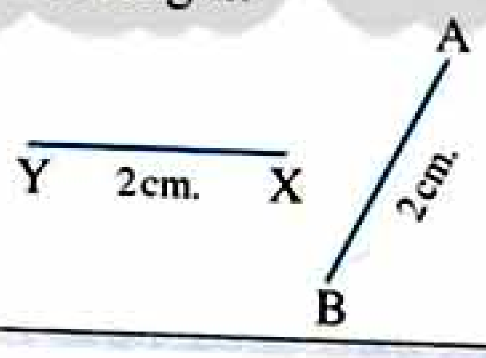


If B ∈ $\overleftrightarrow{AC}$
, then m (∠ ABD) + m (∠ DBC) = 180°

Congruence

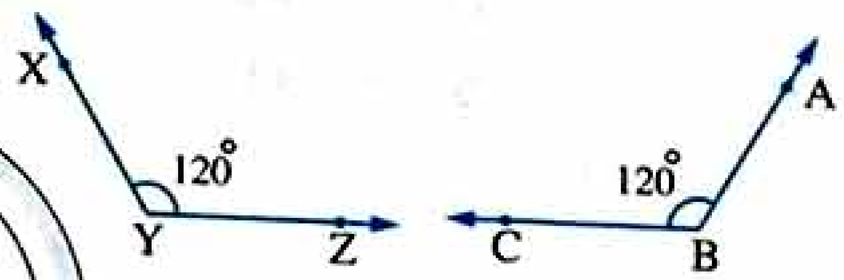
Congruence of two line segments

Two line segments are congruent if they are equal in length.



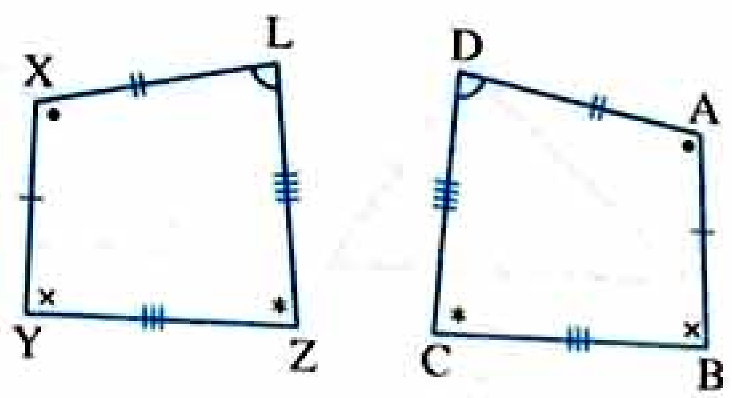
Congruence of two angles

Two angles are congruent if they are equal in measure.



Congruence of two polygons

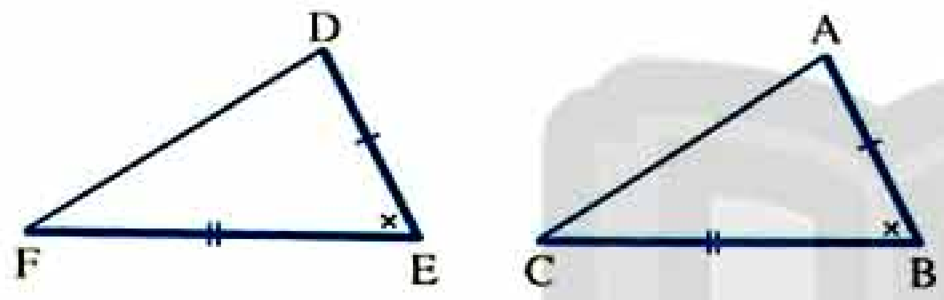
Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.



First case :

Two sides and the included angle (S.A.S.)

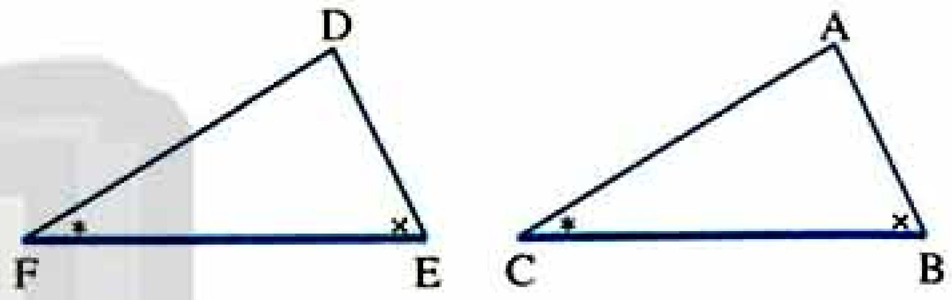
Two triangles are congruent if two sides and the inculded angle of one triangle are congruent to the corresponding parts of the other triangle.



Second case :

Two angles and one side (A.S.A.)

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

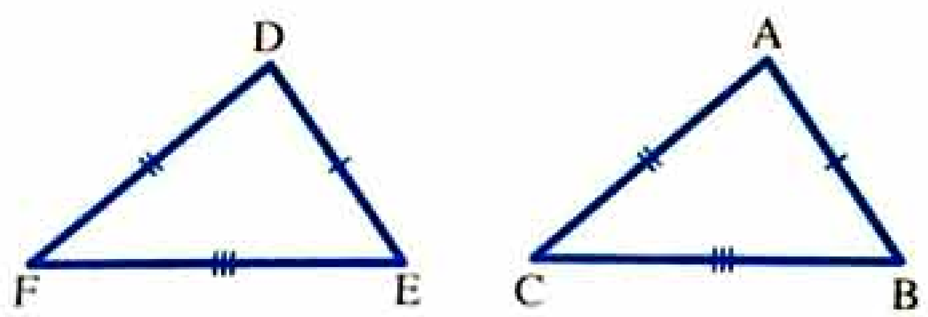


The cases of congruence of two triangles

Third case :

Three sides (S.S.S.)

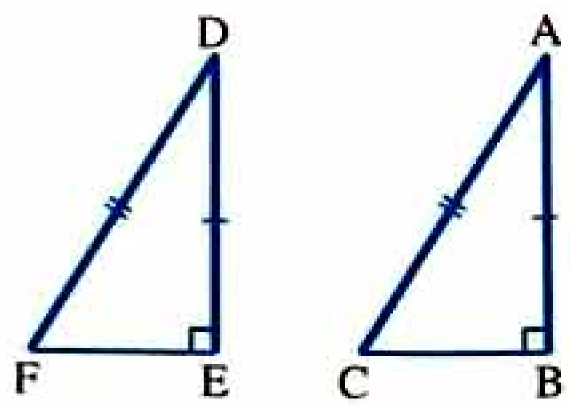
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.



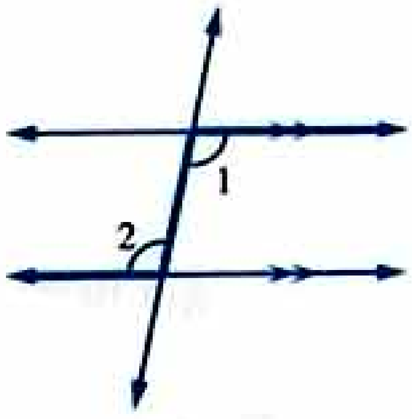
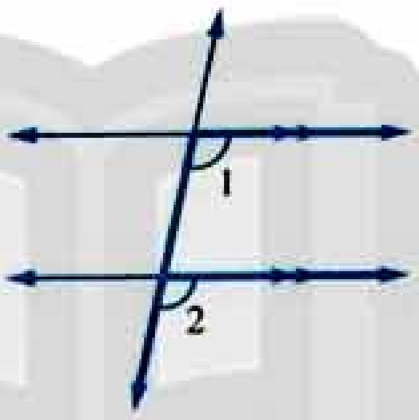
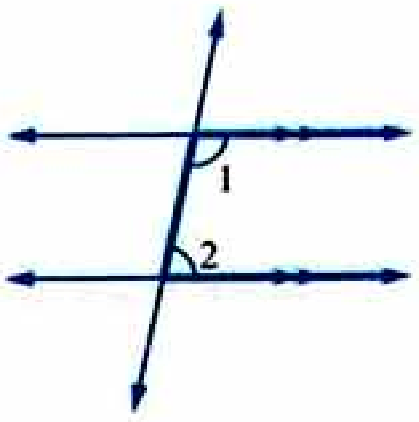
Fourth case :

Hypotenuse and one side in the right-angled triangle (R.H.S.)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.



If a straight line intersects two parallel straight lines , then

Each two alternate angles are equal in measure.	Each two corresponding angles are equal in measure.	Each two interior angles in the same side of the transversal are supplementary.
		
For example : m (∠ 1) = m (∠ 2) (alternate angles)	***For example :*** m (∠ 1) = m (∠ 2) (corresponding angles)	***For example :*** m (∠ 1) + m (∠ 2) = 180°

Remember How to prove the parallelism of two straight lines

The two straight lines are parallel if a third straight line intersects them and **one** of the following cases is satisfied :

1. Two alternate angles have the same measure.

or

2. Two corresponding angles have the same measure.

or

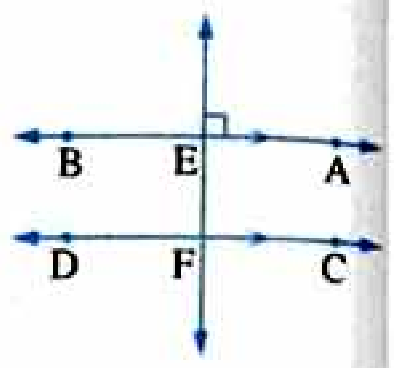
3. Two interior angles in the same side of the transversal are supplementary.

Remember that

The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other.

If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$, $\overleftrightarrow{EF} \perp \overleftrightarrow{AB}$

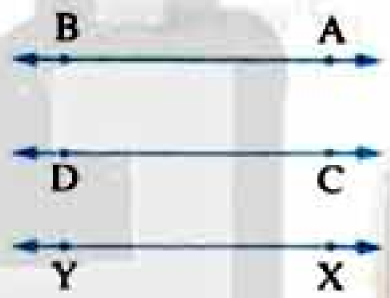
, then $\overleftrightarrow{EF} \perp \overleftrightarrow{CD}$



If two straight lines are parallel to a third straight line , then these two straight lines are parallel.

If $\overleftrightarrow{AB} // \overleftrightarrow{XY}$, $\overleftrightarrow{CD} // \overleftrightarrow{XY}$

, then $\overleftrightarrow{AB} // \overleftrightarrow{CD}$



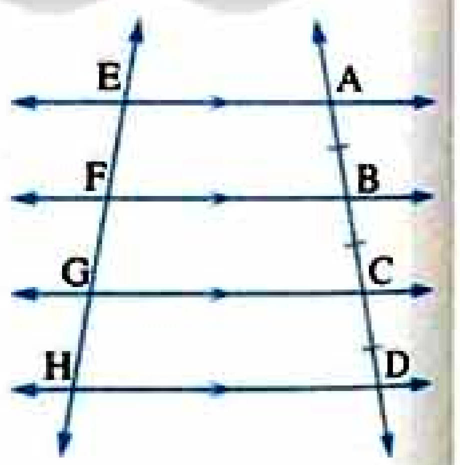
If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.

If $\overleftrightarrow{AE} // \overleftrightarrow{BF} // \overleftrightarrow{CG} // \overleftrightarrow{DH}$

, $\overleftrightarrow{AD}$ and $\overleftrightarrow{EH}$ are transversals to them

and AB = BC = CD

, then EF = FG = GH



Final Examinations

on Geometry

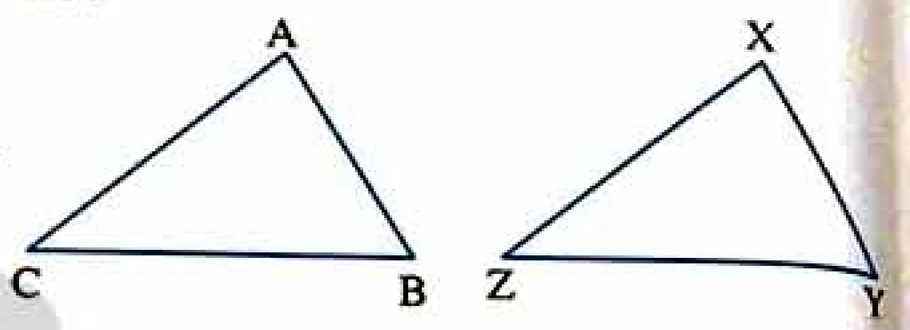
Model Examinations of the School Book on Geometry

Model 1

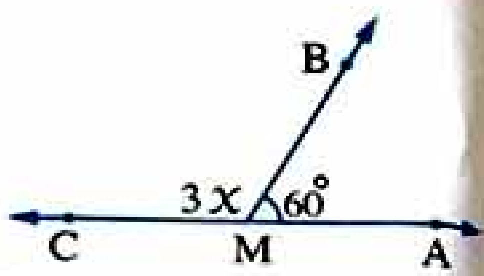
Answer the following questions :

1 Complete each of the following :

1. The perpendicular bisector of a line segment is called ....
2. **In the opposite figure :**
   If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) + m(\angle B) = 140°$
   , then $m(\angle Z) =$ ....°



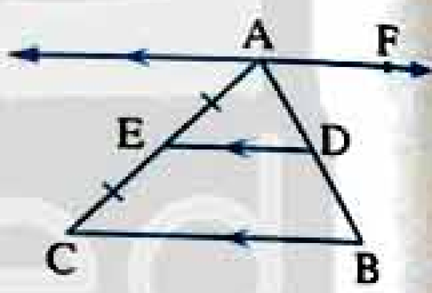
3. If $m(\angle B) = 105°$, then m (reflex $\angle B$) = ...°
4. **In the opposite figure :**
   If $\overrightarrow{MB} \cap \overleftrightarrow{AC} = \{M\}$, $m(\angle AMB) = 60°$
   , then the value of x equals ....°



5. Two right-angled triangles are congruent if ....

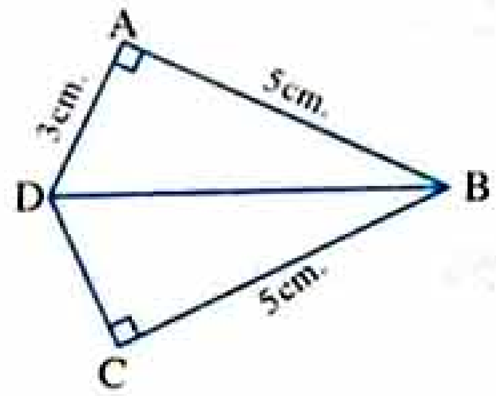
2 Choose the correct answer from those given :

1. If $\angle X \equiv \angle Y$, $\angle X$ and $\angle Y$ are supplementary angles , then $m(\angle X) =$ ....
   (a) 45° (b) 90° (c) 135° (d) 180°
2. **In the opposite figure :**
   If $\overleftrightarrow{AF} \,//\, \overline{DE} \,//\, \overline{BC}$, AE = EC
   , then AD : AB = ...
   (a) 2 : 1 (b) 3 : 2 (c) 1 : 3 (d) 1 : 2



3. The two straight lines that are perpendicular to a third one are ....
   (a) perpendicular. (b) intersecting.
   (c) coincident. (d) parallel.
4. The measure of each of the two equal complementary angles equals ....
   (a) 180° (b) 45° (c) 360° (d) 90°
5. If two straight lines intersect , then each two .... angles have the same measure.
   (a) vertically opposite (b) adjacent
   (c) alternate (d) corresponding
6. If $\Delta ABC \equiv \Delta LMN$, then $m(\angle ACB) = m(\angle$ ....$)$
   (a) LMN (b) MLN (c) LNM (d) NLM

3 **[a] In the opposite figure :**



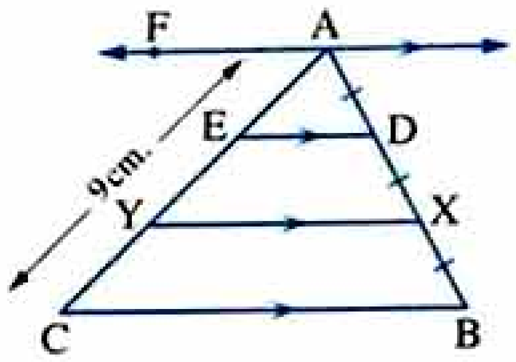
m (∠ BAD) = m (∠ BCD) = 90°

, AB = CB = 5 cm. , AD = 3 cm.

Mention the conditions for Δ ABD , Δ CBD to be congruent

, then find : The length of $\overline{CD}$

[b] In the opposite figure :

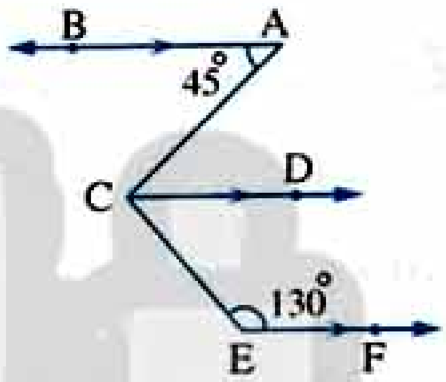


$\overleftrightarrow{AF}$ // $\overline{DE}$ // $\overline{XY}$ // $\overline{BC}$

, AD = DX = XB , AC = 9 cm.

Find : The length of $\overline{AY}$ (Give the reason)

4 **[a] In the opposite figure :**



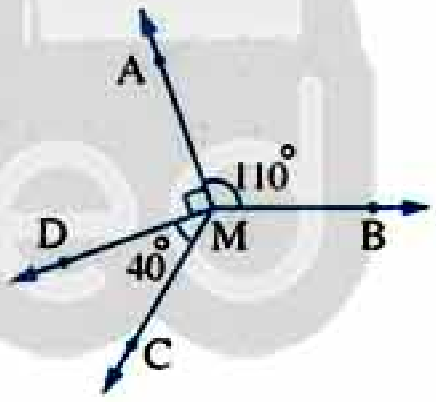
$\overrightarrow{AB}$ // $\overrightarrow{CD}$ // $\overrightarrow{EF}$

, m (∠ A) = 45°

, m (∠ E) = 130°

Find : m (∠ ACE)

[b] In the opposite figure :



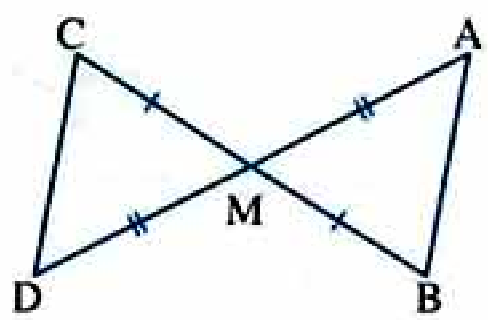
m (∠ AMB) = 110°

, m (∠ AMD) = 90°

, m (∠ DMC) = 40°

Find with steps : m (∠ BMC)

5 **[a] In the opposite figure :**



$\overline{AD}$ ∩ $\overline{BC}$ = {M}

, BM = MC

, AM = MD

Write the conditions for Δ AMB , Δ DMC to be congruent.

[b] Using your geometric instruments , draw ∠ ABC of measure 110° , then draw $\overrightarrow{BF}$ to bisect the angle.

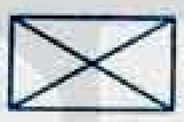
Model 2

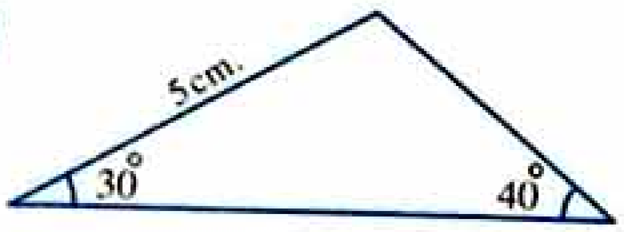
Answer the following questions :

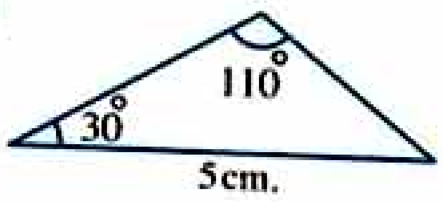
1 Complete each of the following :

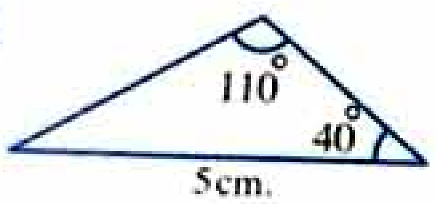
1. The sum of the measures of the accumulative angles at a point equals ....°
2. If a straight line intersects two parallel straight lines , then each two corresponding angles are ....
3. If m (∠ A) = 110° , then m (reflex ∠ A) = ...°
4. Two right-angled triangles are congruent if ....
5. The two adjacent angles formed by the intersection of a straight line and a ray with a starting point on this straight line are ....

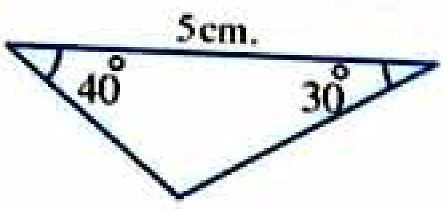
2 Choose the correct answer from those given :

1. If ∠ X complements ∠ Y and ∠ X ≡ ∠ Y , then m (∠ X) = ...
   (a) 45° (b) 90° (c) 180° (d) 360°
2. The number of triangles in the figure  equals ....
   (a) 4 (b) 6 (c) 7 (d) 8
3. If the ratio between the measures of two supplementary angles is 5 : 13 , then the measure of the smaller angle is ....
   (a) 50° (b) 130° (c) 150° (d) 180°
4. If Δ ABC ≡ Δ XYZ , m (∠ A) + m (∠ B) = 100° , then m (∠ Z) = ...
   (a) 50° (b) 80° (c) 90° (d) 100°
5. The two straight lines that are perpendicular to a third one are ....
   (a) perpendicular. (b) parallel. (c) coincident. (d) intersecting.
6. The figure .... is not congruent to the opposite figure.

(a)


(b)


(c)


(d)


3 **[a]** Mention two cases of congruency of two triangles.

[b] In the opposite figure :

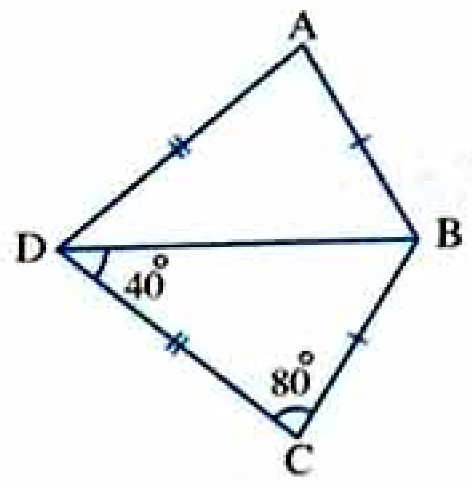
AB = BC , AD = DC

, m (∠ C) = 80°

, m (∠ BDC) = 40°

Prove that : Δ CBD ≡ Δ ABD

, then find : m (∠ ABD)



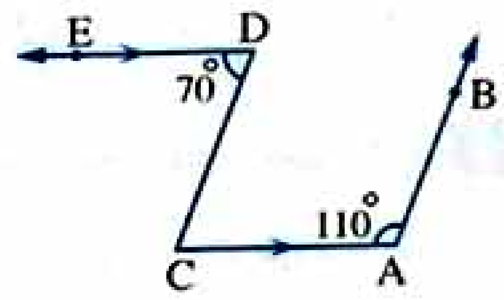
4 **[a] In the opposite figure :**

$\overrightarrow{DE}$ // $\overrightarrow{AC}$, m (∠ A) = 110°

, m (∠ D) = 70°

Find : m (∠ C)

Is $\overrightarrow{AB}$ // $\overrightarrow{CD}$? (Give the reason)



[b] Using the geometric instruments , draw ∠ ABC where m (∠ B) = 80° , then draw $\overrightarrow{BD}$ to bisect it. (Don't remove the arcs).

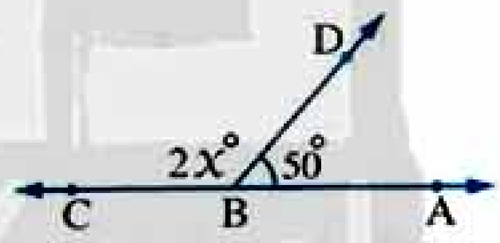
5 **[a] In the opposite figure :**

$\overleftrightarrow{AC} \cap \overrightarrow{BD}$ = {B}

, m (∠ ABD) = 50°

, m (∠ DBC) = 2 x°

Find in degrees the value of x



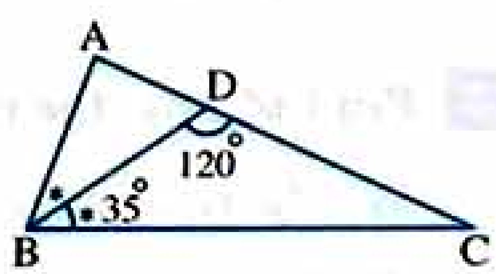
[b] In the opposite figure :

$\overrightarrow{BD}$ bisects ∠ ABC

, m (∠ DBC) = 35°

, m (∠ BDC) = 120°

Find : m (∠ A) in degrees.



Model examination for the merge students

Answer the following questions :

1 Complete each of the following :

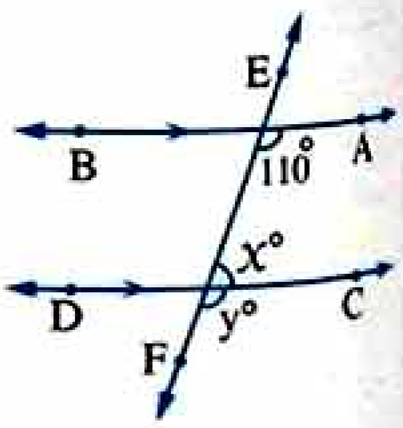
1. If m (∠ A) = 100° , then m (reflex ∠ A) = ...°
2. The angle whose measure is 50° complements an angle of measure ....°
3. The two straight lines parallel to a third are ....
4. Two triangles are congruent if two sides and ....
5. If Δ ABC ≡ Δ XYZ , then m (∠ Z) = m (∠ ....)

2 Choose the correct answer from those given :

1. The sum of the measures of the accumulative angles at a point equals ....
   (a) 630° (b) 180° (c) 90° (d) 360°
2. The axis of symmetry of a line segment is ....
   (a) perpendicular to it from its midpoint. (b) parallel to it.
   (c) equal to it in length. (d) congruent to it.
3. The supplement of the angle whose measure is 30° is an angle of measure ....
   (a) 60° (b) 180° (c) 150° (d) 90°
4. The angle whose measure is more than 90° and less than 180° is .... angle.
   (a) an obtuse (b) an acute (c) a right (d) a straight
5. If Δ ABC ≡ Δ XYZ , then AB = ...
   (a) XY (b) XZ (c) YZ (d) BC

3 Put (✔) for the correct statement and (✘) for the incorrect statement :

1. The right-angled triangle is congruent to the equilateral triangle. ( )
2. The two angles whose measures are 100° and 80° are supplementary. ( )
3. **From the opposite figure :**
   (a) $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$ ( )
   (b) $x = 70°$ ( )
   (c) $y = 180°$ ( )



4 **[a] In the opposite figure :**

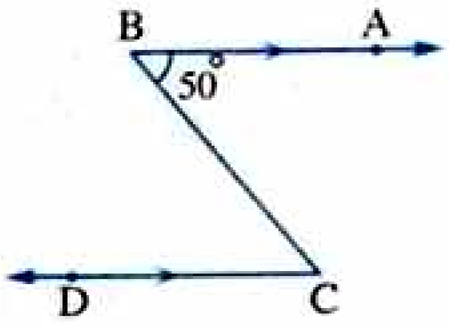
m (∠ ABC) = 50° , $\overrightarrow{BA}$ // $\overrightarrow{CD}$

Complete to find : m (∠ BCD)

$\overrightarrow{BA}$ // ....

, then m (∠ ABC) = m (∠ ....) (............... angles)

, m (∠ BCD) = ...°

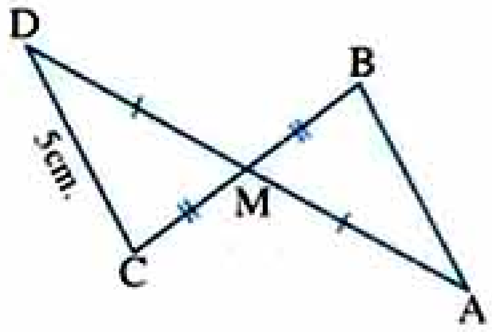


[b] From the opposite figure , complete :

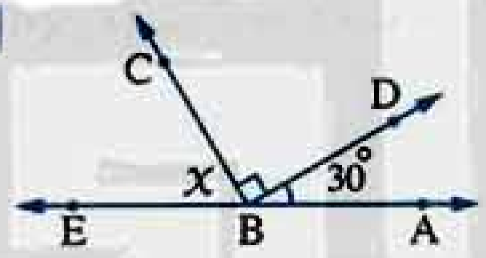
1 Δ ABM ≡ Δ ....

2 AB = ... cm.

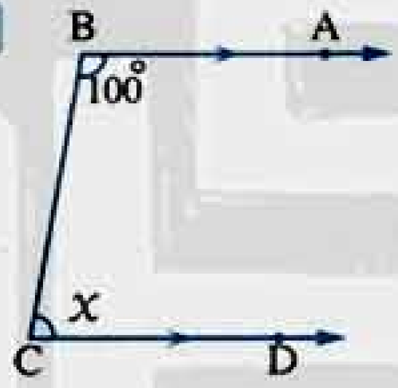
3 m (∠ B) = m (∠ ....)



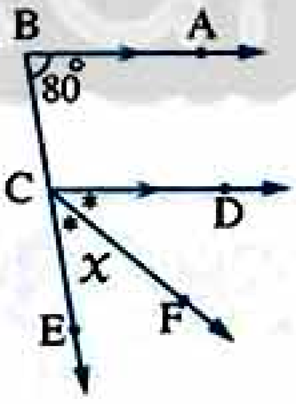
5 **[a] In each of the following figures , find the value of x :**

1


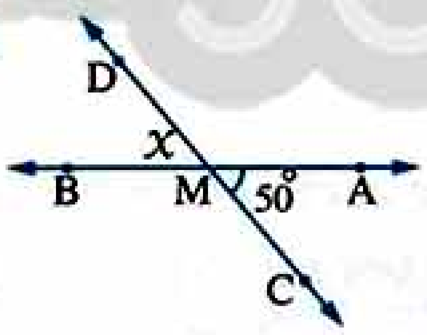
x = ...°

2


x = ...°

3


x = ...°

4


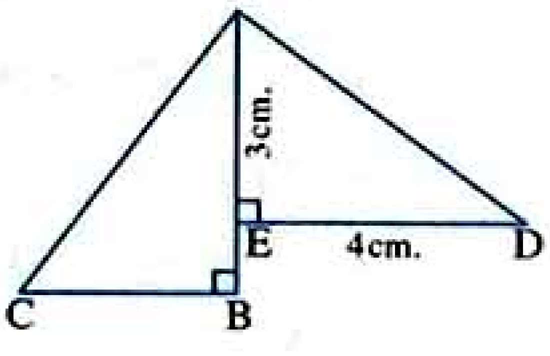
x = ...°

[b] In the opposite figure :

If Δ ABC ≡ Δ DEA ,

AE = 3 cm. and DE = 4 cm.

, complete : BE = ... cm.



Some Schools Examinations on Geometry

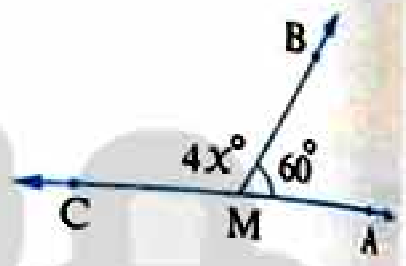
1 Cairo Governorate

Heliopolis Educational Directorate
St.Fatima Language School-Abbassia

Answer the following questions :

1 Complete :

1. The measure of each of two equal complementary angles equals ....°
2. If m (∠ A) = 180° , then m (reflex ∠ A) = ...°
3. The straight line that is perpendicular to one of two parallel lines is also .... to the other.
4. **In the opposite figure :**
   If m (∠ AMB) = 60°
   , then x = ...



5. If a straight line intersects two parallel straight lines , then each two corresponding angles are ....

2 Choose the correct answer :

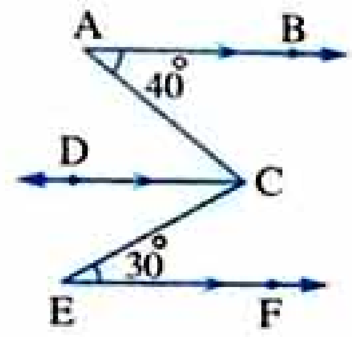
1. If m (∠ X) = 3 m (∠ Y) and ∠ X , ∠ Y are supplementary angles , then m (∠ X) = ...
   (a) 90° (b) 180° (c) 45° (d) 135°
2. If Δ ABC ≡ Δ XYZ and m (∠ X) + m (∠ Y) = 100° , then m (∠ C) = ...
   (a) 50° (b) 100° (c) 90° (d) 80°
3. The supplement of the angle whose measure is 30° is an angle of measure ....
   (a) 60° (b) 180° (c) 150° (d) 20°
4. The ratio between the measures of two complementary angles is 2 : 7 , then the measure of the smaller angle is ....
   (a) 40° (b) 140° (c) 60° (d) 20°
5. If two straight lines intersect , then each two .... angles have the same measure.
   (a) vertically opposite (b) adjacent
   (c) alternate (d) corresponding
6. If Δ ABC ≡ Δ XYZ , then BC = ...
   (a) XY (b) YZ (c) XZ (d) AB

3 **In the opposite figure :**

$\overrightarrow{AB} // \overrightarrow{CD} // \overrightarrow{EF}$

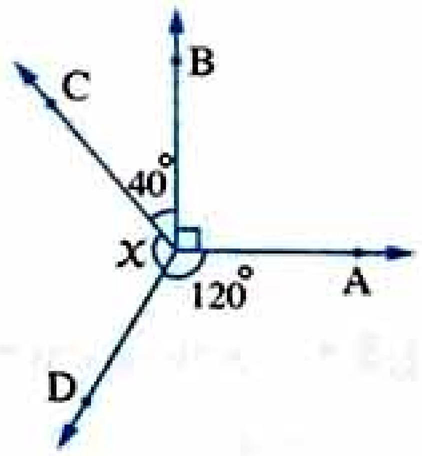
, m (∠ A) = 40° , m (∠ E) = 30°

Find : m (∠ ACE)



4 **[a] In the opposite figure :**

Find the value of x



[b] In the opposite figure :

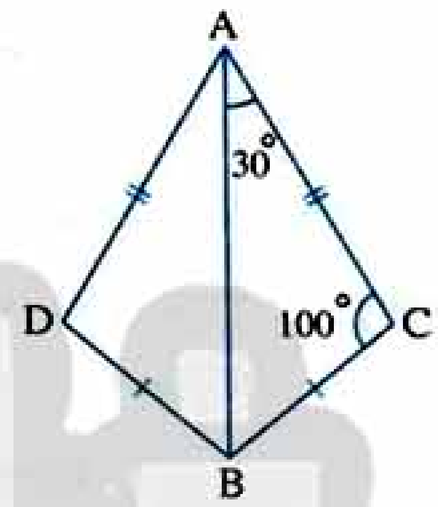
AC = AD , BC = BD

, m (∠ ACB) = 100°

, m (∠ CAB) = 30°

1 **Prove that :** Δ ABC ≡ Δ ABD

2 **Find :** m (∠ ABD)



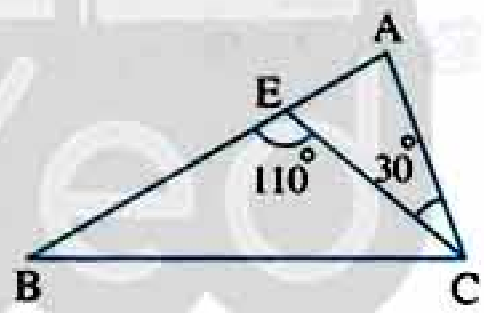
5 **[a]** Mention two cases of congruency of two triangles.

[b] In the opposite figure :

m (∠ ACE) = 30°

, m (∠ CEB) = 110°

Find : m (∠ A)



2 Cairo Governorate

Zietoun Educational Administration
Gomhouria Language School

Answer the following questions :

1 **Choose the correct answer :**

1 If two straight lines intersect , then each two vertically opposite angles are ....

(a) equal in measure. (b) adjacent. (c) supplementary. (d) complementary.

2 If ∠ X ≡ ∠ Y and ∠ X , ∠ Y are complementary angles , then m (∠ X) = ...

(a) 45° (b) 90° (c) 135° (d) 180°

3 The best unit to measure the area of a room is ....

(a) mm^2 (b) cm^2 (c) m^2 (d) km^2

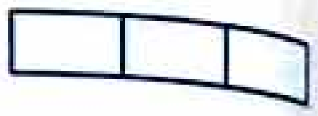
4 If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) = 45°$, $m(\angle C) = 75°$, then $m(\angle Y) =$ ....

(a) 60° (b) 65° (c) 55° (d) 100°

5 If L_1, L_2 and L_3 are straight lines, $L_1 \perp L_3$, $L_2 \perp L_3$, then L_1 .... L_2

(a) // (b) $\perp$ (c) coincides (d) intersects

6 The number of rectangles of the opposite figure is ....



(a) 3 (b) 4 (c) 6 (d) 5

2 Complete each of the following :

1 If $m(\angle A) = 100°$, then $m(\text{reflex } \angle A) =$ ....°

2 Two triangles are congruent if each .... of one triangle is equal to the corresponding part of the other triangle.

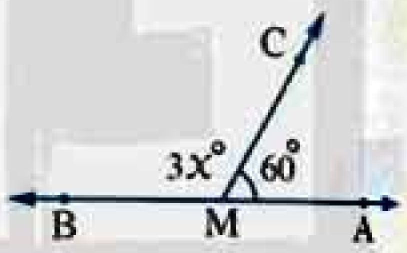
3 The perpendicular to a line segment from its midpoint is called ....

4 If the area of a rectangle is $20\ cm^2$, its width is 4 cm., then the perimeter of the rectangle is .... cm.

5 **In the opposite figure :**

If $\overleftrightarrow{AB} \cap \overrightarrow{MC} = \{M\}$

, then $X =$ ....°



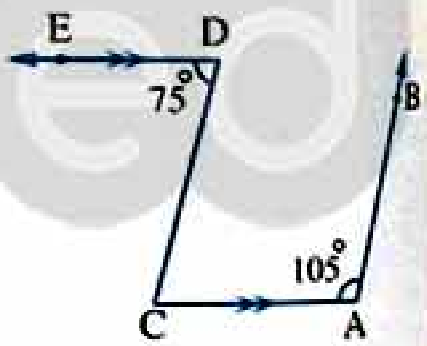
3 [a] In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{AC}$, $m(\angle A) = 105°$

, $m(\angle D) = 75°$

Find : $m(\angle C)$

Is $\overrightarrow{AB} \parallel \overrightarrow{CD}$? Giving the reason.



[b] By using your geometric instruments, draw $\overline{AB}$ of length 6 cm., then draw the straight line L that is the axis of symmetry of $\overline{AB}$ where $\overline{AB} \cap L = \{C\}$

4 [a] In the opposite figure :

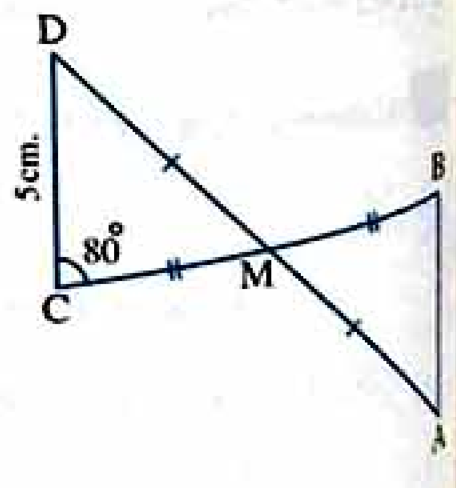
$m(\angle C) = 80°$, $\overline{CB} \cap \overline{AD} = \{M\}$

, MB = MC , MD = MA , CD = 5 cm.

Mention the conditions for

ΔABM, ΔDCM to be congruent

, and find : $m(\angle B)$

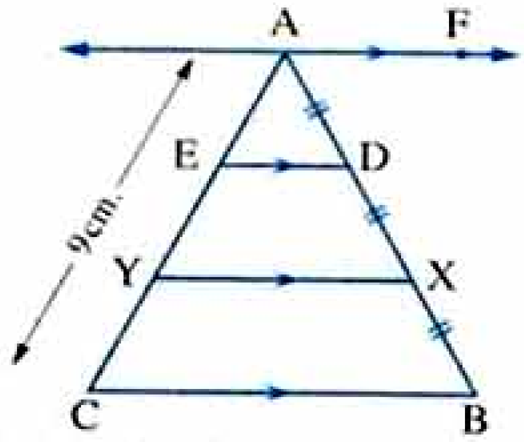


[b] In the opposite figure :

$\overleftrightarrow{AF} // \overline{DE} // \overline{XY} // \overline{BC}$

, AD = DX = XB , AC = 9 cm.

Find : The length of $\overline{AY}$



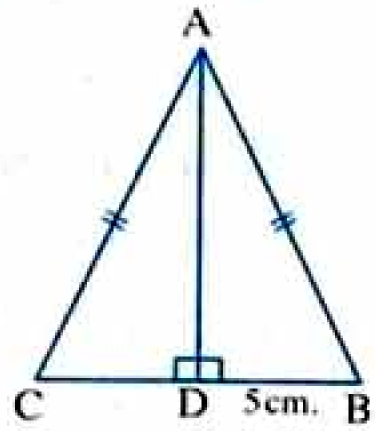
5 **[a] In the opposite figure :**

AB = AC , m (∠ ADB) = m (∠ ADC) = 90° , BD = 5 cm.

Mention the conditions for

Δ ABD , Δ ACD to be congruent

, and find : The length of $\overline{BC}$

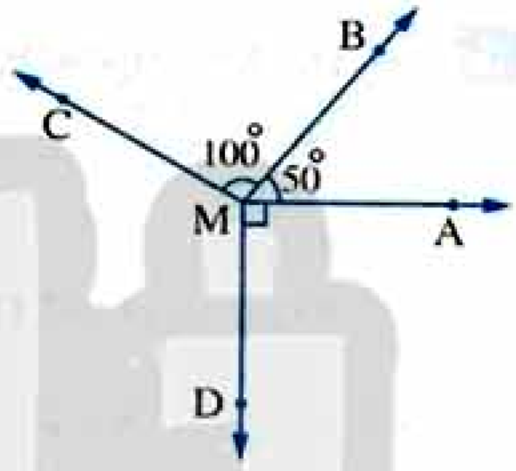


[b] In the opposite figure :

m (∠ AMB) = 50° , m (∠ CMB) = 100°

, m (∠ DMA) = 90°

Find : m (∠ CMD)



3 Cairo Governorate

Dar El-Salam and El-Basatoon Education Zone

Answer the following questions :

1 **Complete :**

1 The two adjacent angles formed by the intersection of a straight line and a ray with a starting point on this straight line are ....

2 The sum of measures of the accumulative angles at a point equals ....°

3 If m (∠ X) = 140° , then the measure of the reflex angle of ∠ X = ...°

4 If two straight lines are perpendicular to a third , then the two straight lines are ....

5 If Δ ABC ≡ Δ XYZ , m (∠ A) + m (∠ B) = 100° , then m (∠ Z) = ...°

2 **Choose the correct answer :**

1 If ∠ X ≡ ∠ Y and ∠ X , ∠ Y are complementary angles , then m (∠ X) = ...°

(a) 45 (b) 90 (c) 180 (d) 30

2 If the ratio between the measures of two supplementary angles is 5 : 13 , then the measure of the smaller angle is ....°

(a) 130 (b) 50 (c) 180 (d) 150

3 If a straight line cuts two parallel lines , then each two corresponding angles are ....

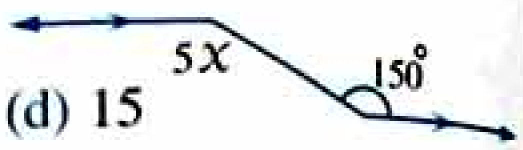
(a) equal in measure. (b) complementary.

(c) supplementary. (d) right.

4 If $\Delta XYZ \equiv \Delta ABC$, then ....

(a) BC = XZ (b) YX = CA (c) ZY = CB (d) AB = YZ

5 **In the opposite figure :** $x =$ ....°



(a) 50 (b) 30 (c) 90 (d) 15

6 If $\overline{XY} \equiv \overline{AB}$, XY = 5 cm. , then XY + 3 AB = ... cm.

(a) 5 (b) 20 (c) 15 (d) 30

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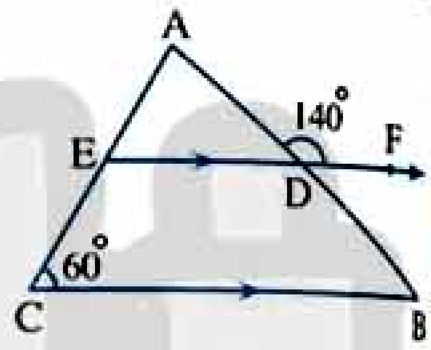
3 [a] **In the opposite figure :**



$\overline{BC} \parallel \overrightarrow{EF}$, m ($\angle$ C) = 60°

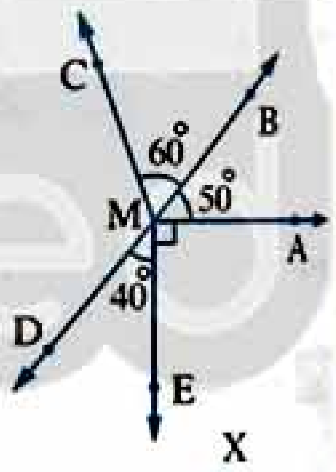
, m ($\angle$ ADF) = 140°

Find each of the following : m ($\angle$ B) and m ($\angle$ A)

[b] Draw $\angle$ XYZ of measure 120° , then bisect it. (Don't remove the arcs)

---

4 [a] **In the opposite figure :**



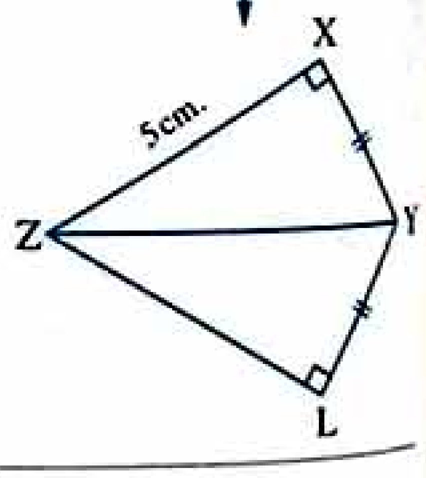
If m ($\angle$ AMB) = 50° , m ($\angle$ BMC) = 60°

, m ($\angle$ DME) = 40°

and $\overrightarrow{MA} \perp \overrightarrow{ME}$

, **find :** m ($\angle$ CMD)

[b] **In the opposite figure :**



m ($\angle$ X) = m ($\angle$ L) = 90° , YX = YL

and ZX = 5 cm.

Prove that : $\Delta XYZ \equiv \Delta LYZ$, **then find :** the length of $\overline{ZL}$

[c] Mention two cases of congruency of two triangles.

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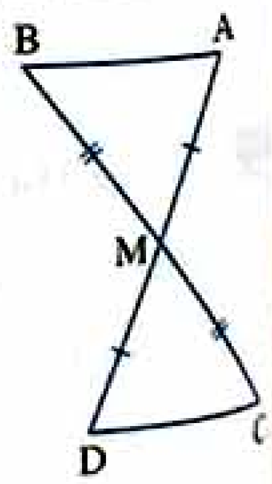
5 [a] **In the opposite figure :**



$\overline{AD} \cap \overline{BC} = \{M\}$

, BM = MC , AM = MD

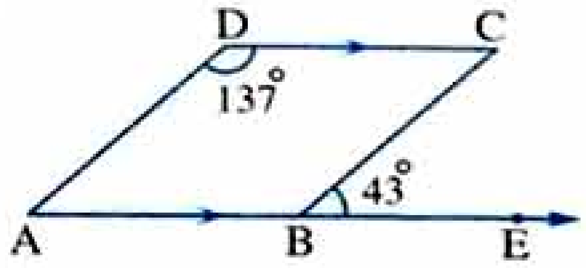
Prove that : $\Delta AMB \equiv \Delta DMC$

[b] In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{DC}$, m (∠ EBC) = 43°

, m (∠ D) = 137°

Is $\overline{BC} \parallel \overline{AD}$? Giving reason.



4 Giza Governorate

El-Dokki Directorate
Math Inspection

Answer the following questions :

1 Choose the correct answer :

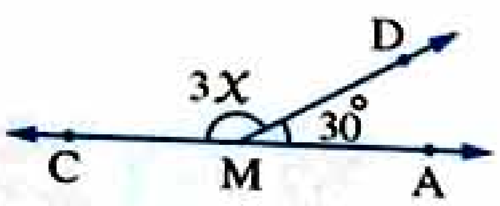
1. The angle whose measure is more than 180° and less than 360° is called ....
   (a) obtuse. (b) straight. (c) reflex. (d) acute.
2. The supplementary angle of the angle of measure 53° is an angle of measure ....
   (a) 53° (b) 37° (c) 127° (d) 180°
3. If Δ MLN ≡ Δ XYZ , then m (∠ N) = m (∠ ....)
   (a) M (b) X (c) Z (d) Y
4. The sum of measures of the accumulative angles at a point equals ....
   (a) 180° (b) 360° (c) 90° (d) 270°
5. The two angles of measures : 40° , 50° are ....
   (a) complementary. (b) supplementary. (c) reflex. (d) obtuse.
6. In Δ XYZ , if m (∠ X) + m (∠ Z) = 95° , then m (∠ Y) = ...
   (a) 180° (b) 95° (c) 90° (d) 85°

2 Complete the following :

1. **In the opposite figure :**

   $\overleftrightarrow{AC} \cap \overrightarrow{MD} = \{M\}$, m (∠ AMD) = 30°

   , m (∠ CMD) = 3 x , then the value of x equals ....°



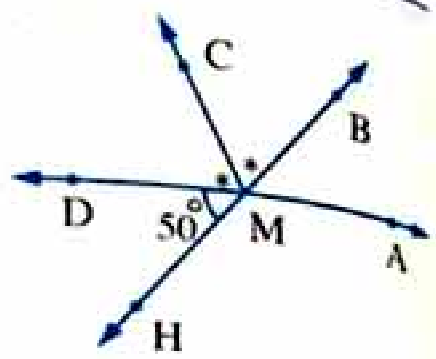
2. Two triangles are congruent if two sides and .... of one triangle are congruent to the corresponding parts of the other triangle.
3. If a straight line intersects two parallel straight lines , then each two corresponding angles are ....
4. If two adjacent angles are supplementary , then their two outer sides are ....
5. If m (∠ B) = 80° , then m (reflex ∠ B) = ...°

3 **[a] In the opposite figure :**

$\overleftrightarrow{AD} \cap \overleftrightarrow{BH} = \{M\}$, m ($\angle$ HMD) = 50°

, $\overrightarrow{MC}$ bisects $\angle$ BMD

Find : m ($\angle$ AMC)

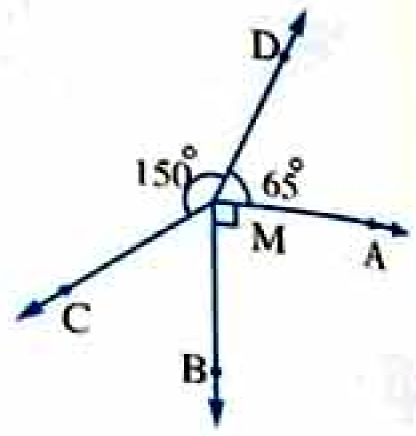


[b] In the opposite figure :

$\overrightarrow{MA} \perp \overrightarrow{MB}$, m ($\angle$ AMD) = 65°

, m ($\angle$ DMC) = 150°

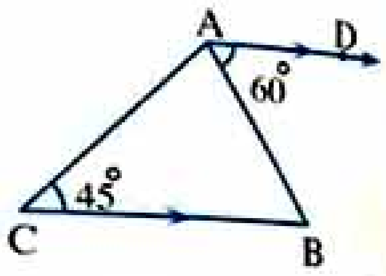
Find : m ($\angle$ BMC)



4 **[a] In the opposite figure :**

$\overrightarrow{AD}$ // $\overline{CB}$, m ($\angle$ BAD) = 60° , m ($\angle$ C) = 45°

Find : m ($\angle$ BAC) , m ($\angle$ B)

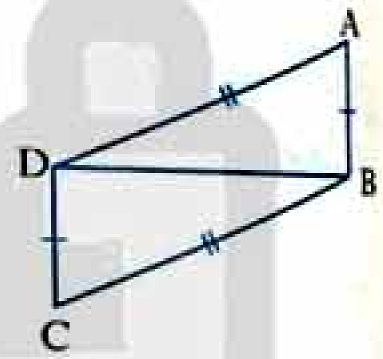


[b] In the opposite figure :

AD = BC , AB = CD

1 Is Δ ABD $\equiv$ Δ CDB ? Why ?

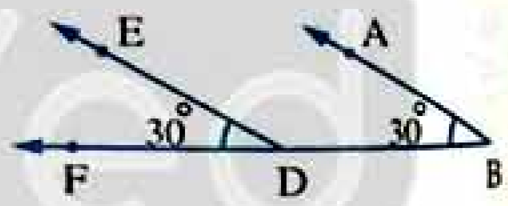
2 **Complete :** m ($\angle$ A) = m ($\angle$ ....)



5 **[a] In the opposite figure :**

m ($\angle$ B) = 30° , m ($\angle$ EDF) = 30°

Is $\overrightarrow{DE}$ // $\overrightarrow{BA}$? Why ?



[b] Using the geometric instruments , draw $\angle$ ABC of measure 115° , then draw $\overrightarrow{BD}$ to bisect it.

(Don't remove the arcs)

5 Giza Governorate

Education Administration of 6 October

Answer the following questions :

1 **Choose the correct answer :**

1 When a transversal cuts two parallel lines , then every two .... angles are equal in measure.

(a) alternate (b) supplementary (c) complementary (d) adjacent

2 The perpendicular bisector of a line segment is called ....

(a) symmetry axis. (b) parallel line. (c) intersecting line. (d) median.

3 If m (∠ A) = 90° , then m (reflex ∠ A) = ...

(a) 90° (b) 270° (c) 180° (d) 0°

4 The measure of the straight angle equals ....

(a) 0° (b) 90° (c) 180° (d) 270°

5 The angle whose measure is 179° , is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

6 If ∠ X ≡ ∠ Y and ∠ X , ∠ Y are supplementary angles , then m (∠ X) = ...

(a) 45° (b) 90° (c) 135° (d) 180°

2 Complete :

1 The angle whose measure is 36° complements an angle of measure ....° and supplements an angle of measure ....°

2 The two right-angled triangles are congruent if ....

3 If Δ ABC ≡ Δ XYZ , then m (∠ A) = m (∠ ....) , and XY = ...

4 The sum of measures of the accumulative angles at a point equals ....°

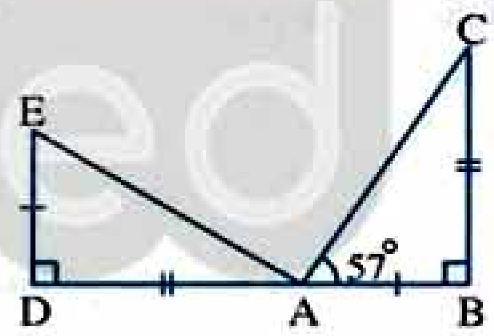
5 The angle whose measure is greater than 180° and less than 360° is called ....

3 [a] In the opposite figure :

AB = DE

, BC = AD , m (∠ CAB) = 57°

Find the measures of the unknown angles in the triangle ADE



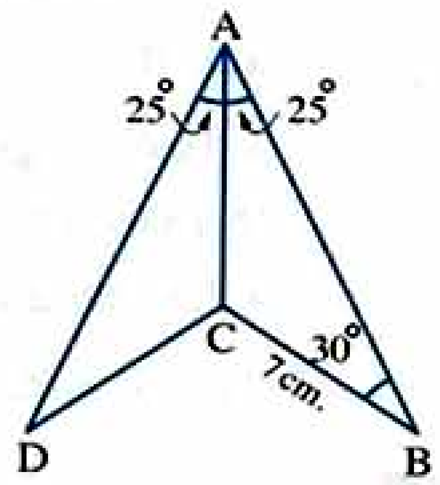
[b] In the opposite figure :

If Δ ACB ≡ Δ ACD , complete :

1 m (∠ D) = ...°

2 CD = ... cm.

3 m (∠ ACD) = ...°



4 [a] Draw the angle ABC where m (∠ ABC) = 70° , then using the ruler and the compasses , draw $\overrightarrow{BD}$ to bisect the angle. (Don't remove the arcs)

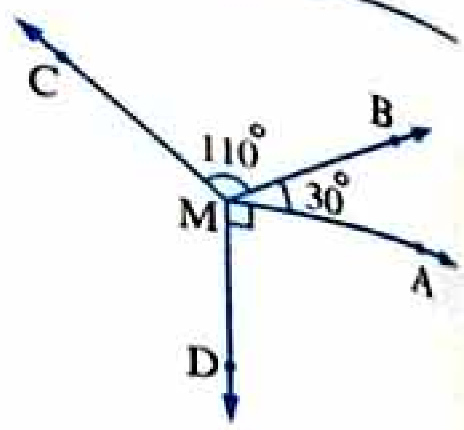
[b] In the opposite figure :

m (∠ AMB) = 30°

, m (∠ BMC) = 110°

and m (∠ AMD) = 90°

Find : m (∠ CMD)

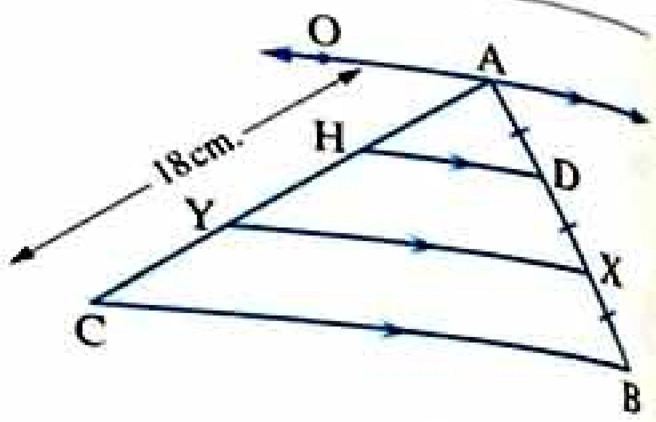


5 **[a] In the opposite figure :**

$\overleftrightarrow{AO}$ // $\overline{HD}$ // $\overline{YX}$ // $\overline{CB}$

, AD = DX = XB and AC = 18 cm.

Find : The length of $\overline{AY}$

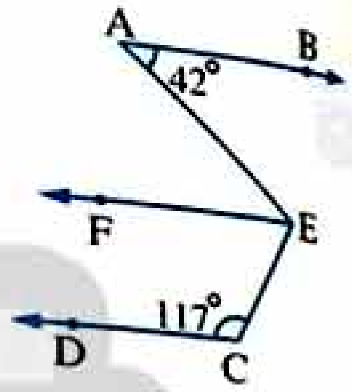


[b] In the opposite figure :

$\overrightarrow{AB}$ // $\overrightarrow{CD}$, $\overrightarrow{EF}$ // $\overrightarrow{CD}$

, m (∠ A) = 42° and m (∠ C) = 117°

Determine : m (∠ AEC)



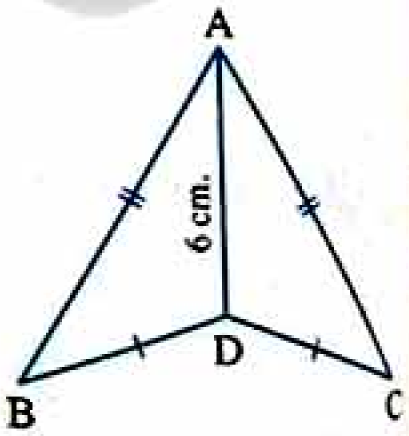
6 Alexandria Governorate

El-Montaza Educational Zone
Maths Supervision

Answer the following questions :

1 **Complete :**

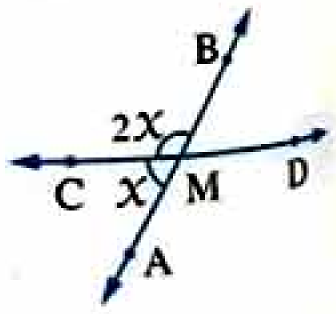
1. The angle of measure .... complements an angle of measure 25°
2. The sum of measures of the accumulative angles at a point is equal to ....°
3. **In the opposite figure :**
   If the perimeter of the shape ABDC = 20 cm.
   and the length of $\overline{AD}$ = 6 cm.
   , then the perimeter of Δ ABD = ... cm.



4. If a straight line intersects two parallel straight lines
   , then every two corresponding angles are .... in measure.
5. An angle has a measure of 120° , then the measure of its reflex angle is ....°

2 **Choose the correct answer :**

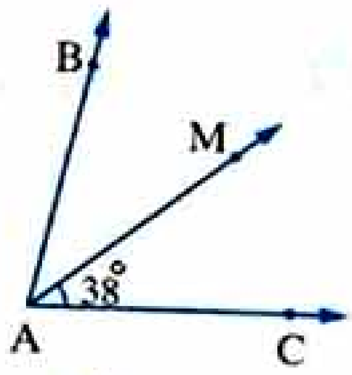
1. **In the opposite figure :**
   If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then x = ...
   (a) 30° (b) 45° (c) 60° (d) 90°



2 If m (∠ B) = 38° , then its supplementary angle is of measure ....

(a) 52° (b) 142° (c) 228° (d) 322°

3 **In the opposite figure :**

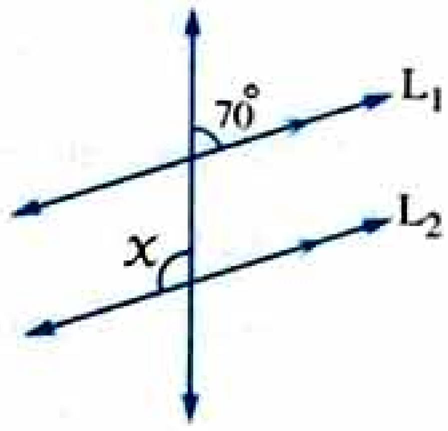


$\overrightarrow{AM}$ bisects ∠ BAC , then m (∠ BAC) = ...

(a) 38° (b) 76°

(c) 142° (d) can't be calculated.

4 **In the opposite figure :**

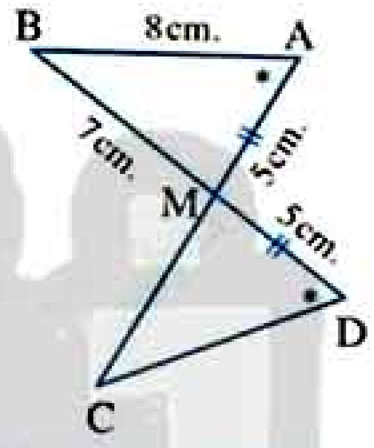


x = ...

(a) 70° (b) 90°

(c) 110° (d) 290°

5 **In the opposite figure :**



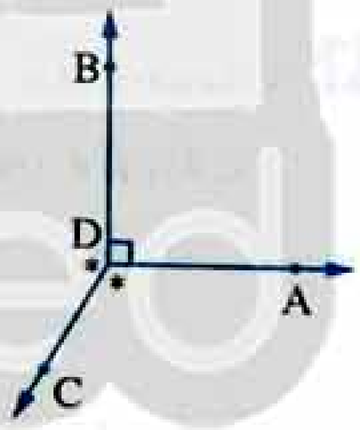
$\overline{AC} \cap \overline{BD} = \{M\}$, AM = MD = 5 cm.

and m (∠ A) = m (∠ D) , then CD = ... cm.

(a) 5 (b) 7

(c) 8 (d) 12

3 **[a] In the opposite figure :**

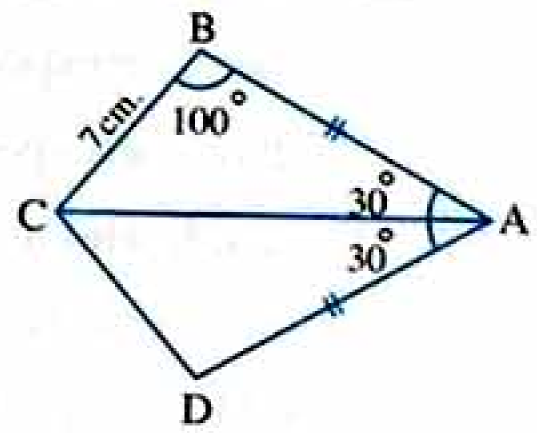


m (∠ ADB) = 90°

, $\overrightarrow{DC}$ bisects the reflex angle BDA

Calculate : m (∠ BDC)

[b] In the opposite figure :



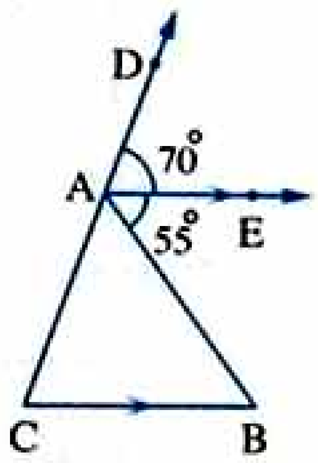
AB = AD , BC = 7 cm. , m (∠ B) = 100°

and m (∠ BAC) = m (∠ DAC) = 30°

1 Is Δ BAC ≡ Δ DAC ? Why ?

2 **Find :** m (∠ ACD) and the length of $\overline{CD}$

4 **[a] In the opposite figure :**



ABC is a triangle where the point A ∈ $\overleftrightarrow{CD}$

, $\overrightarrow{AE}$ // $\overrightarrow{CB}$, m (∠ DAE) = 70°

and m (∠ EAB) = 55°

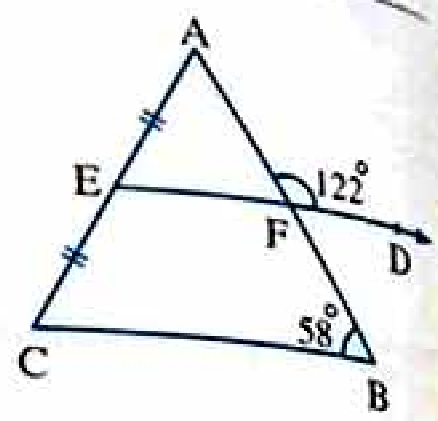
Calculate the measure of each angle in the triangle ABC

[b] Draw a line segment $\overline{AB}$ of length 8 cm. , then draw its line of symmetry. (perpendicular bisector of it) (Don't remove the arcs)

5 **[a] In the opposite figure :**

ABC is a triangle , E is the midpoint of $\overline{AC}$, $\overrightarrow{EF}$ intersects $\overline{AB}$ at F , m (∠ AFD) = 122° and m (∠ B) = 58°

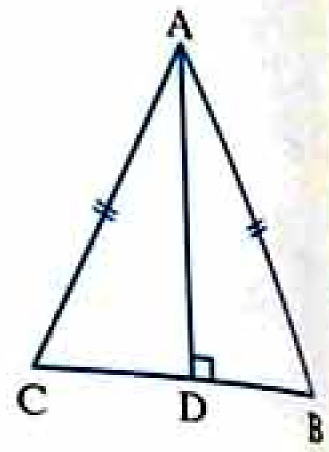
Is $\overleftrightarrow{EF}$ // $\overleftrightarrow{CB}$? Why ?



[b] In the opposite figure :

ABC is an isosceles triangle and $\overline{AD} \perp \overline{BC}$

Why does m (∠ C) = m (∠ B) ?



7 Alexandria Governorate

East Educational Zone
Inspectorate of Mathematics

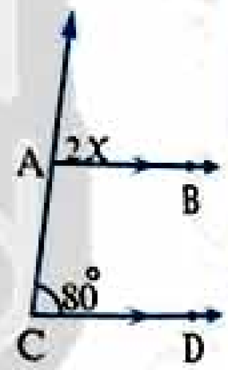
Answer the following questions :

1 **Choose the correct answer :**

1 **In the opposite figure :**

m (∠ C) = 80° , $\overrightarrow{AB}$ // $\overrightarrow{CD}$, then x = ...

(a) 80° (b) 50° (c) 40° (d) 100°



2 Two triangles are congruent if .... are congruent.

(a) two corresponding sides
(b) two corresponding sides and the included angle
(c) a side and an angle with their corresponding
(d) their corresponding angles

3 If Δ ABC ≡ Δ XYZ , then BC = ...

(a) XY (b) AB (c) XZ (d) YZ

4 The acute angle supplements .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

5 If two straight lines intersects , then each two .... angles have the same measure.

(a) vertically opposite (b) adjacent
(c) alternate (d) corresponding

6 The image of the point $(-3, 5)$ by translation $(0, -10)$ is ....

(a) $(3, -5)$ (b) $(-3, -5)$ (c) $(3, 5)$ (d) $(5, -3)$

2 Complete each of the following :

1 If a straight line intersects two parallel straight lines, then each two alternate angles are ....

2 If the ratio between the measures of two supplementary angles is 1 : 2, then the measure of the smaller angle equals ....°

3 If $\angle A \equiv \angle B$, then $m(\angle A) - m(\angle B) =$ ....

4 The perpendicular bisector of a line segment is called ....

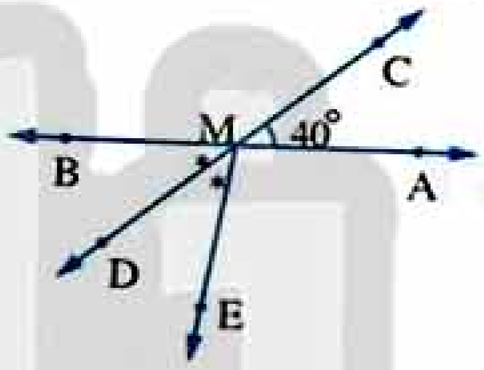
5 The square has .... axes of symmetry.

3 [a] In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, $m(\angle AMC) = 40°$

and $\overrightarrow{MD}$ bisects $\angle BME$

Find : $m(\angle AME)$



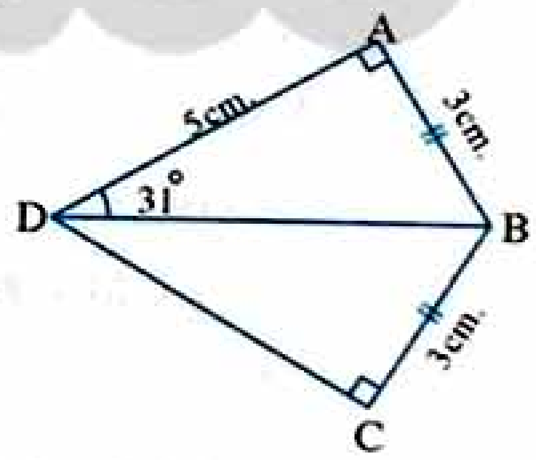
[b] Using the ruler and the compasses, draw ΔABC in which $AB = AC = 6$ cm., $BC = 5$ cm. Bisect $\angle B$, $\angle C$ by two bisectors which intersect at M

(Don't remove the arcs)

4 [a] In the opposite figure :

$m(\angle BAD) = m(\angle BCD) = 90°$

, $m(\angle ADB) = 31°$, $AB = CB = 3$ cm., $AD = 5$ cm.



1 Is $\Delta ABD \equiv \Delta CBD$? Why ?

2 **Find :** The length of $\overline{CD}$

3 **Find :** $m(\angle ADC)$

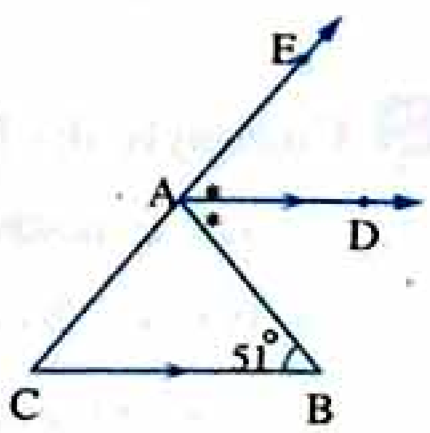
[b] In the opposite figure :

$\overrightarrow{AD} \parallel \overline{BC}$

, $\overrightarrow{AD}$ bisects $\angle EAB$

, $m(\angle ABC) = 51°$

Find : $m(\angle BAD)$ and $m(\angle C)$

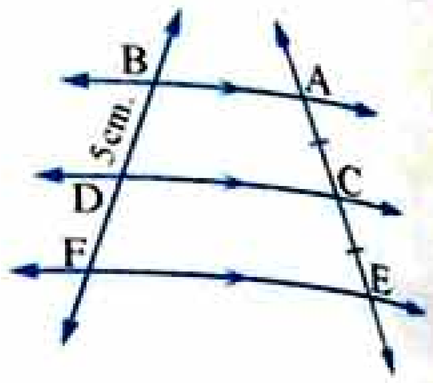


5 **[a] In the opposite figure :**

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, AC = CE , DB = 5 cm.

Find : The length of $\overline{BF}$

, by giving the reason.



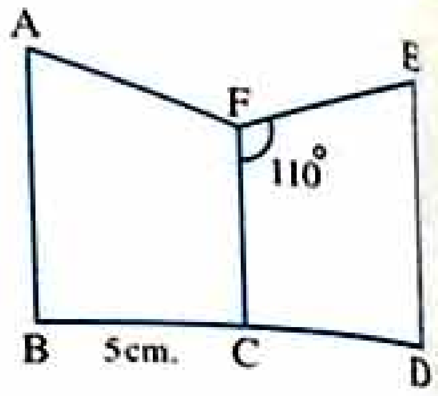
[b] In the opposite figure :

The polygon ABCF ≡ the polygon EDCF

, m (∠ EFC) = 110° , BC = 5 cm.

Find : **1** m (∠ AFC) , m (∠ AFE) , m (∠ FCB)

2 The length of $\overline{BD}$



8 El-Kalyoubia Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If two straight lines intersect , then each two .... angles have the same measure.

(a) corresponding (b) alternate (c) adjacent (d) vertically opposite

2 If two straight lines are perpendicular to a third , then the two straight lines are ....

(a) intersecting. (b) perpendicular. (c) parallel. (d) coincident.

3 The rectangle has .... lines of symmetry.

(a) zero (b) 2 (c) 3 (d) 4

4 If Δ ABC ≡ Δ LMN , then m (∠ BCA) = m (∠ ....)

(a) MNL (b) MLN (c) NML (d) NLM

5 If $\overline{AB} \equiv \overline{CD}$, then AB – CD = ...

(a) 1 (b) 2 (c) zero (d) 5

6 Any two line segments are congruent if they are equal in ....

(a) measure. (b) capacity. (c) weight. (d) length.

2 **Complete the following :**

1 The two straight lines parallel to a third are ....

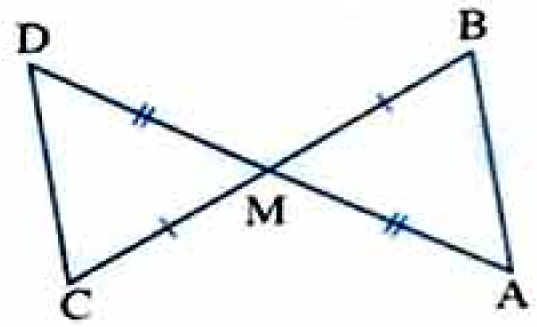
2 If m (∠ B) = 110° , then m (reflex ∠ B) = ...°

3 The two adjacent angles formed by intersecting of a straight line and a ray are ....

4 The two right-angled triangles are congruent if ……………

5 A square of side length 7 cm. , then its area = …………… cm^2.

3 [a] **In the opposite figure :**

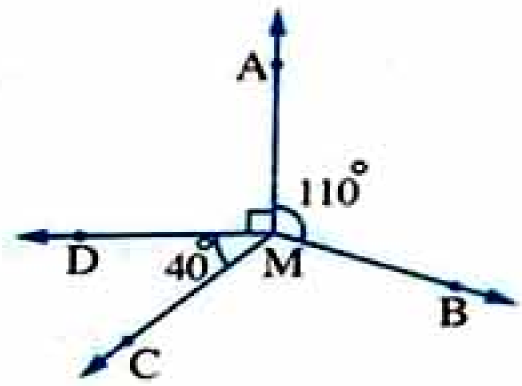


$\overline{AD} \cap \overline{BC} = \{M\}$, MB = MC , MA = MD

Write the conditions for Δ AMB and Δ DMC to be congruent :

1 …………… 2 …………… 3 ……………

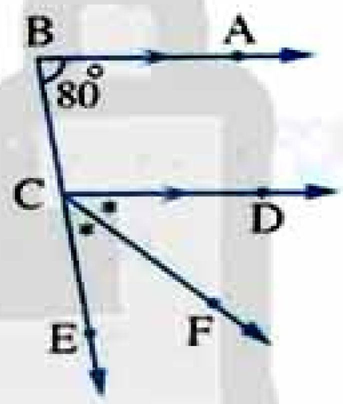
[b] **Using the opposite figure , complete :**



1 m (∠ AMB) + m (∠ BMC) + m (∠ CMD) + m (∠ DMA) = ……………°

2 m (∠ BMC) = …………… − …………… = ……………°

4 [a] **In the opposite figure :**



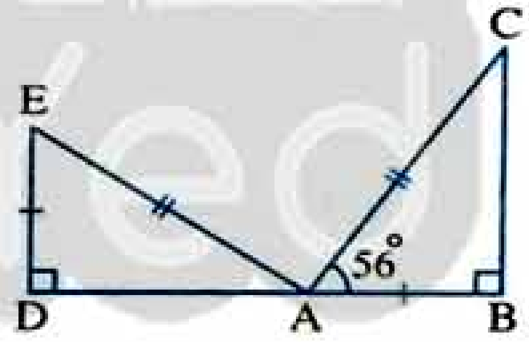
$\overrightarrow{BA}$ // $\overrightarrow{CD}$, m (∠ B) = 80° , $\overrightarrow{CF}$ bisects ∠ DCE

Complete :

1 m (∠ DCE) = m (∠ ……………) = ……………°

2 m (∠ ECF) = ……………°

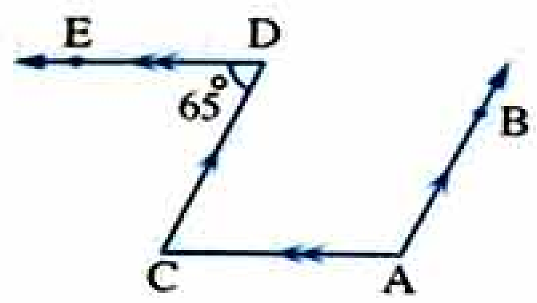
[b] **Using the opposite figure , complete :**



1 Δ ABC ≡ Δ EDA because …………… , …………… , ……………

2 m (∠ EAD) = ……………°

5 [a] **In the opposite figure :**



$\overrightarrow{DE}$ // $\overline{AC}$, $\overrightarrow{AB}$ // $\overline{CD}$, m (∠ D) = 65°

Complete :

1 m (∠ C) = m (∠ ……………) = ……………°

2 m (∠ A) = ……………° because ……………

[b] By using your geometric instruments , draw $\overline{AB}$, where AB = 8 cm. and draw the axis of symmetry of $\overline{AB}$ (Don't remove the arcs).

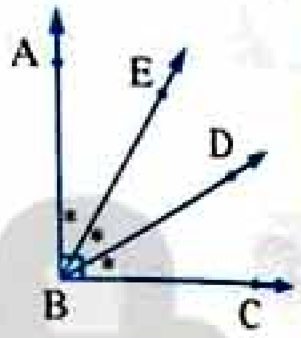
9 El-Sharkia Governorate

Hehia Exp. Lang. School
Department : Math's

Answer the following questions :

1 Complete each of the following :

1. The angle whose measure is 30° complements an angle of measure ....°
2. If Δ ABC $\equiv \Delta$ XYZ , m ($\angle$ A) + m ($\angle$ B) = 110° , then m ($\angle$ Z) = ...°
3. If m ($\angle$ A) = 140° , then m (reflex $\angle$ A) = ...°
4. If a straight line cuts two parallel straight lines , then each two corresponding angles are ....
5. **In the opposite figure :**

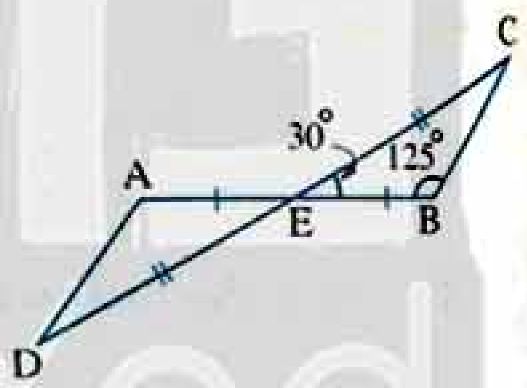
   

   If $\overrightarrow{BA} \perp \overrightarrow{BC}$

   , then m ($\angle$ CBD) = ...°

2 Choose the correct answer :

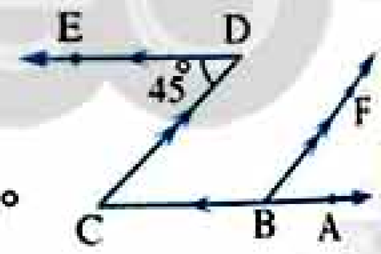
1. **In the opposite figure :**

   m ($\angle$ D) = ...

   (a) 25° (b) 30° (c) 60° (d) 125°

2. **In the opposite figure :**

   m ($\angle$ ABF) = ...

   (a) 45° (b) 90° (c) 135° (d) 40°

3. The angle of measure 98° its type is ....

   (a) acute. (b) right. (c) obtuse. (d) straight.

4. The sum of measures of the accumulative angles at a point equals ....

   (a) 90° (b) 180° (c) 630° (d) 360°

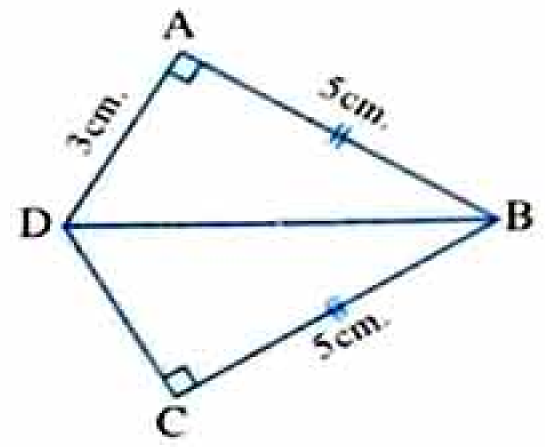
5. If m ($\angle$ A) = 2 m ($\angle$ B) , $\angle$ A supplements $\angle$ B , then m ($\angle$ B) = ...

   (a) 30° (b) 60° (c) 90° (d) 120°

6. The obtuse angle supplements .... angle.

   (a) an acute (b) an obtuse (c) a zero (d) a right

3 **[a] In the opposite figure :**



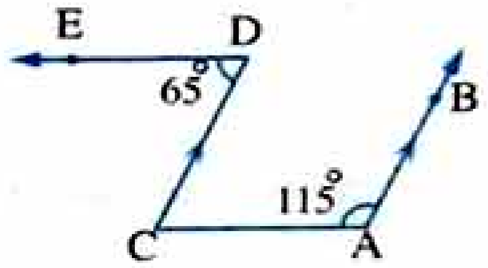
m (∠ BAD) = m (∠ BCD) = 90°

, AB = CB = 5 cm. , AD = 3 cm.

Mention the conditions for Δ ABD , Δ CBD to be congruent

, then find : The length of $\overline{CD}$

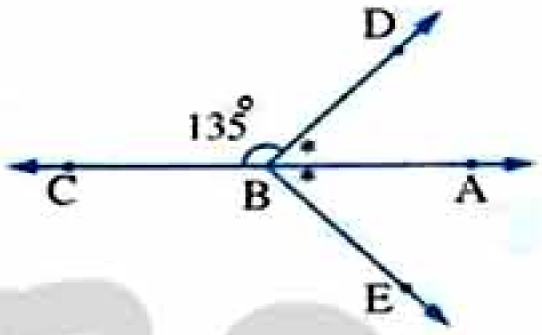
[b] In the opposite figure :



If $\overrightarrow{AB}$ // $\overrightarrow{CD}$, m (∠ D) = 65° , m (∠ A) = 115°

, then prove that : $\overrightarrow{AC}$ // $\overrightarrow{DE}$

4 **[a] In the opposite figure :**



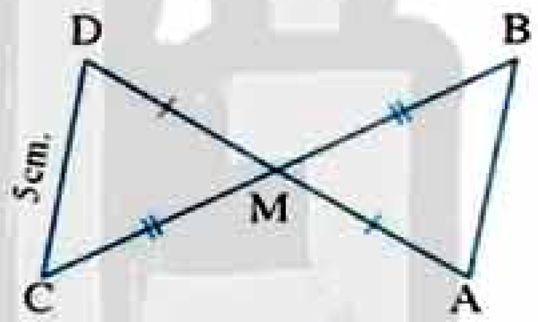
If B ∈ $\overleftrightarrow{AC}$, m (∠ DBC) = 135°

and $\overrightarrow{BA}$ bisects ∠ DBE

, find :

1 m (∠ ABD) 2 m (∠ DBE) 3 m (∠ CBE)

[b] From the opposite figure , complete :

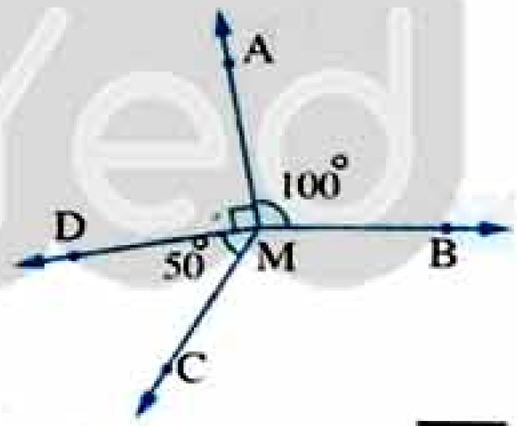


1 Δ ABM ≡ Δ ....

2 AB = ... cm.

3 m (∠ B) = m (∠ ....)

5 **[a] In the opposite figure :**



m (∠ BMA) = 100°

, m (∠ AMD) = 90°

, m (∠ DMC) = 50°

Find with steps : m (∠ BMC)

[b] Draw the line segment AB of length 8 cm. , then construct the axis of symmetry of $\overline{AB}$

(Don't remove the arcs)

10 El-Monofia Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If m (∠ A) = 130° , then m (reflex ∠ A) = ...

(a) 130° (b) 50° (c) 285° (d) 230°

2 If the triangle ABC ≡ the triangle XYZ , then $\overline{AC}$ ≡ ....

(a) $\overline{AB}$ (b) $\overline{XY}$ (c) $\overline{YZ}$ (d) $\overline{XZ}$

3 If two adjacent angles are supplementary , then their outer sides are ....

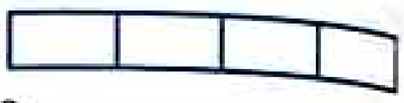
(a) perpendicular . (b) coincident.

(c) skew. (d) on the same straight line.

4 If the perimeter of a square is 24 cm. , then its area is ....

(a) 8 cm^2. (b) 9 cm^2. (c) 3 cm^2. (d) 36 cm^2.

5 **In the opposite figure :** The number of rectangles = ...



(a) 4 (b) 6 (c) 8 (d) 10

6 If L // M , L // N , then the two straight lines M and N are ....

(a) perpendicular. (b) parallel. (c) intersecting. (d) congruent.

2 Complete :

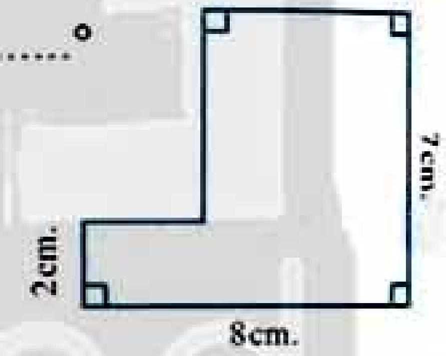
1 Two triangles are congruent if two sides and .... congruent with the corresponding parts from the other triangle.

2 If a straight line cuts two straight lines and two corresponding angles are equal in measure , then the two straight lines are ....

3 The angle of measure 50° complements an angle of measure ....°

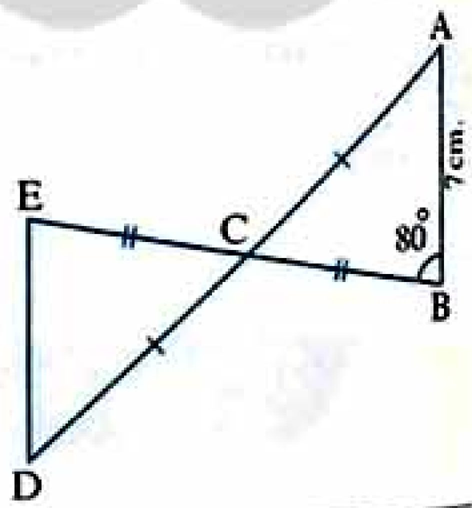
4 Two angles are congruent if ....

5 The perimeter of the opposite figure equals .... cm.



3 [a] Use the geometric instruments to draw ∠ ABC of measure 125° , then bisect it. (Don't remove the arcs)

[b] In the opposite figure :



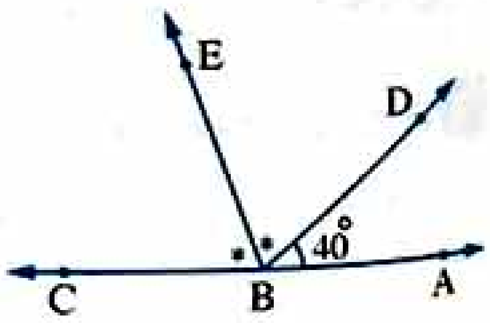
$\overline{AD} \cap \overline{BE} = \{C\}$, AC = CD

, BC = CE , AB = 7 cm. , m (∠ B) = 80°

1 Is Δ ABC ≡ Δ DEC ? Why ?

2 **Find :** The length of $\overline{ED}$, m (∠ E)

4 [a] In the opposite figure :



B ∈ $\overleftrightarrow{AC}$

, $\overrightarrow{BE}$ bisects ∠ DBC , m (∠ ABD) = 40°

Find : m (∠ DBC) , m (∠ ABE)

[b] In the opposite figure :

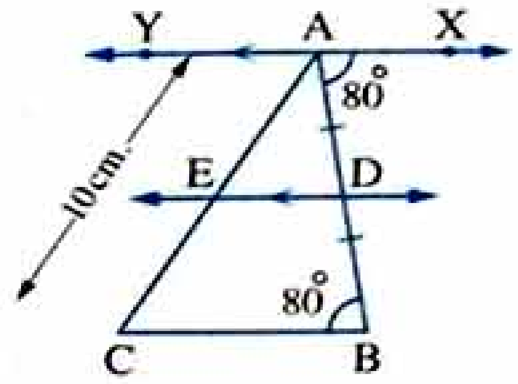
$\overleftrightarrow{XY} \parallel \overleftrightarrow{DE}$

, m ($\angle$ XAB) = 80° , m ($\angle$ B) = 80°

, AD = BD , AC = 10 cm.

Is $\overleftrightarrow{DE} \parallel \overline{BC}$? Why ?

Find : The length of $\overline{AE}$, give reason



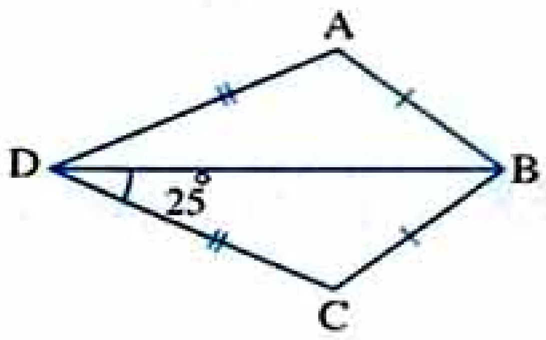
5 [a] In the opposite figure :

AB = CB , AD = CD

, m ($\angle$ CDB) = 25°

Is Δ ABD $\equiv$ Δ CBD ? Why ?

Find : m ($\angle$ ADC)



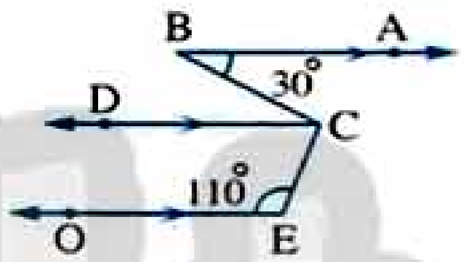
[b] In the opposite figure :

$\overrightarrow{BA} \parallel \overrightarrow{CD} \parallel \overrightarrow{EO}$

, m ($\angle$ ABC) = 30°

, m ($\angle$ CEO) = 110°

Find : m ($\angle$ BCE)



11 El-Gharbia Governorate

The central Maths Supervision
Official Language Schools

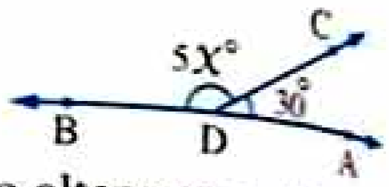
Answer the following questions :

1 Choose the correct answer :

1. If m ($\angle$ A) = 65° , then m (reflex $\angle$ A) = ...
   (a) 305° (b) 295° (c) 25° (d) 115°
2. The acute angle complements .... angle.
   (a) a right (b) an obtuse (c) an acute (d) a straight
3. ABCD is a rectangle , then $\overline{AC} \equiv$ ....
   (a) $\overline{BD}$ (b) $\overline{AD}$ (c) $\overline{DC}$ (d) $\overline{BC}$
4. The sum of measures of the accumulative angles at one point equals ....
   (a) 90° (b) 180° (c) 270° (d) 360°
5. If $\angle$ X supplements $\angle$ Y and m ($\angle$ X) = $\frac{1}{2}$ m ($\angle$ Y) , then m ($\angle$ Y) = ...
   (a) 30° (b) 45° (c) 60° (d) 120°
6. The two straight lines parallel to a third straight line are ....
   (a) intersecting. (b) parallel. (c) coincident. (d) perpendicular.

2 **Complete each of the following :**

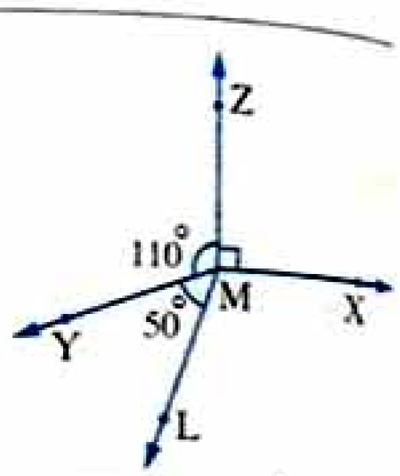
1. The angle whose measure is more than 90° and less than 180° is ....
2. Two angles are congruent if ....
3. If two adjacent angles are complementary , then their outer sides are ....
4. **In the opposite figure :**



m (∠ ADC) = 30° and m (∠ BDC) = 5 X , then X = ...

5. If a straight line intersects two parallel straight lines , then each two alternate angles are ....

3 [a] **In the opposite figure :**



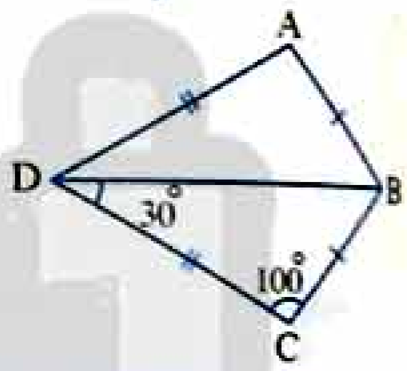
m (∠ XMZ) = 90°

, m (∠ ZMY) = 110°

and m (∠ YML) = 50°

Find by steps : m (∠ XML)

[b] **In the opposite figure :**

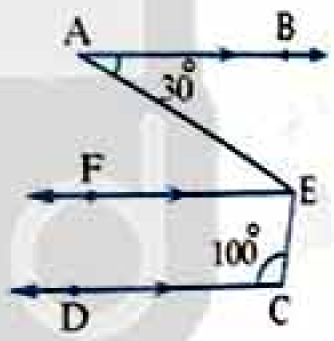


AB = CB , AD = CD , m (∠ C) = 100°

and m (∠ BDC) = 30° Is Δ ABD ≡ Δ CBD ? Why ?

, then find : m (∠ ABD) (Write the steps)

4 [a] **In the opposite figure :**



m (∠ C) = 100° , m (∠ A) = 30°

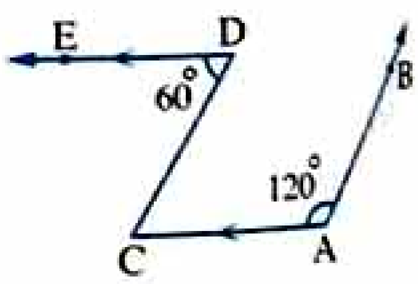
, $\overrightarrow{AB}$ // $\overrightarrow{EF}$ // $\overrightarrow{CD}$

Find by steps : m (∠ AEC)

[b] Draw ∠ ABC of measure 80° , then using the ruler and compasses bisect ∠ B

(Don't remove the arcs)

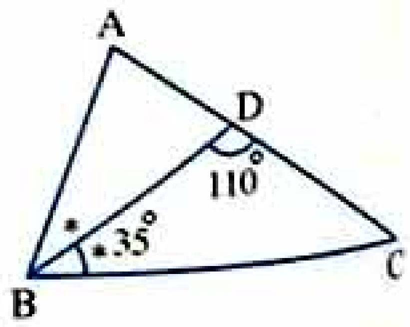
5 [a] **In the opposite figure :**



$\overrightarrow{DE}$ // $\overrightarrow{AC}$, m (∠ A) = 120° , m (∠ D) = 60°

1. **Find :** m (∠ C)
2. Is $\overrightarrow{AB}$ // $\overrightarrow{CD}$? Why ?

[b] **In the opposite figure :**



$\overrightarrow{BD}$ bisects ∠ ABC , m (∠ DBC) = 35°

, m (∠ BDC) = 110°

Find by steps : m (∠ C) and m (∠ A)

12 Ismailia Governorate

Directorate of Education
Al-Manar Language School

Answer the following questions :

1 Choose the correct answer :

1. If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) + m(\angle B) = 100°$, then $m(\angle Z) =$ ……………
   (a) 50° (b) 80° (c) 90° (d) 100°
2. If $\angle M \equiv \angle N$ and $\angle M$, $\angle N$ are supplementary angles , then $m(\angle M) =$ ……………
   (a) 180° (b) 45° (c) 360° (d) 90°
3. The sum of the measures of the accumulative angles at a point is …………… right angles.
   (a) 360 (b) 2 (c) 4 (d) 630
4. If two straight lines are parallel to a third straight line , then they are ……………
   (a) perpendicular. (b) parallel. (c) coincident. (d) intersecting.
5. The measure of the complement of an angle of measure 20° is ……………
   (a) 70° (b) 180° (c) 90° (d) 160°
6. The type of the angle of measure 185° is …………… angle.
   (a) an acute. (b) a reflex. (c) an obtuse. (d) a straight.

2 Complete :

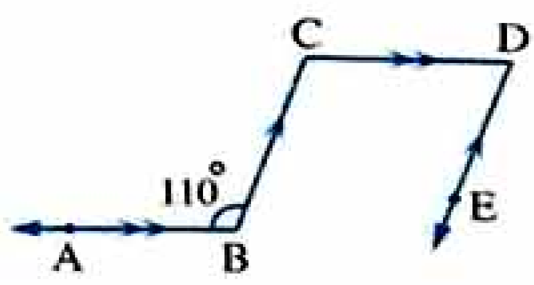
1. If $\Delta ABC \equiv \Delta XYZ$, then $AC - XZ =$ ……………
2. The two adjacent angles formed by intersecting of a straight line and a ray are ……………
3. If a straight line intersects two parallel lines , then each two corresponding angles are ……………
4. Two triangles are congruent if two sides and the …………… angle of one of them are congruent to their corresponding parts of the other.
5. The right angle supplements an angle of measure ……………°

3 [a] In the opposite figure :

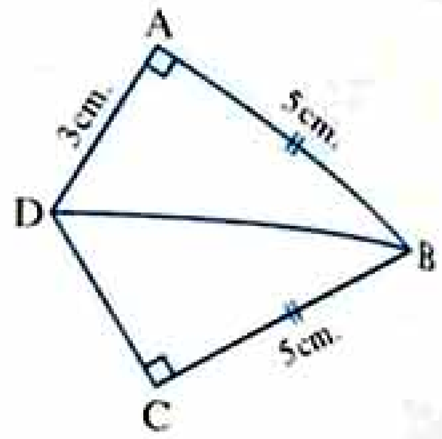
$\overrightarrow{BA} \parallel \overrightarrow{CD}$, $\overrightarrow{CB} \parallel \overrightarrow{DE}$

, $m(\angle B) = 110°$

Find : $m(\angle D)$



[b] In the opposite figure :



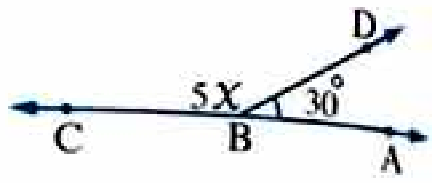
$m(\angle A) = m(\angle C) = 90°$

, $AB = BC = 5$ cm. , $AD = 3$ cm.

1 Mention the conditions for ΔABD , ΔCBD to be congruent.

2 **Find :** The length of $\overline{CD}$

4 [a] In the opposite figure :



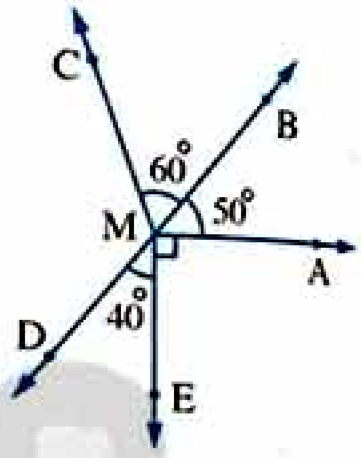
$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{B\}$

, $m(\angle ABD) = 30°$

, $m(\angle DBC) = 5x$

Find in degrees : The value of x

[b] In the opposite figure :

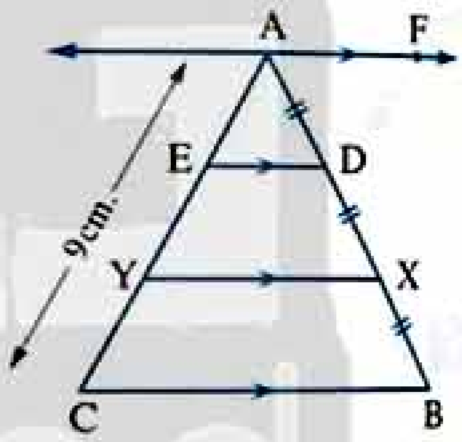


$m(\angle AME) = 90°$, $m(\angle AMB) = 50°$

, $m(\angle BMC) = 60°$, $m(\angle DME) = 40°$

Find : $m(\angle DMC)$

5 [a] In the opposite figure :



$\overleftrightarrow{AF} // \overleftrightarrow{ED} // \overleftrightarrow{YX} // \overleftrightarrow{CB}$

, $AD = DX = XB$

, $AC = 9$ cm.

Find : The length of $\overline{AY}$

[b] Using the geometric tools , draw $\angle ABC$ whose measure is 120° , then draw the bisector of $\angle ABC$

13 Damietta Governorate

Damietta Education Zone
Inspector of Math

Answer the following questions :

1 Choose the correct answer :

1 The angle of measure 95° 60̀ is supplementary to an angle of measure ....°

(a) 75 (b) 84 (c) 90 (d) 100

2 The triangle whose perimeter is 12 cm. and the lengths of its two sides are 2 cm. , 5 cm. , is called ....

(a) isosceles. (b) equilateral. (c) right. (d) scalene.

3 The two vertically opposite angles are ....

(a) corresponding. (b) congruent. (c) supplementary. (d) alternate.

4 If $\overline{AB}$, $\overline{CD}$ are congruent , then AB – CD = ...

(a) zero (b) 1 (c) 2 (d) 3

5 If the two triangles ABC , XYZ are congruent , m (∠ X) = 50° and m (∠ Z) = 60° , then m (∠ B) = ...°

(a) 50 (b) 60 (c) 70 (d) 110

6 If two straight lines are parallel to a third , then they are ....

(a) perpendicular. (b) parallel. (c) coincident. (d) intersecting.

2 Complete :

1 The perpendicular straight line to a line segment from its midpoint , is called ....

2 If a straight line cuts two parallel straight lines , then each two alternate angles are ....

3 If m (∠ B) = 115° , then m (reflex ∠ B) = ...°

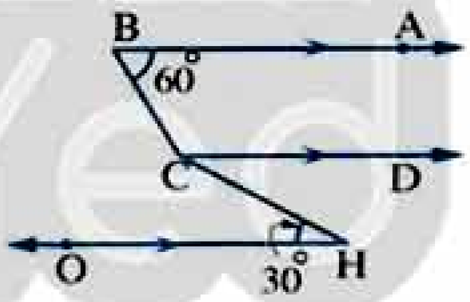
4 The two adjacent angles resulting from intersection of a ray and a straight line are ....

5 If the triangle ABC ≡ the triangle XYZ , then m (∠ C) = m (∠ ....)

3

[a] Draw $\overline{AB}$ of length 6 cm. , then draw its axis of symmetry by using geometrical tools. (Don't remove the arcs)

[b] In the opposite figure :



$\overrightarrow{BA}$ // $\overrightarrow{CD}$ // $\overrightarrow{HO}$

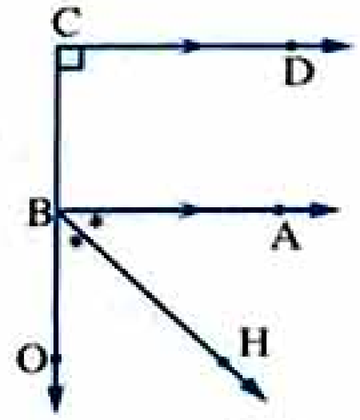
, m (∠ H) = 30°

, m (∠ B) = 60°

Find : m (∠ BCH) , give reason.

4

[a] In the opposite figure :

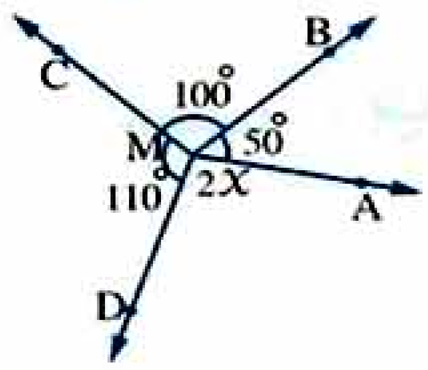


$\overrightarrow{CD}$ // $\overrightarrow{BA}$, m (∠ C) = 90°

, $\overrightarrow{BH}$ bisects ∠ ABO

Find : m (∠ OBH) , give reason.

[b] In the opposite figure :



m (∠ AMB) = 50°

, m (∠ BMC) = 100°

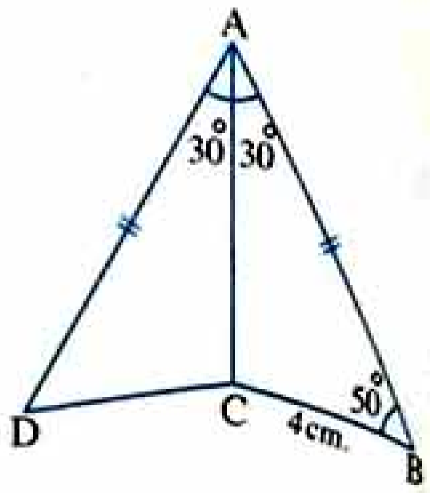
, m (∠ CMD) = 110°

, m (∠ AMD) = 2 x

Find : The value of x , give reason.

5 [a] Mention two cases of congruency of two triangles.

[b] **In the opposite figure :**



If AB = AD , BC = 4 cm.

, m (∠ B) = 50°

, m (∠ BAC) = m (∠ DAC) = 30°

Are the two triangles BAC and DAC congruent ?

Write the conditions and the results.

, then find : m (∠ D) , the length of $\overline{CD}$

14 Beni Suef Governorate

Directorate of Official Language Schools

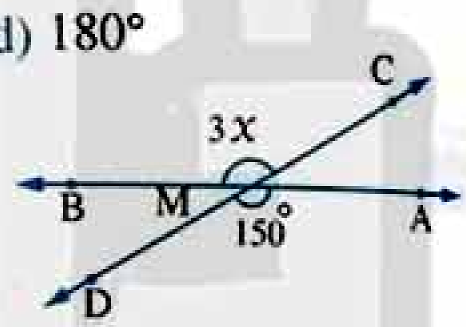
Answer the following questions :

1 **Choose the correct answer :**

1 Two complementary angles are two angles whose sum of their measures is ....

(a) 45° (b) 90° (c) 100° (d) 180°

2 **In the opposite figure :**



If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m (∠ AMD) = 150°

and m (∠ CMB) = 3 X , then the value of X = ...

(a) 25° (b) 50° (c) 100° (d) 150°

3 If Δ ABC ≡ Δ XYZ , then AC = ...

(a) BC (b) YZ (c) XZ (d) XY

4 If two straight lines are parallel to a third straight line , then these two straight lines are .... to each other.

(a) intersecting (b) perpendicular (c) coincident (d) parallel

5 The angle of measure 179° is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

6 $\overline{AB}$ .... $\overrightarrow{AB}$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 **Complete :**

1 The reflex angle is the angle whose measure is more than ....° and less than ....°

2 Two triangles are congruent if two angles and ....

3 If ∠ A ≡ ∠ B and m (∠ A) = 50° , then m (∠ B) = ...°

4 If a straight line intersects two parallel straight lines , then every two interior angles on one side of the transversal are ....

5 In Δ ABC , if m (∠ A) = 40° and m (∠ B) = 80° , then m (∠ C) = ...°

3 [a] Using the geometric instruments , draw ∠ ABC of measure 120° , then draw $\overrightarrow{BF}$ to bisect the angle. (Don't remove the arcs)

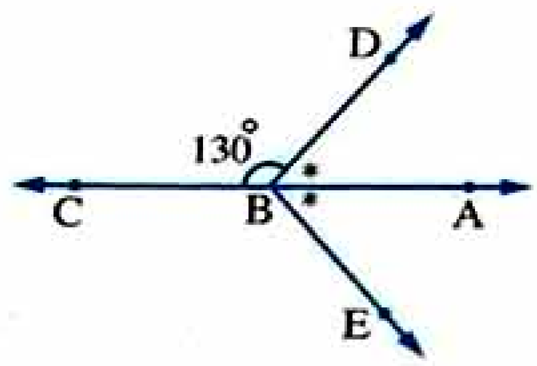
[b] **In the opposite figure :**

If B ∈ $\overleftrightarrow{AC}$

, m (∠ DBC) = 130°

and $\overrightarrow{BA}$ bisects ∠ DBE

, find : m (∠ ABD) and m (∠ DBE) (Give reason)



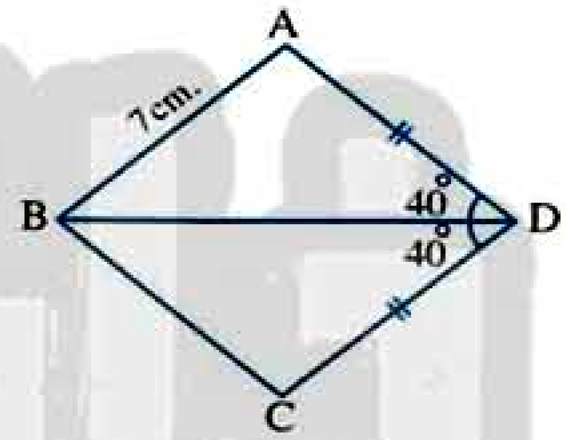
4 [a] **In the opposite figure :**

AD = DC , AB = 7 cm.

and m (∠ ADB) = m (∠ BDC) = 40°

1 **Prove that :** Δ ABD ≡ Δ CBD

2 **Find :** The length of $\overline{BC}$ (Give reason)

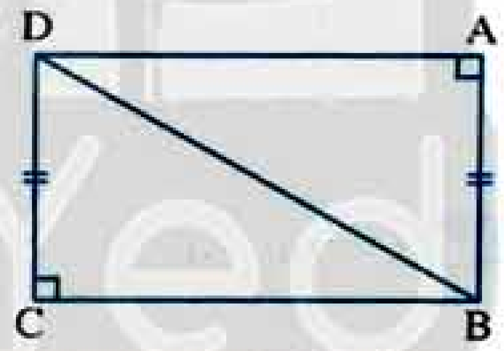


[b] **In the opposite figure :**

m (∠ BAD) = m (∠ BCD) = 90°

and AB = DC

Is Δ ABD ≡ Δ CDB ? Why ?

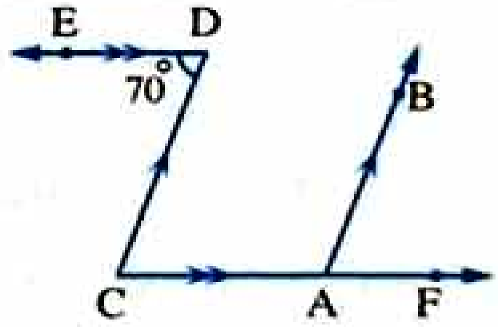


5 [a] **In the opposite figure :**

$\overrightarrow{AB}$ // $\overline{CD}$, $\overrightarrow{DE}$ // $\overrightarrow{CA}$

and m (∠ EDC) = 70°

Find : m (∠ DCA) and m (∠ BAF) (Give reason)

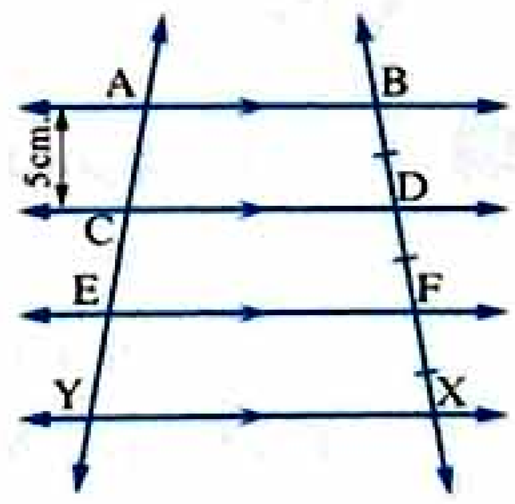


[b] **In the opposite figure :**

$\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$ // $\overleftrightarrow{XY}$, AC = 5 cm.

and BD = DF = FX

Find : The length of $\overline{AY}$ (Give reason)



15 South Sinai Governorate

Tur Sinai Educational Zone

Answer the following questions :

1 Choose the correct answer :

1. The angle whose measure is 30° complements the angle whose measure is ....°
   (a) 90 (b) 180 (c) 60 (d) 150
2. The sum of measures of the two supplementary angles equals ....°
   (a) 90 (b) 100 (c) 360 (d) 180
3. If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) = 60°$, $m(\angle B) = 40°$, then $m(\angle Z) =$ ....°
   (a) 100 (b) 70 (c) 80 (d) 90
4. If $m(\angle X) = 100°$, then m (reflex $\angle X$) = ...°
   (a) 360 (b) 180 (c) 260 (d) 80
5. If two straight lines intersect , then each two .... angles are equal in measure.
   (a) corresponding (b) alternate (c) adjacent (d) vertically opposite
6. The sum of measures of two adjacent angles formed by the intersection of a straight line and a ray with a starting point on this straight line equals ....°
   (a) 90 (b) 180 (c) 270 (d) 360

2 Complete :

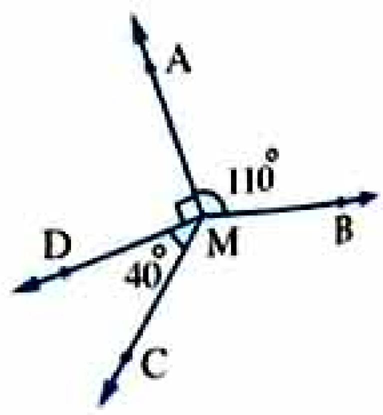
1. The two perpendicular lines on a third are ....
2. A circle of radius length 7 cm. , then its area = ... cm^2. $\left(\text{where } \pi = \frac{22}{7}\right)$
3. The two right-angled triangles are congruent if .... , ... are congruent to their corresponding parts in the other triangle.
4. If the two lines L_1 , L_2 are two parallel lines , then $L_1 \cap L_2 =$ ....
5. The measure of each angle of the two equal complementary angles equals ....°

3 [a] In the opposite figure :

$m(\angle AMB) = 110°$, $m(\angle AMD) = 90°$

, $m(\angle DMC) = 40°$

Find : $m(\angle BMC)$



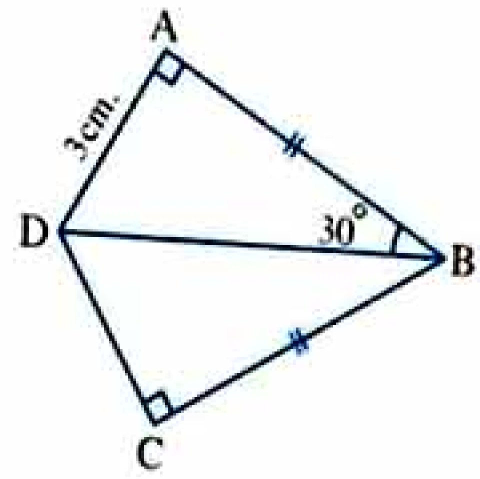
[b] In the opposite figure :

$m(\angle A) = m(\angle C) = 90°$

$, AD = 3$ cm. $, m(\angle ABD) = 30°$ $, AB = BC$

Write the conditions of congruency of the two triangles ABD , CBD

, then find : The length of $\overline{CD}$ and $m(\angle DBC)$



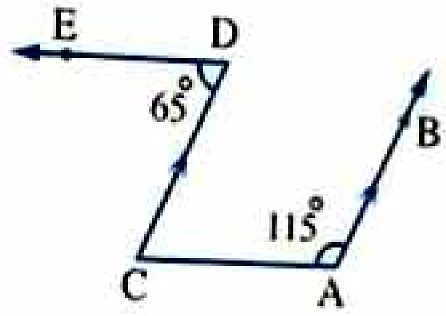
[a] In the opposite figure :

$\overrightarrow{AB} \parallel \overline{CD}$ $, m(\angle A) = 115°$

$, m(\angle D) = 65°$

Find : $m(\angle C)$

Is $\overline{AC} \parallel \overrightarrow{DE}$? Give reason.

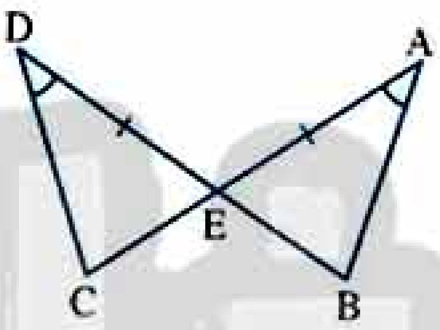


[b] In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{E\}$

$, AE = ED$ $, m(\angle A) = m(\angle D)$

Write the conditions of congruency of the two triangles.



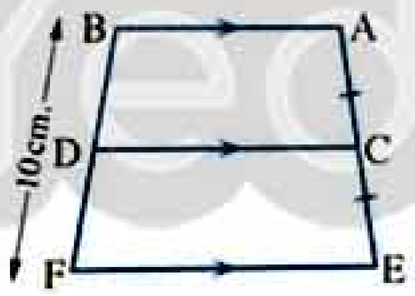
5 **[a]** By using your geometric instruments , draw $\angle ABC$ whose measure is $80°$, then draw $\overrightarrow{BD}$ to bisect the angle.

[b] In the opposite figure :

$\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$

$, AC = CE$ $, BF = 10$ cm.

Find by reason : The length of $\overline{BD}$



Final Examinations 2020

on Geometry

1 Cairo Governorate

Nasr City Educational Zone
St.Fatima Language School

Answer the following questions :

1 Choose the correct answer :

1 If ∠ X ≡ ∠ Y and ∠ X , ∠ Y are supplementary angles , then m (∠ X) = ...

(a) 45° (b) 90° (c) 135° (d) 180°

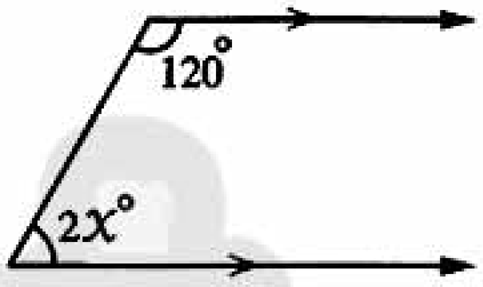
2 If two straight lines are perpendicular to a third , then the two straight lines are ....

(a) perpendicular. (b) parallel. (c) intersecting. (d) congruent.

3 If Δ ABC ≡ Δ XYZ , m (∠ A) + m (∠ B) = 100° , then m (∠ Z) = ...

(a) 90° (b) 100° (c) 50° (d) 80°

4 **From the opposite figure :**

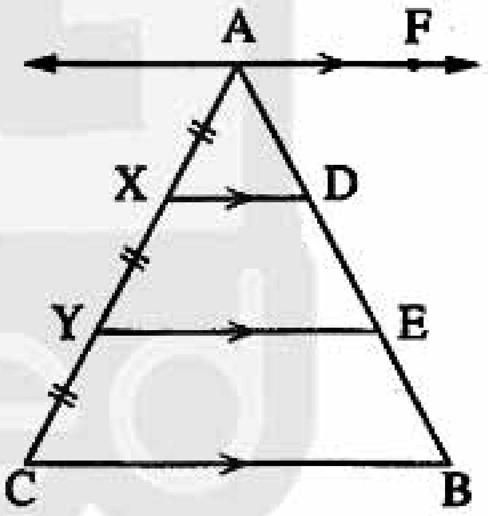


x = ...

(a) 60° (b) 140°

(c) 30° (d) 180°

5 **In the opposite figure :**



$\overleftrightarrow{AF}$ // $\overline{XD}$ // $\overline{YE}$ // $\overline{CB}$

, AX = XY = YC , then AD : AB = ...

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3

6 If Δ ABC ≡ Δ LMN , then m (∠ ACB) = m (∠ ....)

(a) LMN (b) MLN (c) LNM (d) NLM

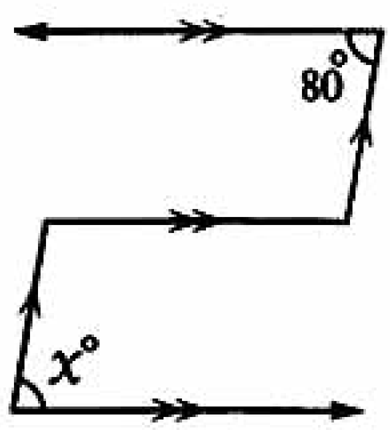
2 Complete :

1 If the ratio between the measures of two adjacent supplementary angles is 1 : 2 , then the measure of the largest angle is ....°

2 If m (∠ A) = 120° , then m (reflex ∠ A) = ...°

3 Two triangles are congruent if each side of ....

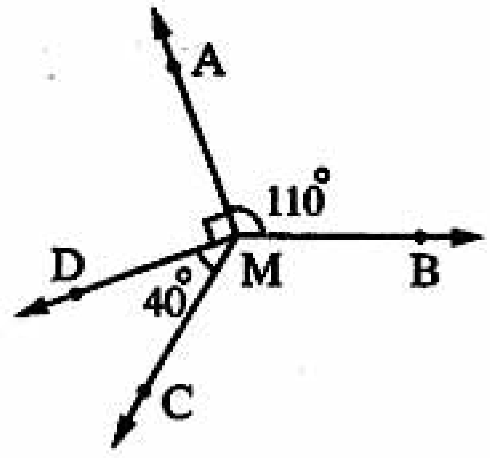
4 **From the opposite figure :**



x = ...°

5 From the opposite figure :

m (∠ BMC) = ……………°



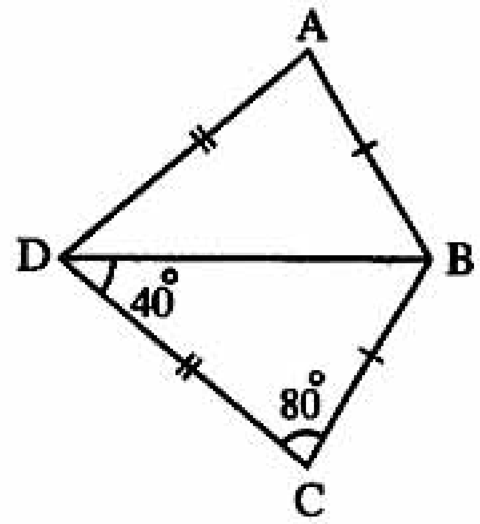
3 [a] In the opposite figure :

AB = BC , AD = CD

, m (∠ C) = 80°

, m (∠ BDC) = 40°

Prove that : Δ CBD ≡ Δ ABD and find : m (∠ ABD)

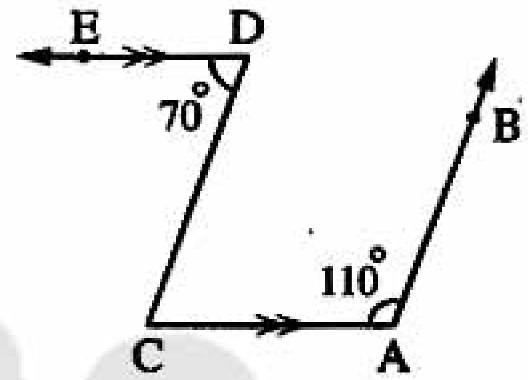


[b] In the opposite figure :

$\overrightarrow{DE}$ // $\overline{AC}$, m (∠ A) = 110°

, m (∠ D) = 70°

Prove that : $\overrightarrow{AB}$ // $\overline{CD}$



4 [a] In each of the following figures , find the value of x and give reason to your answer :

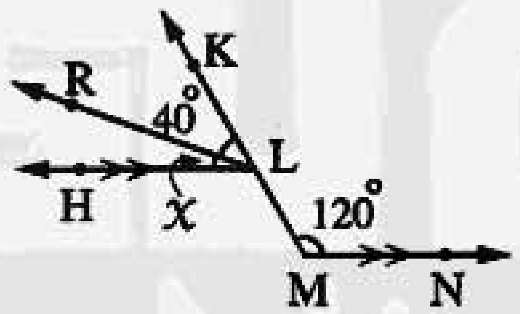


Fig. (1)

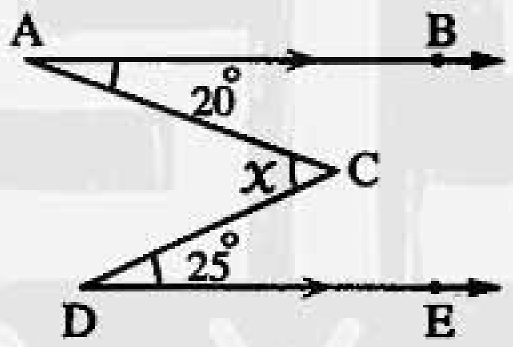


Fig. (2)

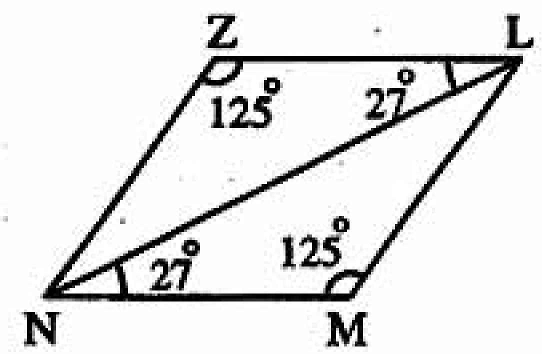
[b] Draw any acute-angled triangle , construct the perpendicular bisector of each side. Do the perpendicular bisectors intersect at one point ?

5 [a] From the opposite figure :

Prove that :

The two triangles LMN and NZL are congruent

, then find : m (∠ LNZ)



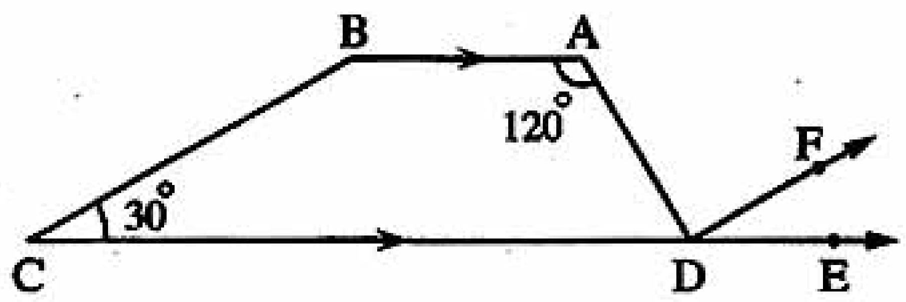
[b] In the opposite figure :

$\overline{AB}$ // $\overrightarrow{CE}$, m (∠ BAD) = 120°

, m (∠ BCD) = 30°

, m (∠ BAD) is four times m (∠ FDE)

Prove that : $\overrightarrow{DF}$ // $\overline{BC}$ and $\overrightarrow{DF}$ ⊥ $\overline{AD}$



2 Cairo Governorate

El-Zaitoun Educational Zone
El-Ma'aref Modern Language School

Answer the following questions :

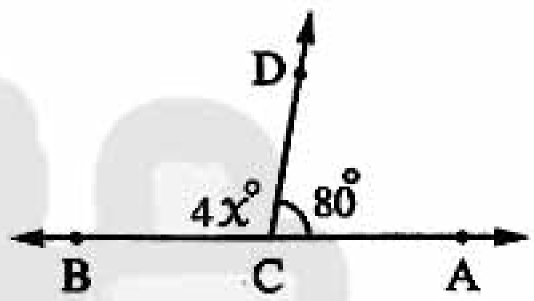
1 Choose the correct answer :

1. If two straight lines are perpendicular to a third , then the two straight lines are ……………
   (a) perpendicular. (b) parallel. (c) congruent. (d) intersecting.

2. If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) + m(\angle B) = 100^\circ$, then $m(\angle Z) =$ ……………
   (a) 50° (b) 90° (c) 80° (d) 100°

3. The image of the point $(-3 , 5)$ by translation of 3 units in the negative direction of the y-axis is ……………
   (a) $(-3 , 2)$ (b) $(-3 , 8)$ (c) $(-6 , 5)$ (d) $(0 , 8)$

4. **In the opposite figure :**
   $\overleftrightarrow{BA} \cap \overrightarrow{CD} = \{C\}$
   , $m(\angle DCA) = 80^\circ$
   , then $x =$ ……………



   (a) 20° (b) 25° (c) 30° (d) 100°

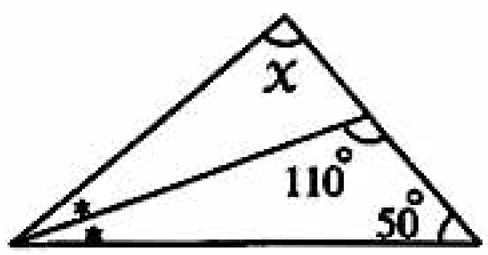
5. If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) = 50^\circ$, $m(\angle Y) = 60^\circ$, then $m(\angle C) =$ ……………
   (a) 50° (b) 60° (c) 70° (d) 80°

6. The measure of the supplement of the angle whose measure is 30° equals ……………
   (a) 60° (b) 180° (c) 90° (d) 150°

2 Complete the following :

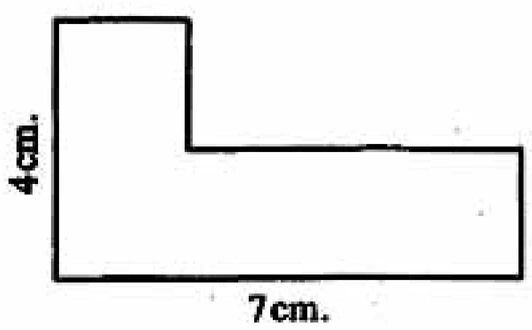
1. If a straight line intersects two parallel straight lines , then each two corresponding angles are ……………

2. **In the opposite figure :**
   $x =$ ……………



3. If $\angle X$ complements $\angle Y$ and $\angle X \equiv \angle Y$, then $m(\angle X) =$ ……………°

4. The perimeter of the opposite figure is …………… cm.

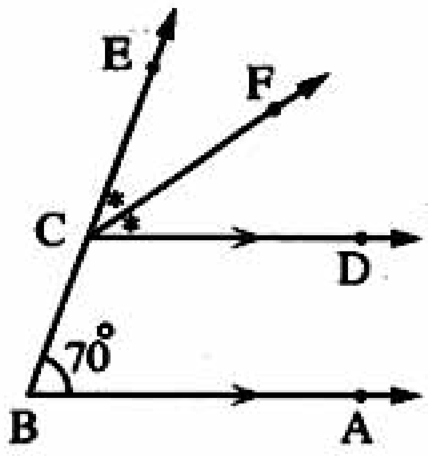


5. The two right-angled triangles are congruent if ……………

3 **[a] From the opposite figure , find :**

m (∠ ECF)

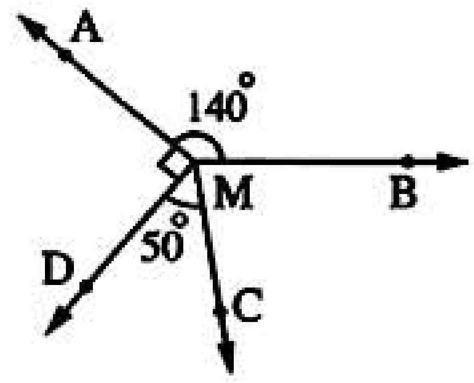
Give the reason.



[b] From the opposite figure , find :

m (∠ BMC)

With steps.



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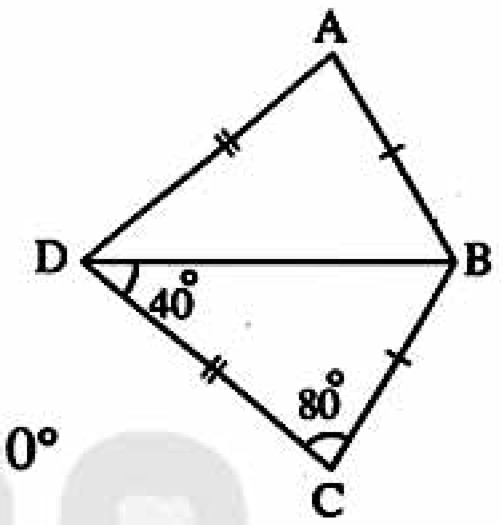
4 **[a] In the opposite figure :**

AB = BC , AD = CD , m (∠ C) = 80° , m (∠ BDC) = 40°

[1] **Prove that :** Δ CBD ≡ Δ ABD

[2] **Find :** m (∠ ABD)



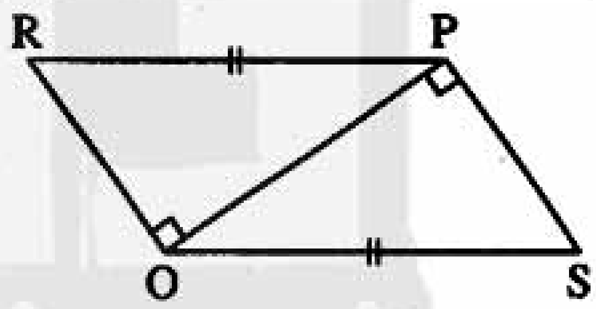
[b] By using your geometric instruments , draw ∠ ABC of measure 110° , then draw $\overrightarrow{BF}$ to bisect the angle.

---

5 **[a] From the opposite figure :**

Prove that : [1] Δ ROP ≡ Δ SPO

[2] m (∠ RPS) = m (∠ SOR)

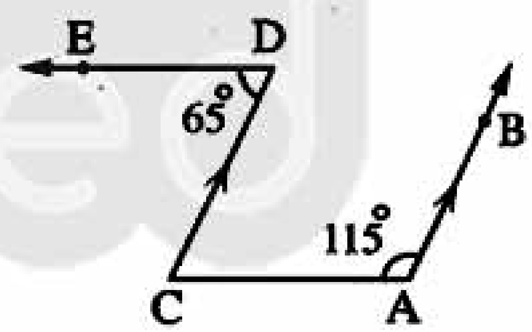


[b] In the opposite figure :

If $\overrightarrow{AB}$ // $\overrightarrow{CD}$, m (∠ D) = 65° , m (∠ A) = 115°

, then prove that :

$\overline{AC}$ // $\overrightarrow{DE}$



3 Cairo Governorate

Zone Educative Abdine
Lycee Bab El-Louk

Answer the following questions :

1 **Choose the correct answer :**

[1] If ∠ X complements ∠ Y and ∠ X ≡ ∠ Y , then m (∠ X) = ...

(a) 45° (b) 90° (c) 180° (d) 360°

[2] If Δ ABC ≡ Δ XYZ , m (∠ A) + m (∠ B) = 100° , then m (∠ Z) = ...

(a) 50° (b) 80° (c) 90° (d) 100°

3 If two straight lines are perpendicular to a third , then the two straight lines are ....

(a) perpendicular. (b) parallel. (c) congruent. (d) intersecting.

4 The sum of the measures of the accumulative angles at a point is ....

(a) 630° (b) 180° (c) 90° (d) 360°

5 The measure of the supplement of the angle whose measure is 30° equals ....

(a) 60° (b) 180° (c) 150° (d) 90°

6 The angle whose measure is more than 90° and less than 180° is .... angle.

(a) an obtuse (b) an acute (c) a right (d) a straight

2 Complete the following :

1 The two triangles are congruent if two sides and .... are congruent with the corresponding parts of the other.

2 If $\Delta ABC \equiv \Delta XYZ$, then m ($\angle$ Z) = m ($\angle$ ....)

3 The sum of the measures of the accumulative angles at a point equals ....°

4 If m ($\angle$ A) = 110° , then m (reflex $\angle$ A) = ...°

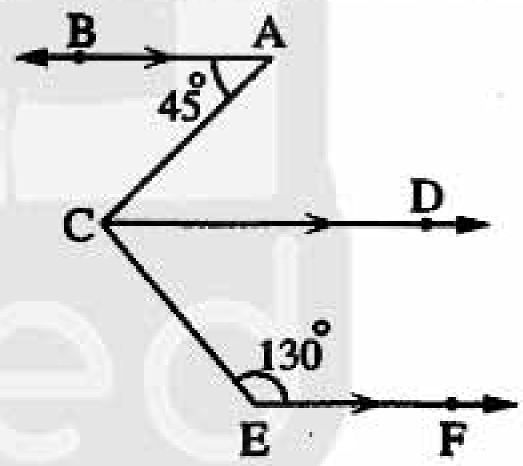
5 The two adjacent angles formed by intersecting of a straight line and a ray are ....

3 [a] In the opposite figure :

$\overrightarrow{AB} // \overrightarrow{CD} // \overrightarrow{EF}$, m ($\angle$ A) = 45°

, m ($\angle$ E) = 130°

Find : m ($\angle$ ACE)

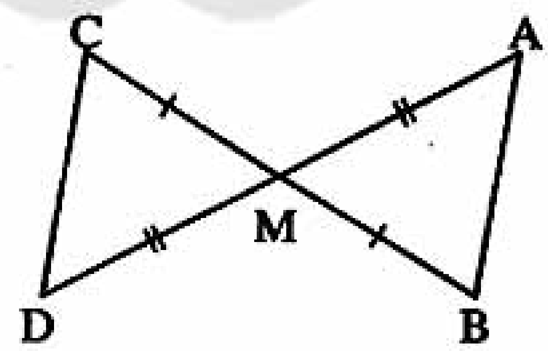


[b] In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{M\}$, BM = MC , AM = MD

, write the conditions

for Δ AMB , Δ DMC to be congruent.

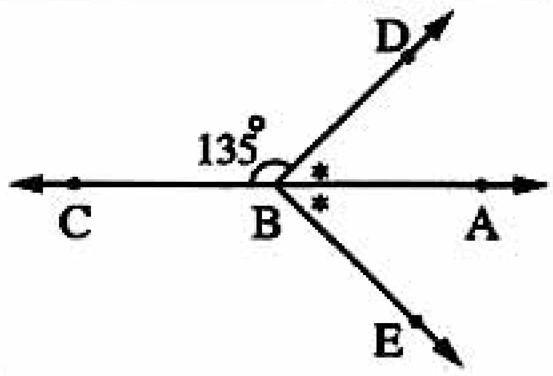


4 [a] In the opposite figure :

If B $\in \overleftrightarrow{AC}$, m ($\angle$ DBC) = 135°

and $\overrightarrow{BA}$ bisects $\angle$ DBE

Find : 1 m ($\angle$ ABD) 2 m ($\angle$ DBE) 3 m ($\angle$ CBE)



[b] By using your geometric instruments , draw $\angle$ ABC whose measure is 130° , then draw $\overrightarrow{BF}$ to bisect the angle.

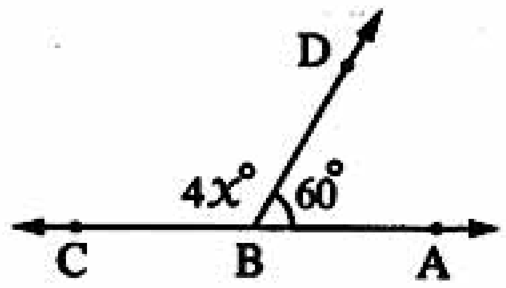
5 **[a] In the opposite figure :**

$\overleftrightarrow{AC} \cap \overrightarrow{BD} = \{B\}$

, m (∠ ABD) = 60°

, m (∠ DBC) = 4 x°

Find in degrees : The value of x

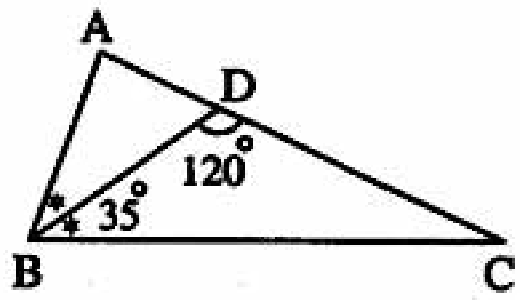


[b] In the opposite figure :

$\overrightarrow{BD}$ bisects ∠ ABC , m (∠ DBC) = 35°

, m (∠ BDC) = 120°

Find : m (∠ A) in degrees.



4 Giza Governorate

El-Haram Zone

El-Maarefa Exp. Lang. School

Answer the following questions :

1 **Choose the correct answer :**

1 If Δ ABC ≡ Δ XYZ , m (∠ A) = 50° , m (∠ B) = 60° , then m (∠ Z) = ...

(a) 50° (b) 60° (c) 70° (d) 120°

2 The sum of measures of the accumulative angles at a point equals ....

(a) 180° (b) 630° (c) 360° (d) 603°

3 The angle whose measure is 78° 60̀ , is .... angle.

(a) a right (b) an acute (c) an obtuse (d) a straight

4 If ∠ A ≡ ∠ B and ∠ A complements ∠ B , then m (∠ A) = ...

(a) 45° (b) 90° (c) 100° (d) 180°

5 If two straight lines are parallel to a third straight line , then they are ....

(a) perpendicular. (b) parallel. (c) congruent. (d) intersecting.

6 The measure of the supplement of an angle of measure 35° equals ....

(a) 65° (b) 165° (c) 180° (d) 145°

2 **Complete the following :**

1 The perpendicular bisector of a line segment is called ....

2 If m (∠ A) = 160° , then m (reflex ∠ A) = ...°

3 The two adjacent angles formed by a straight line and a ray with a start point on this straight line are ....

4 If two straight lines intersect , then each two vertically opposite angles are ....

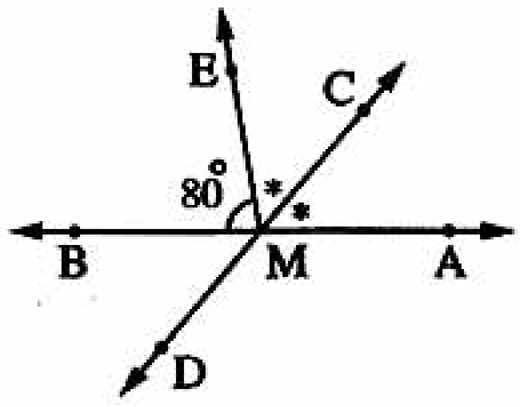
5 If $L_1 \perp L_2$ and $L_2 // L_3$, then L_1 .... L_3

3 **[a] In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m (∠ BME) = 80°

, $\overrightarrow{MC}$ bisects ∠ AME

Find : 1 m (∠ AMC)   2 m (∠ BMD)

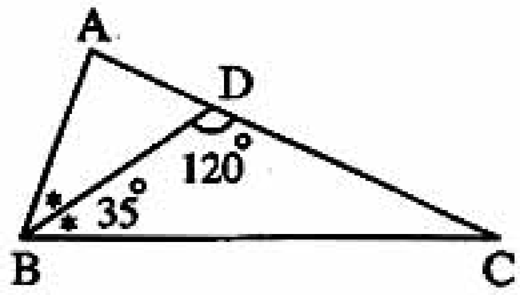


[b] In the opposite figure :

$\overrightarrow{BD}$ bisects ∠ ABC , m (∠ DBC) = 35°

, m (∠ BDC) = 120°

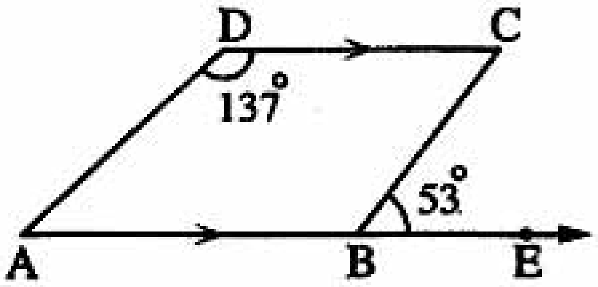
Find : m (∠ A) in degrees.



4 **[a] In the opposite figure :**

$\overrightarrow{AB} // \overline{DC}$, m (∠ EBC) = 53° , m (∠ D) = 137°

Is $\overline{BC} // \overline{AD}$? "State the reason"

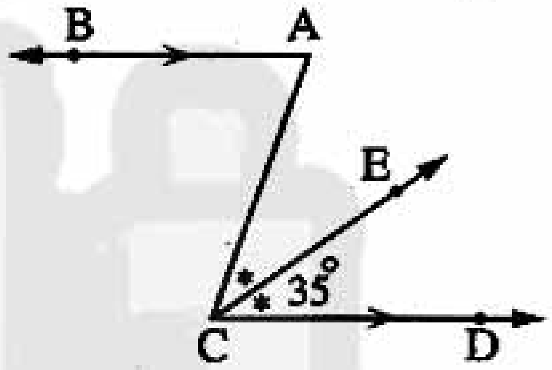


[b] In the opposite figure :

$\overrightarrow{AB} // \overrightarrow{CD}$, $\overrightarrow{CE}$ bisects ∠ ACD

, m (∠ DCE) = 35°

Find : m (∠ A)



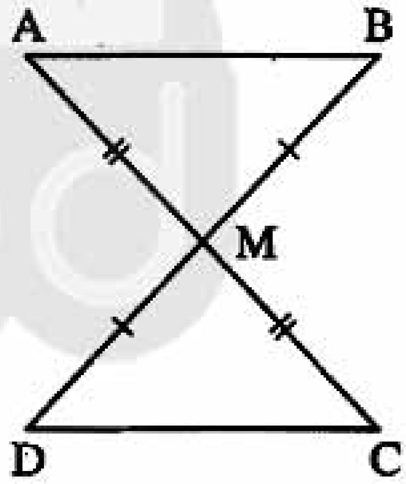
5 [a] Draw ∠ ABC of measure 85° , then bisect it. (Don't remove the arcs)

[b] In the opposite figure :

AM = CM

, BM = DM

Show with the reason if Δ ABM ≡ Δ CDM or not.



5 Giza Governorate

Boulaq El-Dakrour Dire. of Edu.
Dar El-Hanan Lang. Sch. for Girls

Answer the following questions :

1 **Choose the correct answer :**

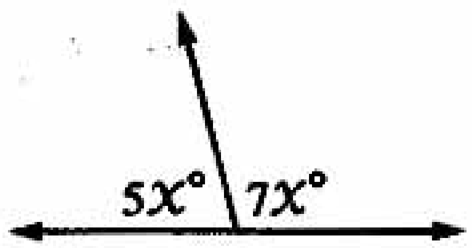
1 The supplement of the angle whose measure is 30° is an angle whose measure is ....

(a) 60°   (b) 180°   (c) 150°   (d) 90°

2 If Δ ABC ≡ Δ XYZ and m (∠ A) + m (∠ B) = 110° , then m (∠ Z) = ...

(a) 50°   (b) 60°   (c) 70°   (d) 80°

3 **From the opposite figure :**

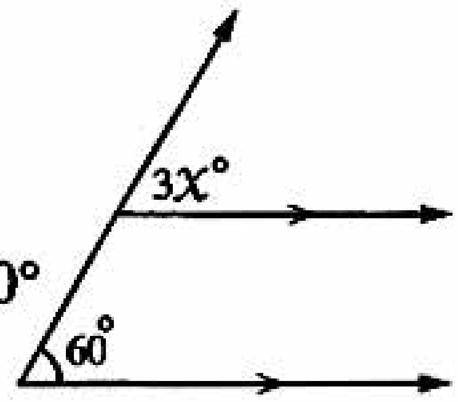


The value of x = ...

(a) 30° (b) 15°

(c) 45° (d) 18°

4 **From the opposite figure :**



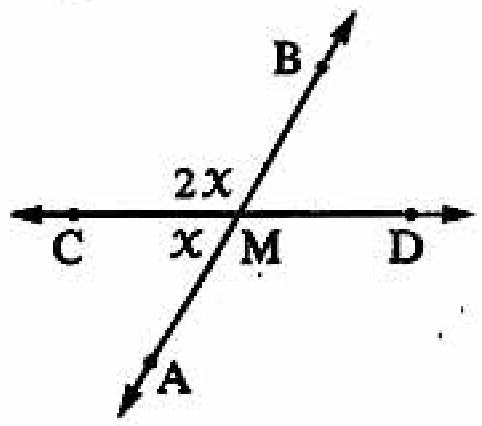
x = ...

(a) 20° (b) 30° (c) 40° (d) 120°

5 The angle of measure 179° is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

6 **In the opposite figure :**



$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then x = ...

(a) 30° (b) 60°

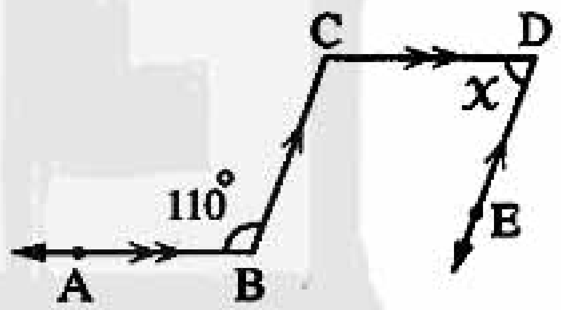
(c) 45° (d) 90°

2 Complete the following :

1 The complement of an angle of measure 65° is an angle of measure ....°

2 If m (∠ B) = 160° , then m (reflex ∠ B) = ...°

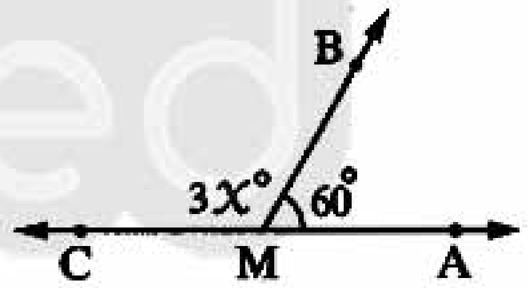
3 **In the opposite figure :**



$\overline{CD} \mathbin{//} \overrightarrow{BA}$, $\overrightarrow{DE} \mathbin{//} \overline{CB}$

, then x = ...°

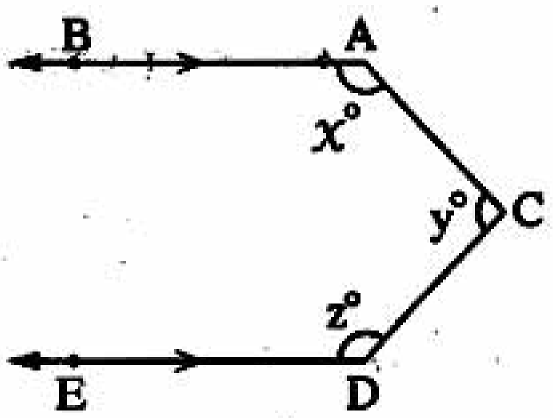
4 **In the opposite figure :**



If $\overrightarrow{MB} \cap \overleftrightarrow{AC} = \{M\}$, m (∠ AMB) = 60°

, then the value of x equals ....°

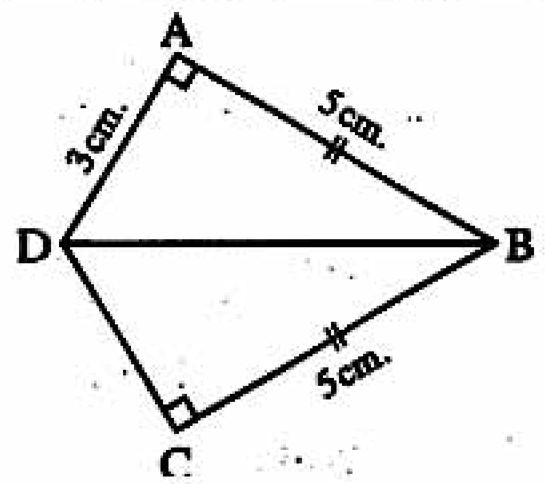
5 **In the opposite figure :**



$\overrightarrow{AB} \mathbin{//} \overrightarrow{DE}$

, then x + y + z = ...

3 [a] In the opposite figure :



m (∠ A) = m (∠ C) = 90°

, AB = BC = 5 cm. , AD = 3 cm.

1 Mention the conditions for Δ ABD , Δ CBD to be congruent.

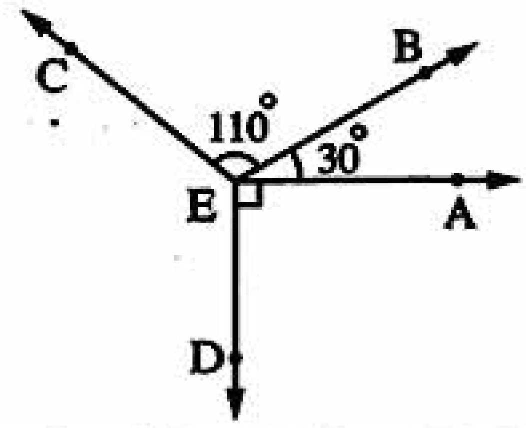
2 **Find :** The length of $\overline{CD}$.

[b] **In the opposite figure :**

$m(\angle AEB) = 30^\circ$, $m(\angle BEC) = 110^\circ$

, $m(\angle AED) = 90^\circ$

Find : $m(\angle DEC)$

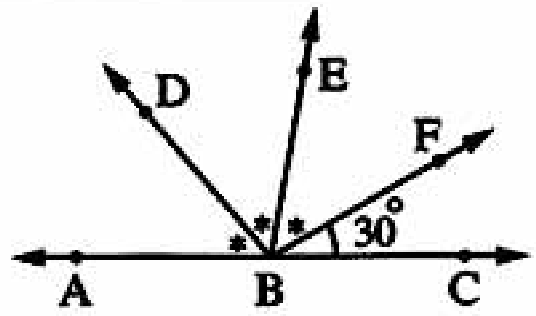


4 [a] **In the opposite figure :**

$B \in \overleftrightarrow{AC}$, $m(\angle FBC) = 30^\circ$

, $m(\angle ABD) = m(\angle DBE) = m(\angle EBF)$

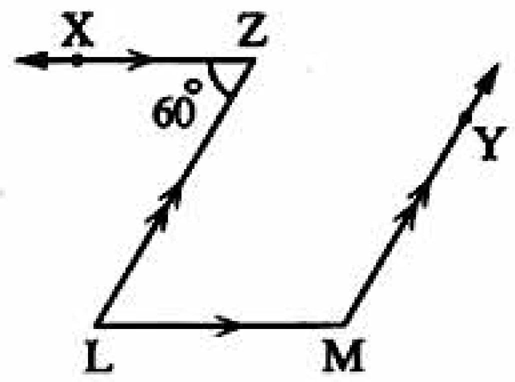
Find : $m(\angle ABE)$



[b] **In the opposite figure :**

$\overrightarrow{ZX} \parallel \overline{LM}$, $\overline{LZ} \parallel \overrightarrow{MY}$, $m(\angle Z) = 60^\circ$

Find : 1 $m(\angle L)$ 2 $m(\angle M)$

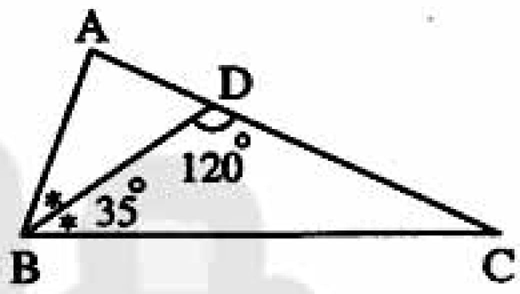


5 [a] **In the opposite figure :**

$\overrightarrow{BD}$ bisects $\angle ABC$, $m(\angle DBC) = 35^\circ$

, $m(\angle BDC) = 120^\circ$

Find : $m(\angle A)$

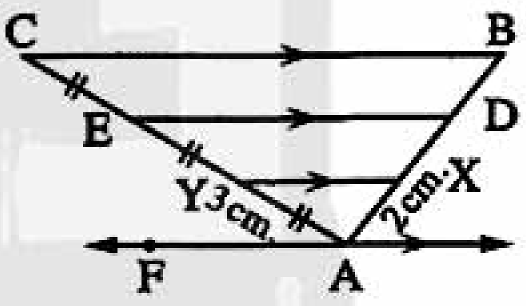


[b] **In the opposite figure :**

$\overleftrightarrow{AF} \parallel \overline{XY} \parallel \overline{DE} \parallel \overline{BC}$ and $AY = YE = EC$, $AY = 3$ cm.

, $AX = 2$ cm. and the perimeter of $\Delta ABC = 23$ cm.

Find : The length of $\overline{BC}$



[c] Draw $\angle ABC$ of measure 100° and bisect it. (Don't remove the arcs)

6 Alexandria Governorate

East Educational Zone
Sidi Gaber Lang. Sch. for boys

Answer the following questions :

1 **Complete the following :**

1 If $m(\angle A) = 120^\circ$, then the measure of the reflex angle of $\angle A = \dots\dots\dots\dots\dots^\circ$

2 The two adjacent angles formed by intersecting a straight line and a ray are $\dots\dots\dots\dots\dots$

3 If $\angle A$ supplements $\angle B$ and $\angle A$ supplements $\angle C$
, then $\angle B$ and $\angle C$ are $\dots\dots\dots\dots\dots$

4 Two triangles are congruent if the lengths of two sides and the measure of $\dots\dots\dots\dots\dots$ are congruent with the corresponding parts of the other.

5 If ∠ A and ∠ B are complementary angles , m (∠ A) = 2 m (∠ B) , then m (∠ B) = ...°

2 **Choose the correct answer :**

1 If two straight lines are perpendicular to a third , then the two straight lines are ....

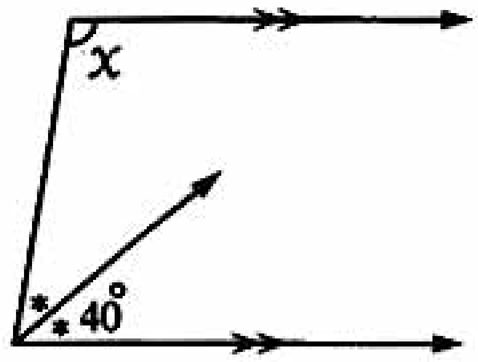
(a) perpendicular. (b) congruent. (c) parallel. (d) intersecting.

2 The axis of symmetry of a line segment is ....

(a) perpendicular from its midpoint. (b) equal to it.

(c) parallel to it. (d) congruent to it.

3 **In the opposite figure :**

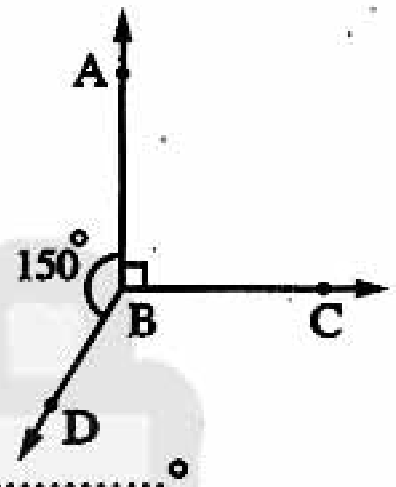


x = ...°

(a) 80 (b) 120

(c) 100 (d) 180

4 **In the opposite figure :**



m (∠ CBD) = ...°

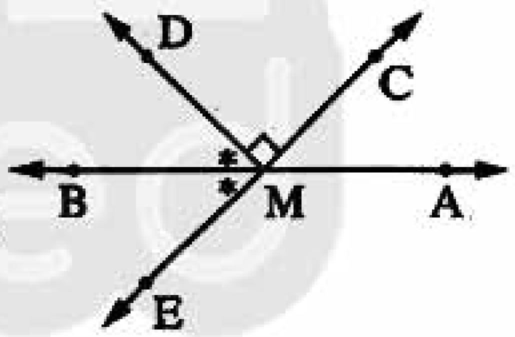
(a) 100 (b) 120

(c) 140 (d) 240

5 If Δ ABC ≡ Δ XYZ , m (∠ Z) = 55° , then m (∠ A) + m (∠ B) = ...°

(a) 110 (b) 115 (c) 120 (d) 125

3 **[a] In the opposite figure :**

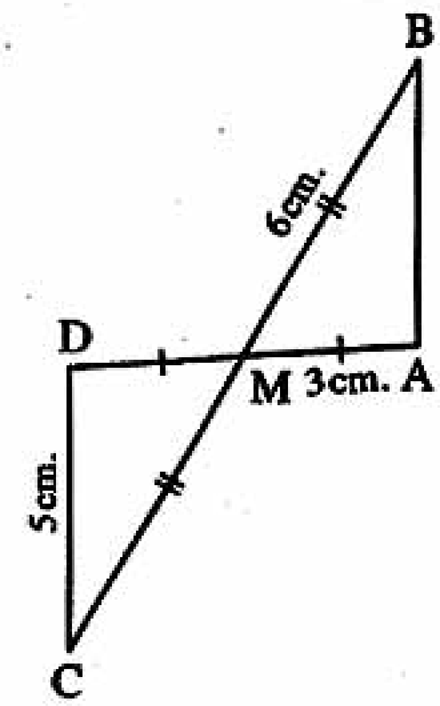


$\overleftrightarrow{AB} \cap \overleftrightarrow{CE} = \{M\}$, $\overrightarrow{MD} \perp \overrightarrow{MC}$, $\overrightarrow{MB}$ bisects ∠ DME

Find showing the reason : 1 m (∠ BME) 2 m (∠ AMC) 3 m (∠ AME)

[b] Draw the line segment AB of length 8 cm. , then construct the axis of symmetry of $\overline{AB}$ (Don't remove the arcs)

4 **[a] In the opposite figure :**



Complete :

1 Δ ABM ≡ Δ ....

2 m (∠ B) = m (∠ ....)

3 m (∠ A) = m (∠ ....)

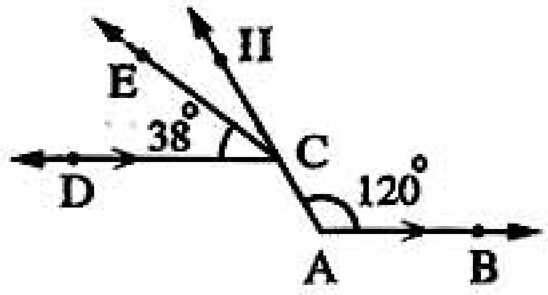
4 The perimeter of Δ DMC = ... cm.

[b] In the opposite figure :

$\overleftrightarrow{AB} \,/\!/\, \overleftrightarrow{DC}$, m (∠ A) = 120° , H ∈ $\overrightarrow{AC}$

, m (∠ ECD) = 38°

Find : m (∠ ACD) , m (∠ HCE) (showing the reason)



5 **In the opposite figure :**

$\overleftrightarrow{OR}$ is the axis of symmetry of the shape NERAM , O ∈ $\overleftrightarrow{MN}$

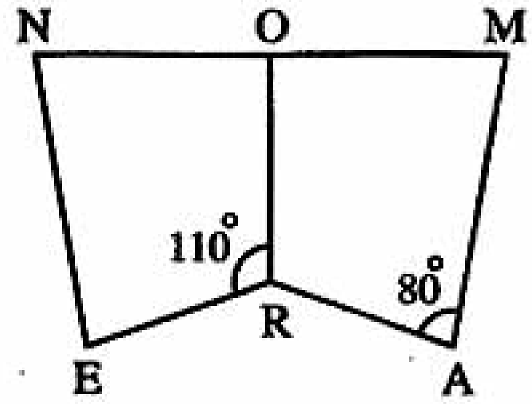
Complete : 1 Quad AMOR ≡ Quad ....

2 m (∠ NOR) = m (∠ ....)

3 m (∠ AMO) = m (∠ ....)

4 m (∠ ORA) = m (∠ ....) = ...°

5 m (∠ NER) = m (∠ ....) = ...°



7 Alexandria Governorate

Borg El-Arab Educational Zone
Al-Safwa Integrated Schools

Answer the following questions : (Calculator is allowed)

1 **Complete each of the following :**

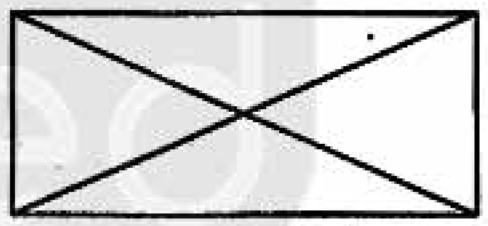
1 The complement of the angle of measure 55° is an angle of measure ....°

2 The sum of measures of the accumulative angles at a point equals ....°

3 If m (∠ B) = 160° , then m (reflex ∠ B) = ...°

4 The perpendicular bisector of a line segment is called ....

5 The number of triangles in the opposite figure is ....



2 **Choose the correct answer :**

1 If $L_1 /\!/ L_2$ and $L_2 \perp L_3$, then ....

(a) $L_1 \perp L_2$ (b) $L_3 /\!/ L_2$ (c) $L_1 \perp L_3$ (d) $L_3 /\!/ L_1$

2 If Δ ABC ≡ Δ XYZ and m (∠ A) + m (∠ B) = 110° , then m (∠ Z) = ...°

(a) 50 (b) 60 (c) 70 (d) 80

3 If the ratio between the measures of two supplementary angles is 5 : 13 , then the measure of the smaller angle is ....°

(a) 50 (b) 130 (c) 150 (d) 180°

4 The type of the angle of measure 89° 60̀ is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

5 The two diagonals are perpendicular and equal in length in the ....

(a) rectangle. (b) rhombus. (c) square. (d) parallelogram.

6 If $\Delta ABC \equiv \Delta LMN$, then $\overline{AC}$ .... $\overline{LN}$

(a) = (b) ≡ (c) < (d) >

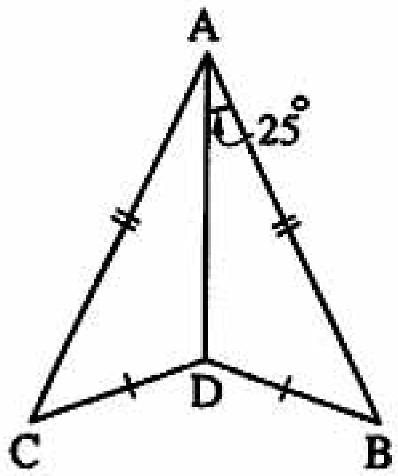
3 **[a] In the opposite figure :**

AB = AC , BD = CD

, m (∠ BAD) = 25°

Is Δ ADC ≡ Δ ADB ? Why ?

Find : m (∠ CAB)



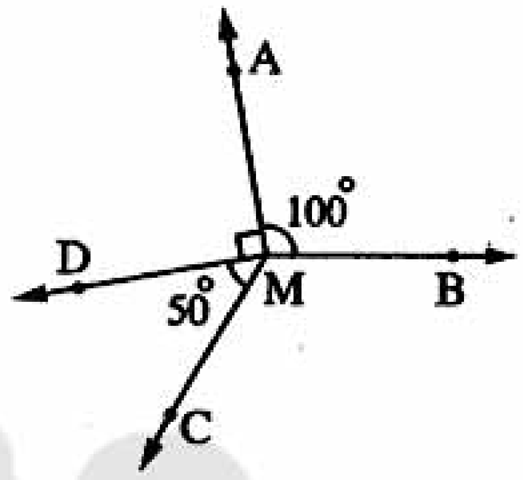
[b] In the opposite figure :

m (∠ BMA) = 100°

, m (∠ AMD) = 90°

, m (∠ DMC) = 50°

Find with steps : m (∠ BMC)

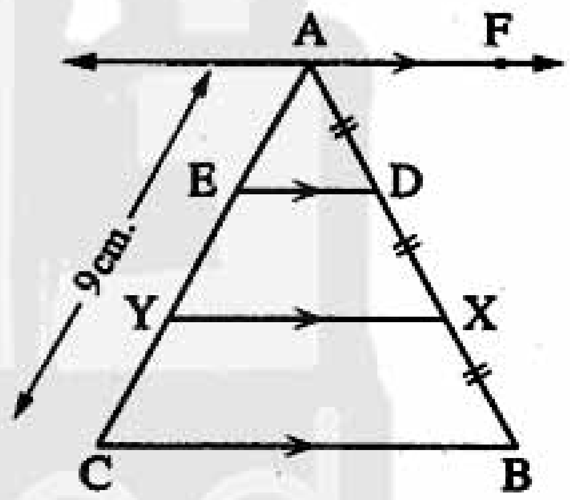


4 **[a] In the opposite figure :**

$\overleftrightarrow{AF} \parallel \overline{ED} \parallel \overline{YX} \parallel \overline{CB}$

, AD = DX = XB , AC = 9 cm.

Find : The length of $\overline{AY}$ (Give reason)



[b] Draw ∠ ABC of measure 100° and bisect it.

5 **[a] In the opposite figure :**

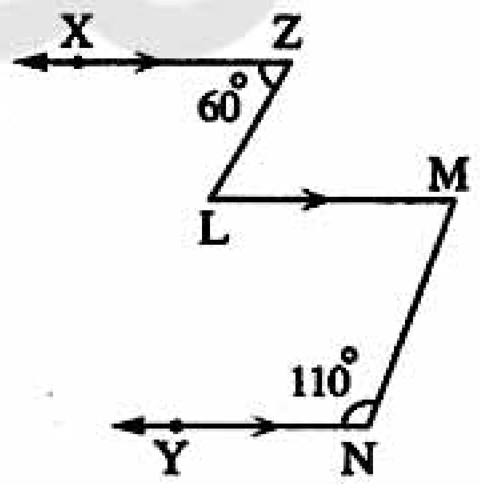
$\overrightarrow{ZX} \parallel \overline{LM}$

, $\overline{LM} \parallel \overrightarrow{NY}$

, m (∠ N) = 110°

, m (∠ Z) = 60°

Find : 1 m (∠ L) 2 m (∠ M)



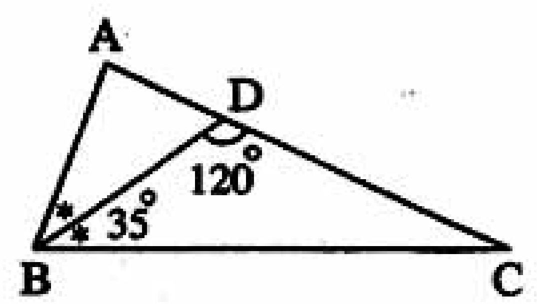
[b] In the opposite figure :

$\overrightarrow{BD}$ bisects ∠ ABC

, m (∠ DBC) = 35°

, m (∠ BDC) = 120°

Find : m (∠ A)



8 El-Kalyoubia Governorate

Directorate of Education
Mathematics Supervision

Answer the following questions :

1 Choose the correct answer :

1 If $\Delta ABC \equiv \Delta XYZ$, then AC = ...

(a) XY (b) XZ (c) YZ (d) AB

2 If m ($\angle$ B) = 105° , then m (reflex $\angle$ B) = ...

(a) 255° (b) 75° (c) 105° (d) 50°

3 If $\overline{AB} \equiv \overline{CD}$ and AB = 4 cm. , then AB + 2 CD = ... cm.

(a) 10 (b) 4 (c) 8 (d) 12

4 The measure of the supplementary of the angle whose measure is 30° equals ....°

(a) 60 (b) 80 (c) 150 (d) 90

5 A cube is of volume 125 cm^3. , then the area of its base = ... cm^2.

(a) 5 (b) 15 (c) 25 (d) 10

6 The measure of the right angle is ....°

(a) 60 (b) 90 (c) 180 (d) 70

2 Complete the following :

1 The two diagonals are equal in length in .... and ....

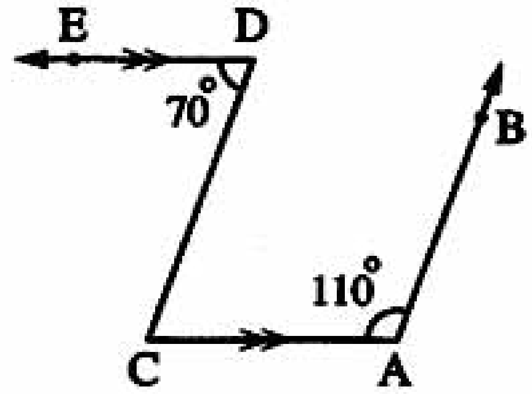
2 The perpendicular bisector of a line segment is called ....

3 The sum of the measures of the accumulative angles at a point equals ....°

4 If $\Delta ABC \equiv \Delta XYZ$, m ($\angle$ A) + m ($\angle$ B) = 100° , then m ($\angle$ Z) = ...°

5 If two straight lines are perpendicular to a third , then the two straight lines are ....

3 [a] In the opposite figure :



$\overrightarrow{DE}$ // $\overrightarrow{AC}$, m ($\angle$ A) = 110° , m ($\angle$ D) = 70°

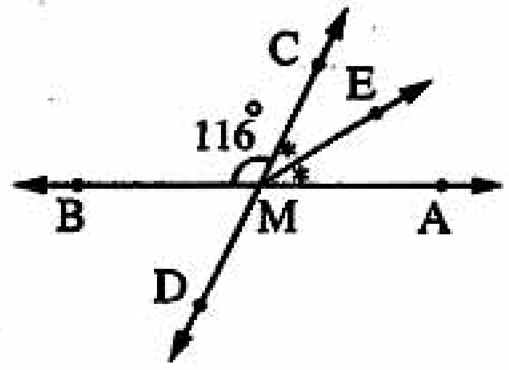
Complete the following :

1 m ($\angle$ C) = ... because ....

2 Is $\overrightarrow{AB}$ // $\overrightarrow{CD}$? (..............) because ....

[b] Using the geometric instruments , draw $\angle$ ABC where m ($\angle$ B) = 120° , then draw $\overrightarrow{BD}$ to bisect the angle. (Don't remove the arcs)

4 **[a] In the opposite figure :**



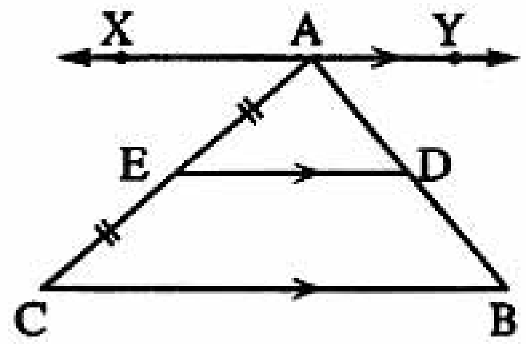
$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, $\overrightarrow{ME}$ bisects $\angle$ AMC , m ($\angle$ BMC) = 116°

Complete the following :

1 m ($\angle$ AMC) = ...° 2 m ($\angle$ AMD) = ...°

3 m ($\angle$ AME) = ...°

[b] In the opposite figure :

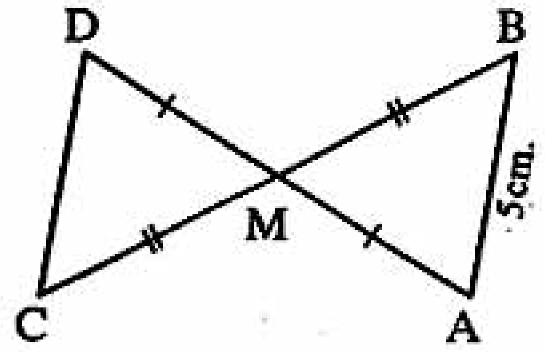


$\overleftrightarrow{XY}$ // $\overline{ED}$ // $\overline{BC}$, AE = EC

Complete the following :

1 AD = ... 2 AD : AB = ... : ....

5 **[a] From the opposite figure complete the following :**



1 Δ ABM $\equiv \Delta$ ....

2 CD = ... cm.

3 m ($\angle$ B) = m ($\angle$ ....)

[b] Mention two cases of congruency of two triangles.

9 El-Sharkia Governorate

West Zagazig Zone
Zagazig English Lang. Sch. for Girls

Answer the following questions :

1 **Choose the correct answer :**

1 If $\angle$ X complements $\angle$ Y and $\angle$ X $\equiv$ $\angle$ Y , then m ($\angle$ X) = ...°

(a) 45 (b) 90 (c) 20 (d) 180

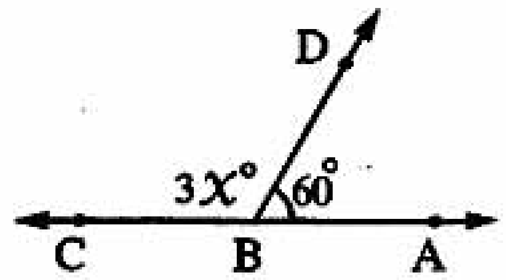
2 A square is of perimeter 20 cm. , then its area = ... cm^2.

(a) 4 (b) 5 (c) 25 (d) 400

3 The two diagonals are equal in length in the ....

(a) rhombus. (b) parallelogram. (c) trapezium. (d) rectangle.

4 **In the opposite figure :**



B $\in \overleftrightarrow{AC}$, then X = ...

(a) 30 (b) 120

(c) 40 (d) 150

5 If m ($\angle$ A) = 110° , then m (reflex $\angle$ A) = ...

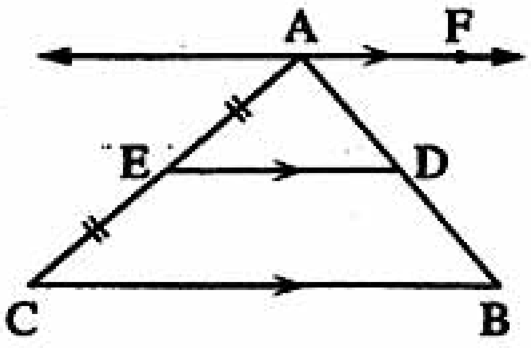
(a) 70° (b) 360° (c) 250° (d) 150°

6 In the opposite figure :

If $\overleftrightarrow{AF} \parallel \overline{ED} \parallel \overline{CB}$, AE = EC , then AD : AB = ...

(a) 2 : 1 (b) 3 : 2

(c) 1 : 3 (d) 1 : 2



2 Complete each of the following :

1 If Δ ABC $\equiv \Delta$ XYZ , m ($\angle$ A) + m ($\angle$ B) = 120° , then m ($\angle$ Z) = ...°

2 If a straight line intersects two parallel lines , then each two corresponding angles are ....

3 If Δ ABC $\equiv \Delta$ XYZ , then AC = ...

4 Two right-angled triangles are congruent if ....

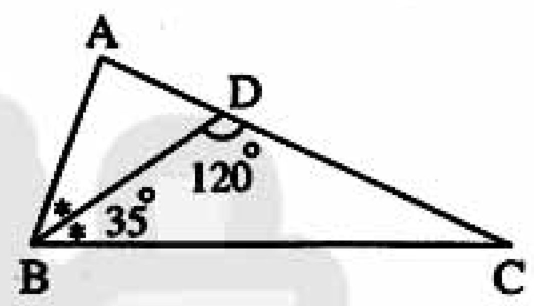
5 If two straight lines intersect , then the measures of each two vertically opposite angles are ....

3 [a] In the opposite figure :

$\overrightarrow{BD}$ bisects $\angle$ ABC , m ($\angle$ DBC) = 35°

, m ($\angle$ BDC) = 120°

Find : m ($\angle$ C) , m ($\angle$ ABC) and m ($\angle$ A)



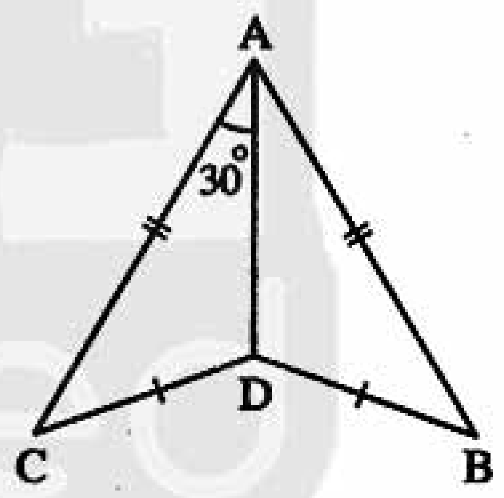
[b] In the opposite figure :

AC = AB , DC = DB

, m ($\angle$ CAD) = 30°

1 **Prove that :** Δ ABD $\equiv \Delta$ ACD

2 **Find :** m ($\angle$ CAB)



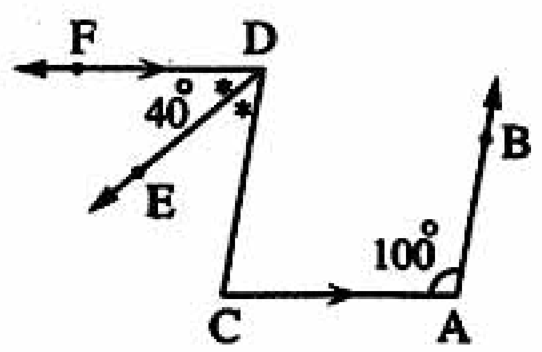
4 [a] In the opposite figure :

$\overrightarrow{DF} \parallel \overrightarrow{AC}$, m ($\angle$ A) = 100°

, $\overrightarrow{DE}$ bisects $\angle$ FDC , m ($\angle$ FDE) = 40°

1 **Find :** m ($\angle$ FDC) and m ($\angle$ C)

2 **Prove that :** $\overline{CD} \parallel \overrightarrow{AB}$



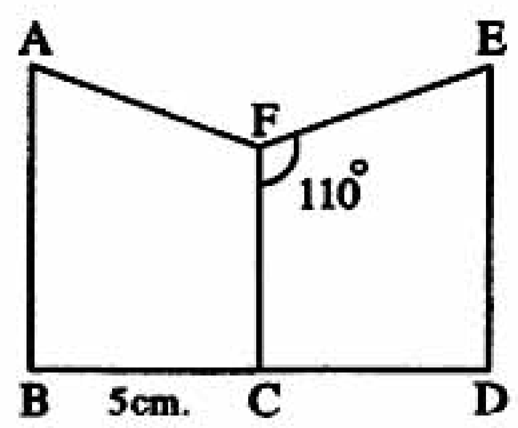
[b] In the opposite figure :

The polygon ABCF $\equiv$ the polygon EDCF

, m ($\angle$ EFC) = 110° , BC = 5 cm.

Find : 1 m ($\angle$ AFC) , m ($\angle$ AFE) and m ($\angle$ FCB)

2 The length of $\overline{BD}$

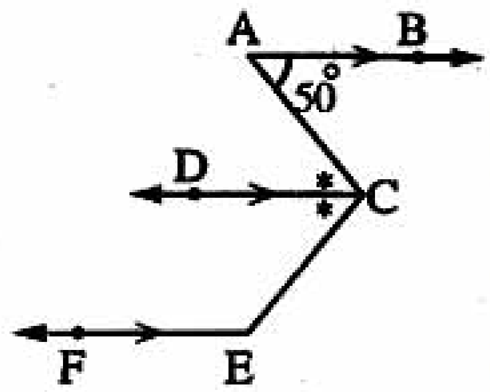


5 **[a] In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{EF}$, $\overrightarrow{CD}$ bisects $\angle$ ACE

, m ($\angle$ A) = 50°

Find : m ($\angle$ ACE) and m ($\angle$ E)



[b] Using the ruler and compasses , draw the triangle ABC in which BC = 6 cm.

, AB = AC = 5 cm. Draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

(Don't remove the arcs)

10 El-Monofia Governorate

Kwesna Educational Directorate
Mathematics Supervision

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer :**

1. The sum of the measures of the accumulative angles at a point equlas ....°

   (a) 90 (b) 180 (c) 270 (d) 360

2. If two triangles ABC and XYZ are congruent , then ....

   (a) BC = XZ (b) YX = CA (c) ZY = CB (d) AB = YZ

3. If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are ....

   (a) equal. (b) supplementary. (c) corresponding. (d) complementary.

4. If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ A) + m ($\angle$ B) = 115° , then m ($\angle$ Z) = ...°

   (a) 115 (b) 65 (c) 15 (d) 70

5. If m ($\angle$ A) = 90° , then m (reflex $\angle$ A) = ...

   (a) 270 (b) 180 (c) 90 (d) 360

6. If $\angle$ A supplements $\angle$ B and $\angle$ A $\equiv$ $\angle$ B , then m ($\angle$ B) = ...°

   (a) 45 (b) 90 (c) 120 (d) 60

2 **Complete each of the following :**

1. The angle whose measure is 40° complements an angle of measure ....°

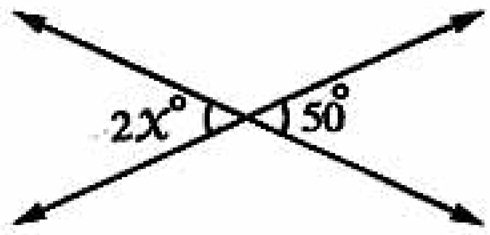
2. Two triangles are congruent if two sides and the .... in one of them are congruent to their corresponding parts of the other.

3. If two straight lines are perpendicular to a third line , then these two straight lines are ....

4. If $L_1 \parallel L_2$ and $L_1 \perp L_3$, then L_3 .... L_2

5 In the opposite figure :

$x =$ ……………



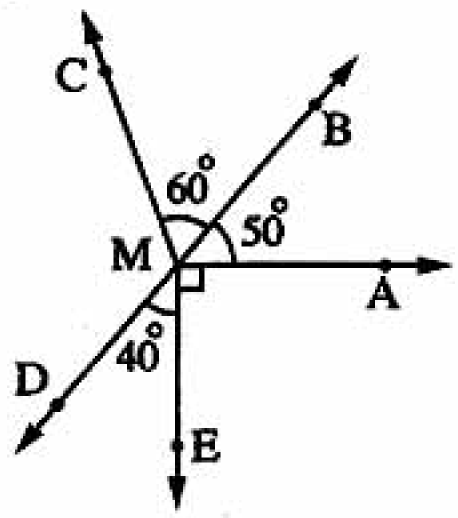
3 [a] In the opposite figure :

m (∠ AMB) = 50°

, m (∠ BMC) = 60°

, m (∠ DME) = 40° and $\overrightarrow{MA} \perp \overrightarrow{ME}$

Find : m (∠ DMC)



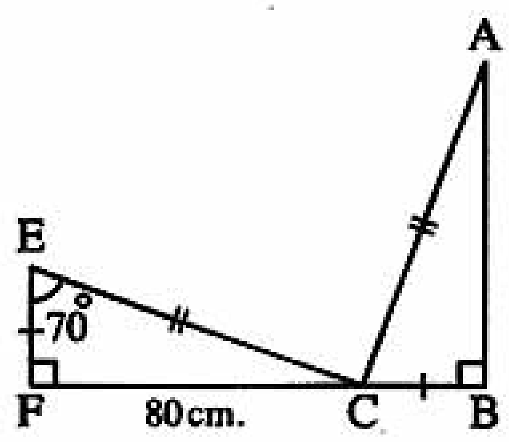
[b] In the opposite figure :

CB = FE , AC = EC

, m (∠ B) = m (∠ F) = 90°

, m (∠ E) = 70° and FC = 80 cm.

Find : m (∠ A) and the length of $\overline{AB}$



4 [a] Draw the angle ABC where m (∠ B) = 130° , using the ruler and the compasses bisect ∠ B

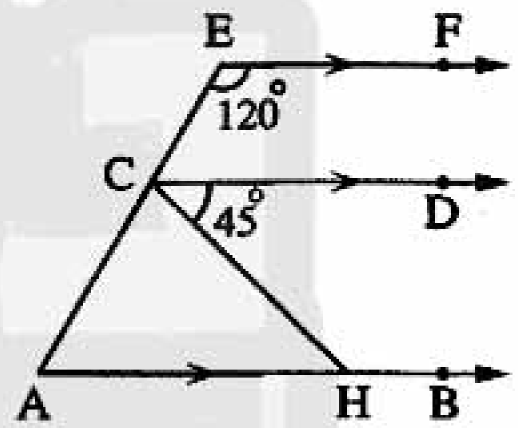
[b] In the opposite figure :

$\overrightarrow{EF} \parallel \overrightarrow{CD} \parallel \overrightarrow{AB}$

, m (∠ CEF) = 120°

, m (∠ HCD) = 45°

Find : The measures of the angles of Δ AHC



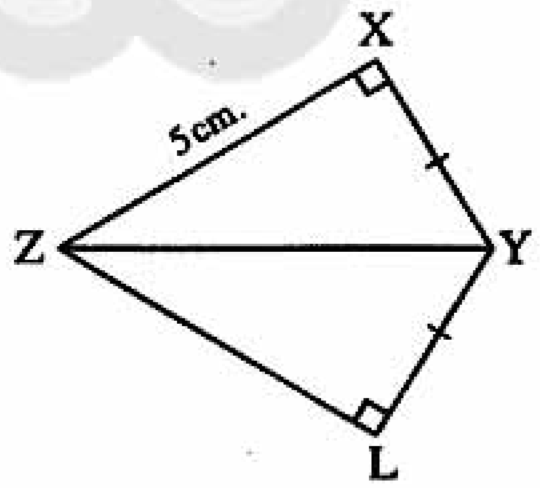
5 [a] In the opposite figure :

m (∠ ZXY) = m (∠ ZLY) = 90°

, XY = LY and ZX = 5 cm.

1 Is Δ YXZ ≡ Δ YLZ ? Why ?

2 Find : The length of $\overline{ZL}$



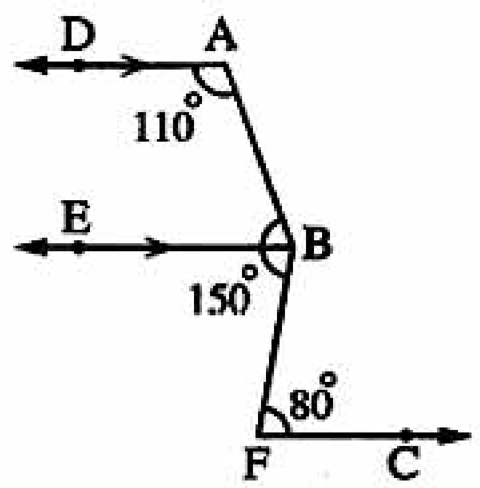
[b] In the opposite figure :

$\overrightarrow{AD} \parallel \overrightarrow{BE}$

, m (∠ F) = 80°

, m (∠ A) = 110° and m (∠ ABF) = 150°

Is $\overrightarrow{BE} \parallel \overrightarrow{FC}$? (Give reason)



11 El-Dakahlia Governorate

Talkha Educational Directorate
A.M.D.L School

Answer the following questions :

1 Choose the correct answer :

1 The sum of measures of the accumulative angles at a point is ....

(a) 180° (b) 90° (c) 360° (d) 60°

2 The acute angle supplements .... angle.

(a) an acute (b) an obtuse (c) a right (d) a reflex

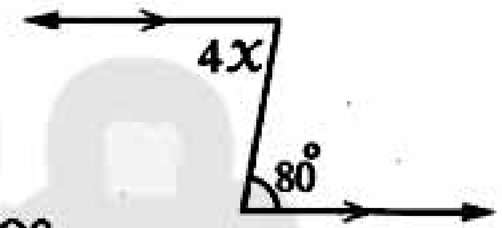
3 The two straight lines parallel to a third straight line are ....

(a) intersecting. (b) congruent. (c) parallel. (d) perpendicular.

4 If $\Delta ABC \equiv \Delta DEF$, $m(\angle A) + m(\angle B) = 110°$, then $m(\angle F) =$ ....

(a) 180° (b) 110° (c) 80° (d) 70°

5 **In the opposite figure :**



$x =$ ....

(a) 80° (b) 100° (c) 20° (d) 40°

6 $\overrightarrow{AB} \cup \overrightarrow{AC} =$ ....

(a) $\overrightarrow{AB}$ (b) $\angle ABC$ (c) $\angle BAC$ (d) $\varnothing$

2 Complete the following :

1 The complement of an angle of measure 75° is an angle of measure ....

2 If $m(\angle A) = 160°$, then $m(\text{reflex } \angle A) =$ ....°

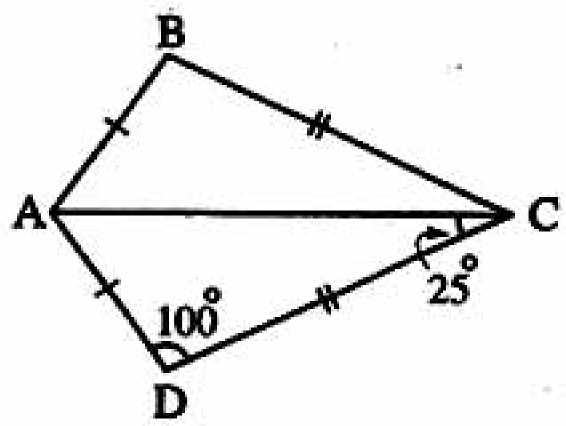
3 If two straight lines intersect , then the measures of each two vertically opposite angles are ....

4 If $\overline{AB} \equiv \overline{XY}$, then $AB - XY =$ ....

5 If $\angle A$ supplements $\angle B$ and $\angle A \equiv \angle B$, then $m(\angle B) =$ ....°

3 [a] State any two cases of congruency of two triangles.

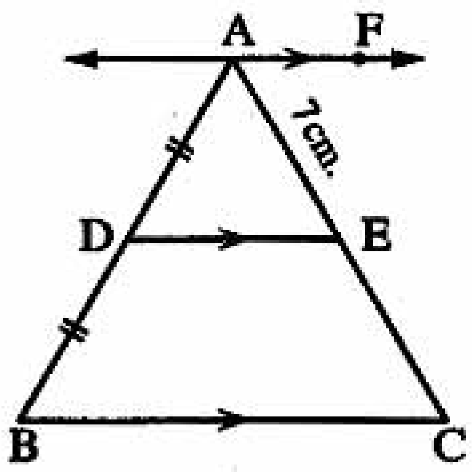
[b] From the opposite figure :



1 **Prove that :** $\Delta ABC \equiv \Delta ADC$

2 **Find :** $m(\angle BAC)$

4 **[a] In the opposite figure :**



$\overleftrightarrow{AF} \parallel \overline{DE} \parallel \overline{BC}$

, D is the midpoint of $\overline{AB}$

, AE = 7 cm.

Find : AC

[b] Using the geometric instruments , draw Δ ABC in which BC = 6 cm. , AB = AC = 5 cm.

, then draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$, **Find by measuring :** AD

(Don't remove the arcs)

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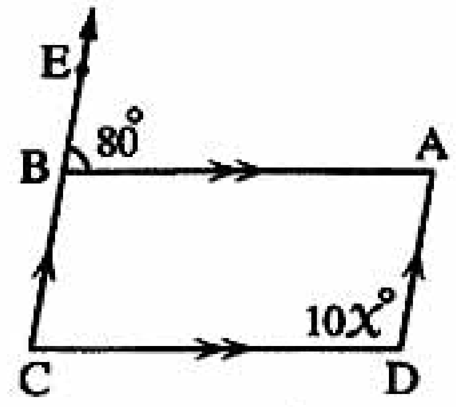
5 **[a] In the opposite figure :**



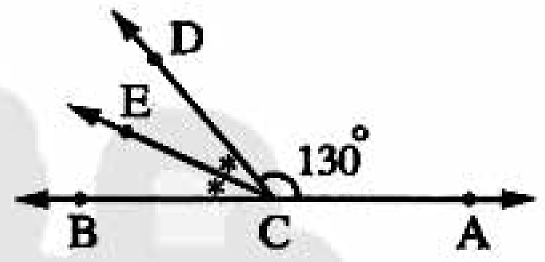
$\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$

, E $\in \overleftrightarrow{BC}$, m ($\angle$ D) = 10 x°

, m ($\angle$ ABE) = 80°

Find : The value of x

[b] In the opposite figure :



C $\in \overleftrightarrow{AB}$, m ($\angle$ ACD) = 130° , $\overrightarrow{CE}$ bisects $\angle$ BCD

Find : m ($\angle$ DCE)

12 Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The angle of measure 60° supplements an angle of measure ....°

   (a) 40 (b) 30 (c) 120 (d) 90

2. If two straight lines are perpendicular to a third , then the two straight lines are ....

   (a) perpendicular. (b) intersecting. (c) parallel. (d) congruent.

3. If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ A) + m ($\angle$ B) = 140° , then m ($\angle$ Z) = ...°

   (a) 60 (b) 40 (c) 80 (d) 140

4. The number of axes of symmetry of the square equals ....

   (a) 1 (b) 2 (c) 3 (d) 4

5. If a straight line cuts two parallel lines , then each two corresponding angles are ....

   (a) equal in measure. (b) complementary.

   (c) supplementary. (d) right.

6 If m (∠ A) = 100° , then m (reflex ∠ A) = ...°

(a) 80 (b) 260 (c) 50 (d) 100

2 Complete the following :

1 If two adjacent angles are complementary , then their outer sides are ....

2 If Δ ABC ≡ Δ XYZ , then AC = ...

3 If ∠ C ≡ ∠ D , m (∠ C) = 90° , then m (∠ D) = ...°

4 The measure of the straight angle equals ....°

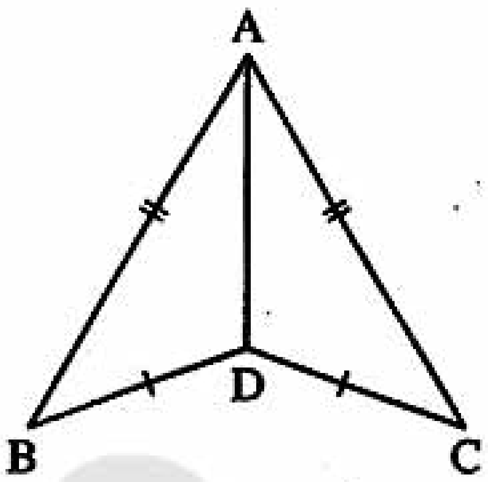
5 The perimeter of a square is 40 cm. , then its side length is .... cm.

3 [a] In the opposite figure :

AC = AB

, DC = DB

Is Δ ADB ≡ Δ ADC ? Why ?



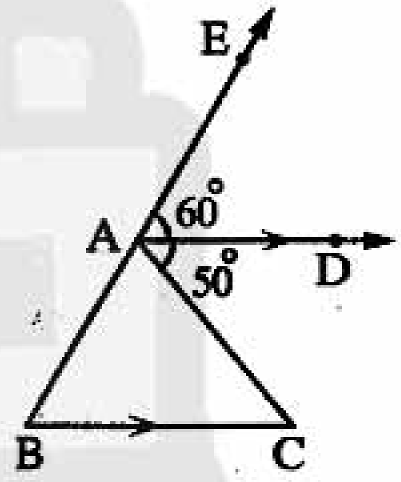
[b] In the opposite figure :

$\overrightarrow{AD}$ // $\overline{BC}$

, m (∠ EAD) = 60°

, m (∠ CAD) = 50°

Find : 1 m (∠ C) 2 m (∠ B) 3 m (∠ BAC)



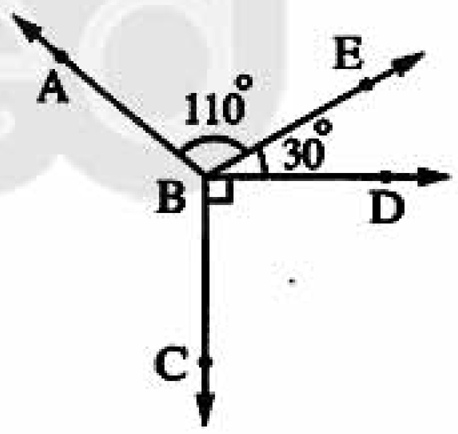
4 [a] In the opposite figure :

m (∠ DBE) = 30°

, ∠ CBD is a right angle

, m (∠ EBA) = 110°

Find : m (∠ ABC)



[b] Draw $\overline{AB}$ of length 6 cm. and bisect it. (Don't remove the arcs)

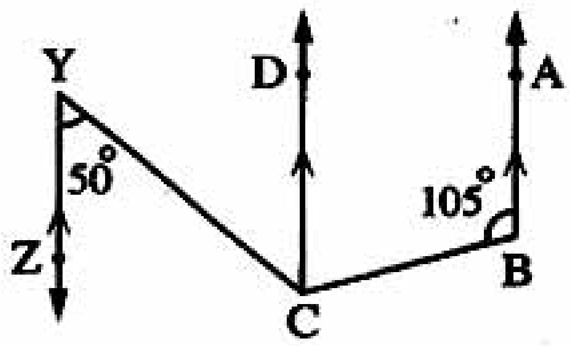
5 [a] In the opposite figure :

$\overrightarrow{BA}$ // $\overrightarrow{CD}$ // $\overrightarrow{YZ}$

, m (∠ ABC) = 105°

, m (∠ ZYC) = 50°

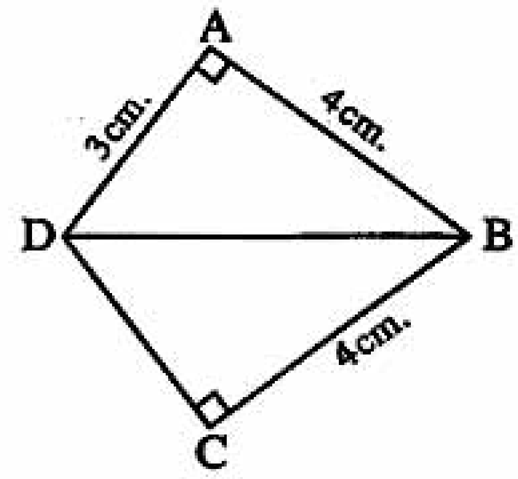
Find : 1 m (∠ YCD) 2 m (∠ BCD) 3 m (∠ BCY)



[b] In the opposite figure :

AB = BC = 4 cm. , AD = 3 cm.

, m (∠ A) = m (∠ C) = 90°



[1] Is Δ ABD ≡ Δ CBD ? Why ?

[2] **Find :** The length of $\overline{CD}$

13 Damietta Governorate

Damietta Inspection of Mathematics
Official Language Schools

Answer the following questions :

1 Choose the correct answer :

[1] If ∠ X supplements ∠ Y and ∠ X ≡ ∠ Y , then m (∠ X) = ...°

(a) 45 (b) 90 (c) 180 (d) 360

[2] If Δ ABC ≡ Δ XYZ , then ....

(a) AB = YZ (b) BC = XZ (c) YX = CA (d) ZY = CB

[3] The centimeter cube is a unit for measuring the ....

(a) perimeter. (b) area. (c) volume. (d) length.

[4] Two straight lines are perpendicular to a third line , then the two straight lines are ....

(a) perpendicular. (b) parallel. (c) congruent. (d) intersecting.

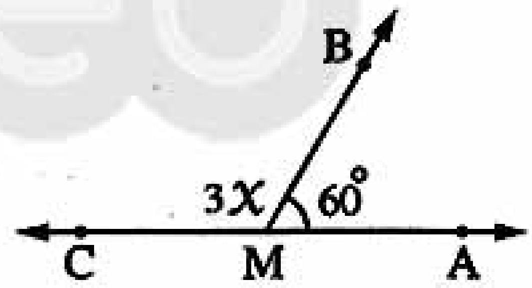
[5] $\overline{XY}$ .... $\overleftrightarrow{XY}$

(a) $\notin$ (b) $\in$ (c) $\subset$ (d) $\not\subset$

[6] **In the opposite figure :**

If $\overleftrightarrow{AC} \cap \overrightarrow{MB} = \{M\}$

, then the value of x = ...°



(a) 20 (b) 30 (c) 40 (d) 60

2 Complete each of the following :

[1] If m (∠ A) = 120° , then m (reflex ∠ A) = ...°

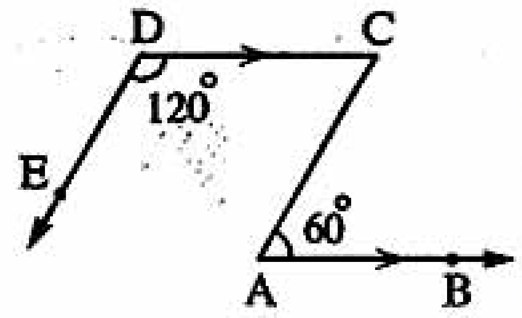
[2] If the perimeter of a square is 20 cm. , then its area equals .... cm^2.

[3] The number of edges of the cuboid is ....

[4] If a straight line cuts two parallel straight lines , then each two alternate angles are ....

[5] If $\overline{AB} \equiv \overline{CD}$, then AB – CD = ...

3 **[a] In the opposite figure :**



$\overrightarrow{AB} \parallel \overline{DC}$

, m ($\angle$ A) = 60°

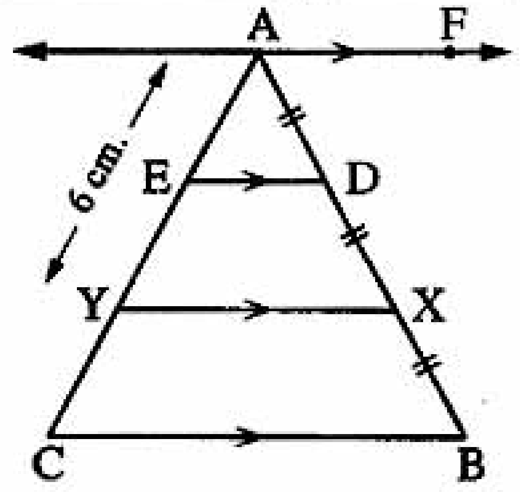
, m ($\angle$ D) = 120°

1 **Find :** m ($\angle$ C) **2** Is $\overline{AC} \parallel \overrightarrow{DE}$? Why ? (Write the steps)

[b] Draw $\angle$ ABC where m ($\angle$ B) = 115° Using the ruler and compasses bisect $\angle$ B by $\overrightarrow{BD}$ (Don't remove the arcs)

---

4 **[a] In the opposite figure :**



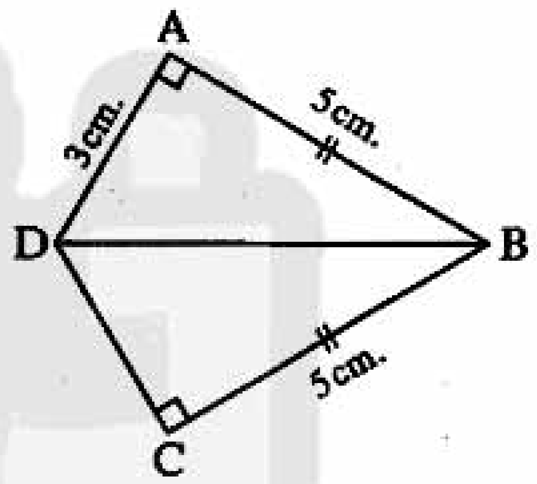
$\overleftrightarrow{AF} \parallel \overline{DE} \parallel \overline{XY} \parallel \overline{BC}$

, AD = DX = XB

, AY = 6 cm.

Find : The length of $\overline{AC}$ (Give the reason)

[b] In the opposite figure :



m ($\angle$ BAD) = m ($\angle$ BCD) = 90°

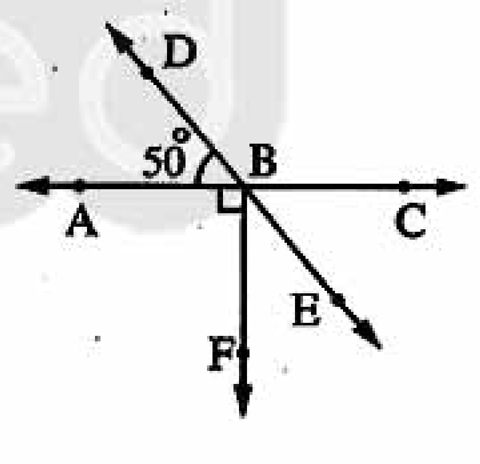
, AB = CB = 5 cm. , AD = 3 cm.

Mention the conditions for Δ ABD , Δ CBD to be congruent

, then find : The length of $\overline{CD}$

---

5 **[a] In the opposite figure :**



$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$

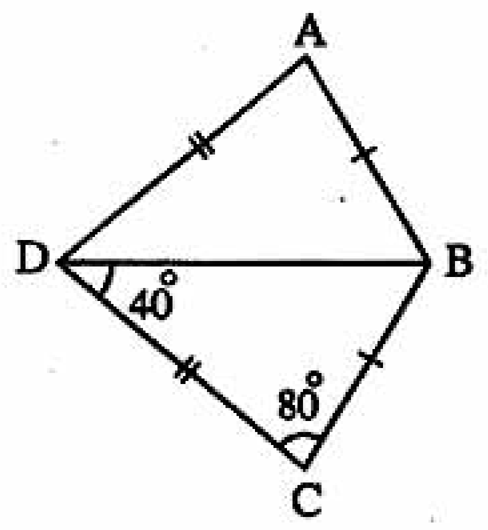
, m ($\angle$ ABD) = 50°

, m ($\angle$ ABF) = 90°

Find showing the steps :

1 m ($\angle$ DBC) **2** m ($\angle$ CBE) **3** m ($\angle$ FBE)

[b] In the opposite figure :



AB = BC , AD = CD

, m ($\angle$ C) = 80°

, m ($\angle$ BDC) = 40°

Is Δ CBD $\equiv$ Δ ABD ? Why ?

and find : m ($\angle$ ABD)

14 Souhag Governorate

Maths Supervision

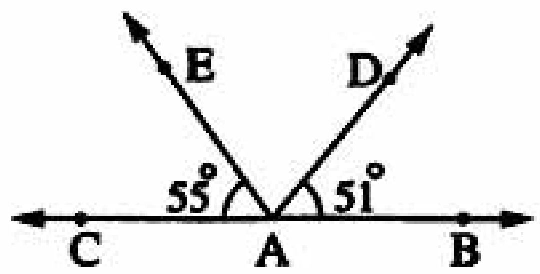
Answer the following questions :

1 Choose the correct answer :

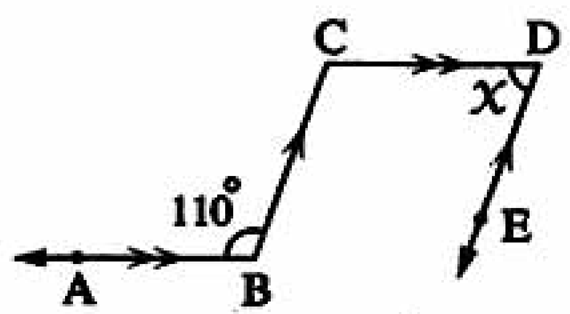
1. If ∠ X ≡ ∠ Y and ∠ X , ∠ Y are supplementary angles , then m (∠ X) = ...
   (a) 45° (b) 90° (c) 135° (d) 180°
2. If two straight lines are perpendicular to a third line , then the two straight lines are ....
   (a) perpendicular. (b) parallel. (c) congruent. (d) intersecting.
3. If Δ XYZ ≡ Δ ABC and m (∠ A) + m (∠ B) = 100° , then m (∠ Z) = ...
   (a) 50° (b) 80° (c) 100° (d) 360°
4. The angle whose measure is more than 90° and less than 180° is ....
   (a) obtuse. (b) acute. (c) right. (d) straight.
5. If m (∠ X) = 2 m (∠ Y) , ∠ X and ∠ Y are two complementary angles , then m (∠ Y) = ...
   (a) 90° (b) 45° (c) 30° (d) 15°
6. The sum of the measures of the accumulative angles at a point is ....
   (a) 45° (b) 90° (c) 180° (d) 360°

2 Complete each of the following :

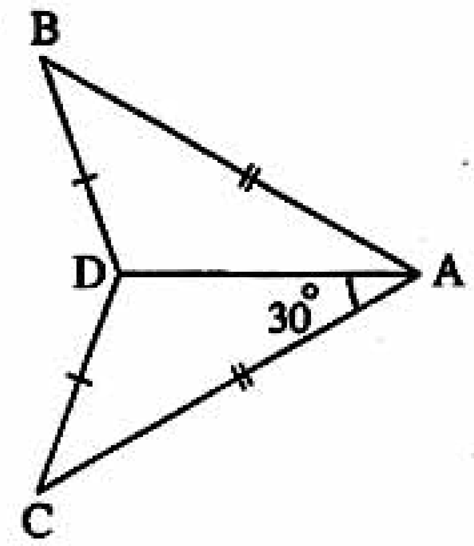
1. If two straight lines intersects , then each two vertically opposite angles are ....
2. If Δ ABC ≡ Δ XYZ , then XZ = ...
3. If ∠ A supplements ∠ B , m (∠ A) = 100° , then m (reflex ∠ B) = ...°
4. **In the opposite figure :**
   $A \in \overleftrightarrow{CB}$
   , then m (∠ DAE) = ...°



5. **In the opposite figure :**
   $\overline{CD} \mathbin{//} \overrightarrow{BA}$
   , $\overrightarrow{DE} \mathbin{//} \overline{CB}$
   , then x = ...°



3 **[a] In the opposite figure :**



AB = AC

, BD = DC

, m (∠ CAD) = 30°

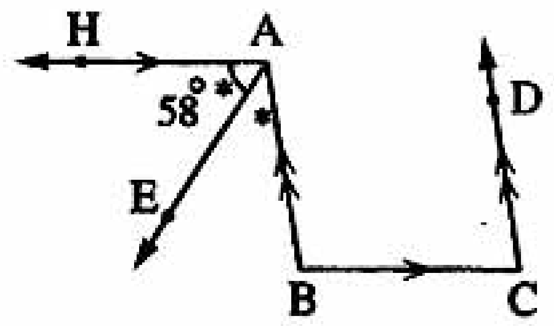
1 **Prove that :** Δ ABD ≡ Δ ACD

2 **Find :** m (∠ CAB)

[b] Using the ruler and the compasses , draw the angle ABC where m (∠ ABC) = 110° and draw $\overrightarrow{BD}$ to bisect the angle. (Don't remove the arcs)

---

4 **[a] In the opposite figure :**

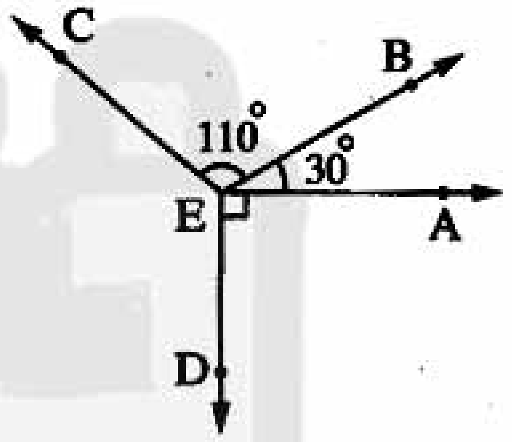


$\overrightarrow{CD}$ // $\overrightarrow{BA}$, $\overrightarrow{CB}$ // $\overrightarrow{AH}$

, $\overrightarrow{AE}$ bisects ∠ BAH , m (∠ EAH) = 58°

Find : m (∠ C)

[b] In the opposite figure :



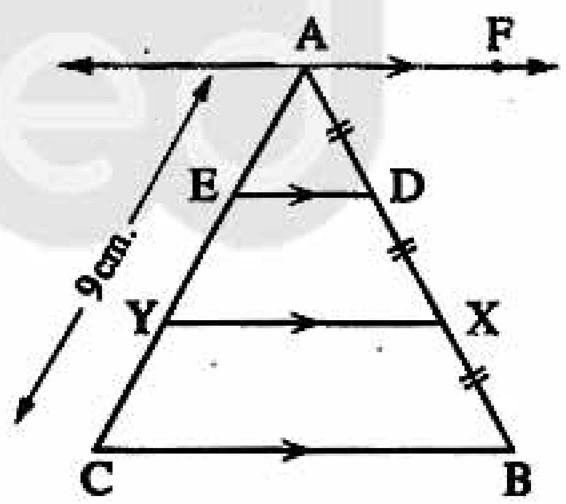
m (∠ AEB) = 30° , m (∠ BEC) = 110°

, m (∠ AED) = 90°

Find : m (∠ DEC)

---

5 **[a] In the opposite figure :**



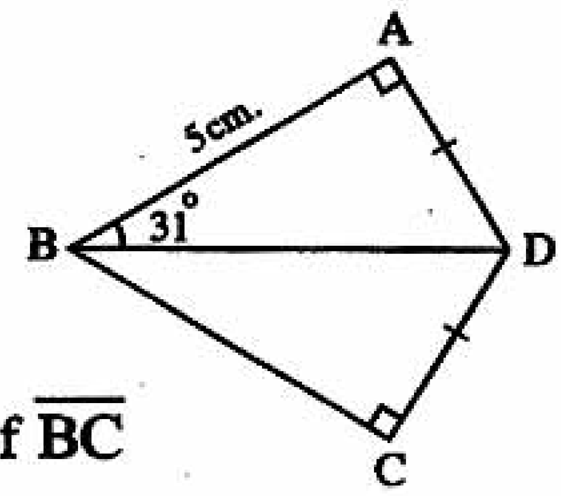
$\overleftrightarrow{AF}$ // $\overline{ED}$ // $\overline{YX}$ // $\overline{CB}$

, AD = DX = XB

, AC = 9 cm.

Find : The length of $\overline{AY}$

[b] In the opposite figure :



m (∠ A) = m (∠ C) = 90° , m (∠ ABD) = 31°

, AB = 5 cm.

, AD = CD

1 **Prove that :** Δ ABD ≡ Δ CBD

2 **Find :** The length of $\overline{BC}$

3 **Find :** m (∠ CBD)

15 Luxor Governorate

Luxor Directorate
El-Salam Language School

Answer the following questions :

1 Choose the correct answer :

1 A square is of side length 7 cm. , then its perimeter = ... cm.

(a) 14 (b) 21 (c) 24 (d) 28

2 The circumference of the circle = ...

(a) 2π (b) $2\pi r$ (c) πr (d) πr^2

3 The sum of measures of the accumulative angles at a point equals ....°

(a) 360 (b) 180 (c) 603 (d) 150

4 If $L_1 // L_3$, $L_2 // L_3$, then ....

(a) $L_1 // L_2$ (b) $L_1 \perp L_2$ (c) $L_2 \perp L_3$ (d) $L_1 \perp L_3$

5 The measure of the supplement of the angle whose measure is 30° equals ....°

(a) 60 (b) 180 (c) 150 (d) 90

6 If $\angle X$ complements $\angle Y$ and $\angle X \equiv \angle Y$, then m $(\angle X)$ = ...°

(a) 45 (b) 90 (c) 180 (d) 360

2 Complete :

1 Two triangles are congruent if two sides and .... of one triangle are congruent to their corresponding parts of the other triangle.

2 If m $(\angle A) = 105°$, then m (reflex $\angle A$) = ...°

3 If $\Delta ABC \equiv \Delta XYZ$, then $\overline{AC} \equiv$ ....

4 If a straight line intersects two parallel lines , then each two corresponding angles are ....

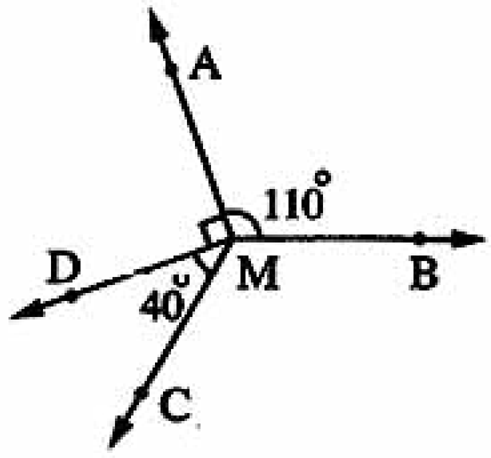
5 In ΔABC , if m $(\angle A) = 50°$, m $(\angle B) = 40°$, then m $(\angle C)$ = ...°

3 [a] In the opposite figure :

m $(\angle AMB) = 110°$, m $(\angle AMD) = 90°$

, m $(\angle DMC) = 40°$

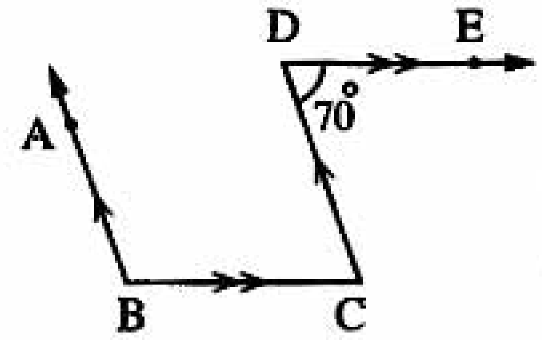
Find : m $(\angle BMC)$ (With steps)



[b] Using the geometric tools , draw ∠ ABC whose measure is 90°
, then draw $\overrightarrow{BF}$ to bisect the angle. (Don't remove the arcs)

---

4 **[a] In the opposite figure :**



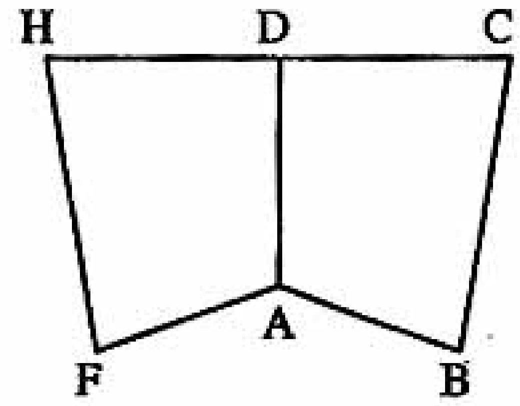
$\overrightarrow{DE}$ // $\overline{BC}$

, $\overline{DC}$ // $\overrightarrow{BA}$

, m (∠ D) = 70°

Find : m (∠ C) , m (∠ B) (Give reason)

[b] In the opposite figure :



The polygon ABCD ≡ the polygon AFHD

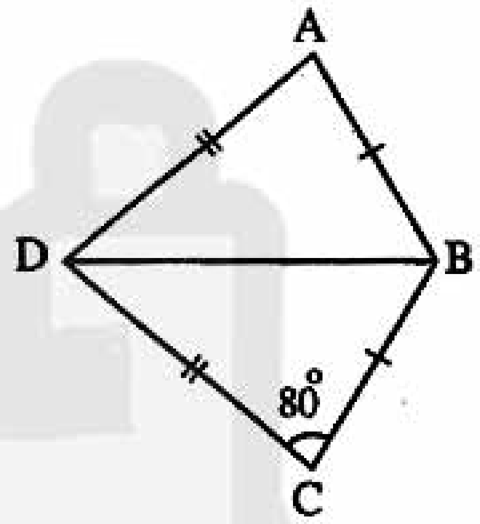
Complete :

1 AB = ... 2 BC = ...

3 m (∠ C) = m (∠ ....) 4 m (∠ F) = m (∠ ....)

---

5 **[a] In the opposite figure :**



AB = BC

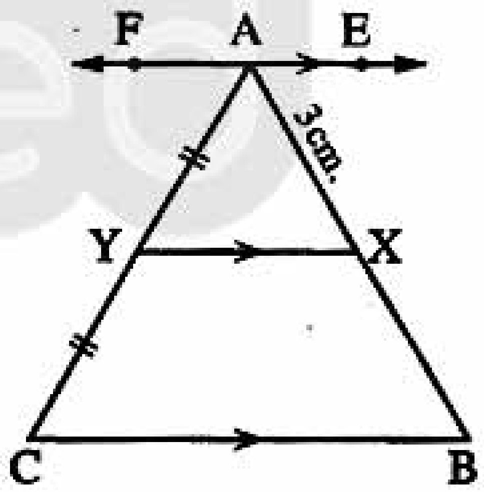
, AD = DC

, m (∠ C) = 80°

1 **Prove that :** Δ ABD ≡ Δ CBD

2 **Find :** m (∠ A)

[b] In the opposite figure :

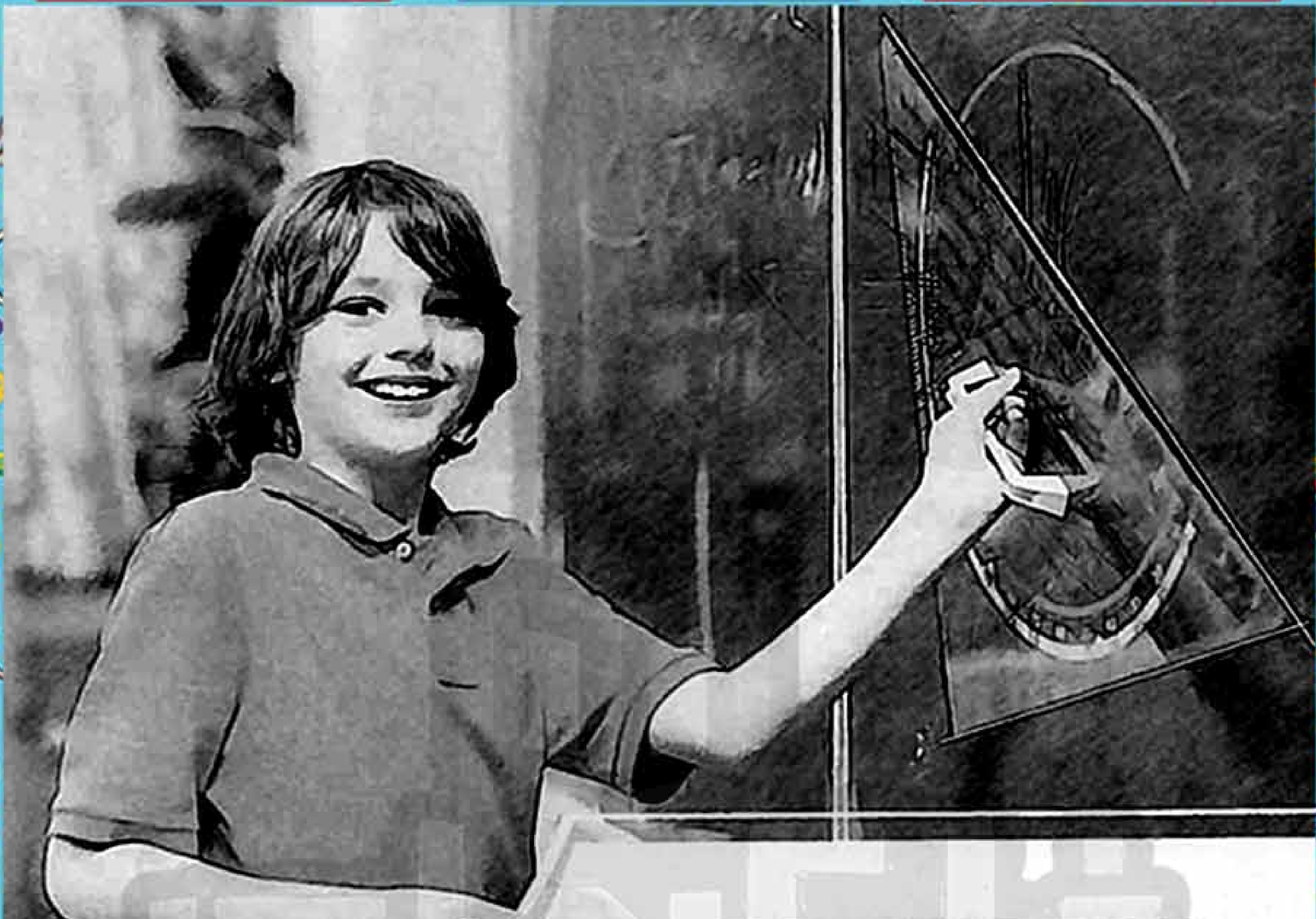


$\overleftrightarrow{AF}$ // $\overline{XY}$ // $\overline{BC}$

, AY = YC

, AX = 3 cm.

Find : The length of $\overline{AB}$ (Give reason)



EL-MOASSER

In Mathematics

For 1st Prep.

First Term

By
A group of supervisors

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Guide Answers

of The Exercises

Answers of unit one

Answers of Exercise 1

1 All numbers are rational numbers except $\frac{4}{5-5}$

2 The integers are : $\frac{15}{5}$, $-\frac{35}{7}$, $-\frac{14}{14}$, $\frac{0}{5}$

3

(1) 12 , 6 (2) 8 , 20

4

(1) $\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$ (2) $-\frac{24}{56} = -\frac{24 \div 8}{56 \div 8} = -\frac{3}{7}$

(3) $\frac{45}{20} = \frac{45 \div 5}{20 \div 5} = \frac{9}{4}$ (4) $-\frac{132}{88} = -\frac{132 \div 44}{88 \div 44} = -\frac{3}{2}$

5 The required rational numbers are : $\frac{7}{20}$, $\frac{5}{8}$, $2\frac{2}{5}$

6

(1) $-\frac{5}{1}$ (2) $\frac{\text{zero}}{1}$ (3) $\frac{75}{100}$ (4) $-\frac{1}{100}$

(5) $\frac{54}{10}$ (6) $\frac{30}{100}$ (7) $\frac{45}{1000}$ (8) $\frac{26}{3}$

There are other solutions

7

(1) $0.1\dot{6} \simeq 16.7\ \%$ (2) $2.5 = 250\ \%$

(3) $-0.15 = -15\%$ (4) $0.\dot{5} \simeq 55.6\ \%$

(5) $7.1875 = 718.75\ \%$ (6) $5.\dot{3} \simeq 533.3\ \%$

8 Because division by zero is meaningless.

9

(1) $\frac{b}{a} = \frac{6}{2} = 3 \in Q$ (2) $-\frac{2}{a} = -\frac{2}{2} = -1 \in Q$

(3) $\frac{0}{a+b} = \frac{0}{2+6} = \frac{0}{8} = 0 \in Q$

(4) $\frac{2b}{a-2} = \frac{2 \times 6}{2-2} = \frac{12}{0} \notin Q$

10

(1) zero (2) 2 (3) zero

(4) 4 (5) 5 (6) 25

(7) 2.1 (8) 40

11

(1) (b) (2) (b) (3) (c)

(4) (d) (5) (d) (6) (c)

(7) (a) (8) (a) (9) (c)

12

Since : $\frac{3}{5} = \frac{9}{15}$, $9 + 15 = 24$

, then the number is : $\frac{9}{15}$

13

(1) $X = 1$ *or* 3 *or* 5 *or* 15 *or* 25 *or* 75

(2) $X =$ zero *or* 2 *or* 4 *or* 14

Answers of Exercise 2

1

The missing numbers from right to left :

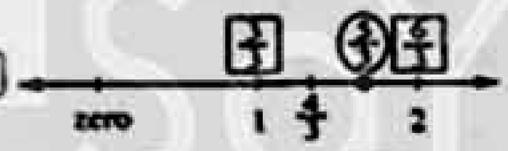
(1) $\frac{4}{3}$, $\frac{1}{3}$, $-\frac{2}{3}$, $-\frac{5}{3}$

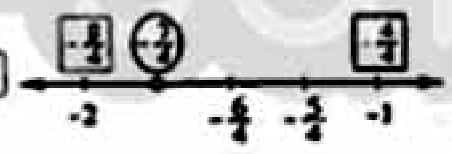
(2) $\frac{5}{6}$, $\frac{4}{6}$, $\frac{2}{6}$, $\frac{1}{6}$, $-\frac{2}{6}$, $-\frac{3}{6}$, $-\frac{4}{6}$

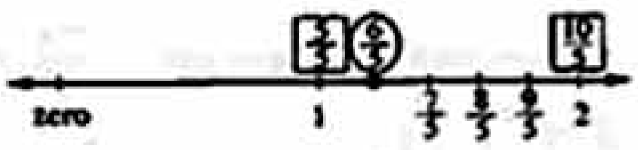
2

(1) 

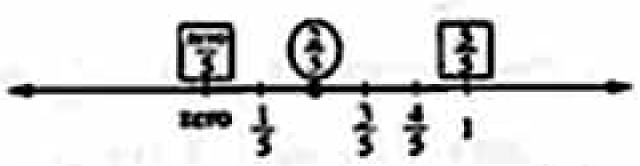
(2) 

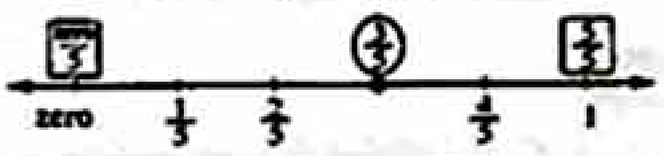
(3) 

(4) 

(5) $1\frac{1}{5} = \frac{6}{5}$ 

(6) $-3\frac{1}{2} = -\frac{7}{2}$ 

(7) $0.4 = \frac{4}{10} = \frac{2}{5}$ 

(8) $|-\frac{3}{5}| = \frac{3}{5}$ 

3

(1) < (2) < (3) >

(4) < (5) > (6) =

4

1 > 2 > 3 >
4 < 5 > 6 =

5

L.C.M. of denominators = 30 , then

$\frac{3}{10}=\frac{9}{30}$, $-\frac{1}{3}=-\frac{10}{30}$, $-\frac{1}{5}=-\frac{6}{30}$, $\frac{4}{15}=\frac{8}{30}$

The descending order is :

$\frac{3}{10}$, $\frac{4}{15}$, $\frac{7}{30}$, $-\frac{1}{5}$ and $-\frac{1}{3}$

6

L.C.M. = 24

$\frac{3}{4}=\frac{18}{24}$, $-\frac{5}{8}=-\frac{15}{24}$, $-\frac{7}{12}=-\frac{14}{24}$, $\frac{2}{3}=\frac{16}{24}$

The ascending order is : $-\frac{5}{8}$, $-\frac{7}{12}$, $\frac{2}{3}$ and $\frac{3}{4}$

7

1 $\frac{1}{2}$ 2 $-\frac{1}{2}$
3 $\frac{3}{16}$ 4 $-\frac{7}{28}=-\frac{1}{4}$

(There are other solutions)

8

1 L.C.M. of the denominators = 10

$\frac{1}{2}=\frac{5}{10}$, $\frac{4}{5}=\frac{8}{10}$

The two numbers are : $\frac{6}{10}$ and $\frac{7}{10}$

2 L.C.M. of the denominators = 12

$-\frac{3}{4}=-\frac{9}{12}$, $-\frac{2}{3}=-\frac{8}{12}$

$-\frac{9}{12}=-\frac{27}{36}$, $-\frac{8}{12}=-\frac{24}{36}$

The two numbers are : $-\frac{25}{36}$ and $-\frac{26}{36}$

3 $0.3=\frac{3}{10}$

L.C.M. of the denominators = 10

$\frac{3}{5}=\frac{6}{10}$

The two numbers are : $\frac{4}{10}$ and $\frac{5}{10}$

(There are other solutions)

9

1 L.C.M. of the denominators = 12

$\frac{1}{2}=\frac{6}{12}$

The numbers are : $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$ and $\frac{10}{12}$

2 L.C.M. of the denominators = 18

$-\frac{4}{9}=-\frac{8}{18}$, $-\frac{5}{6}=-\frac{15}{18}$

The numbers are : $-\frac{9}{18}$, $-\frac{10}{18}$, $-\frac{11}{18}$ and $-\frac{12}{18}$

3 $0=\frac{0}{2}$, $3=\frac{6}{2}$

The numbers are : $\frac{1}{2}$, 1 , $\frac{3}{2}$ and 2

(There are other solutions)

10

The left numbers are : $\frac{6}{15}$, $\frac{7}{15}$, $\frac{8}{15}$ and $\frac{9}{15}$

11

L.C.M. of the denominators = 4

, then $\frac{3}{2}=\frac{6}{4}$, then $\frac{6}{4}=\frac{12}{8}$, $\frac{3}{4}=\frac{6}{8}$

Then the numbers are : $\frac{7}{8}$, 1 , $\frac{9}{8}$ and $\frac{10}{8}$

(There are other solutions)

12

L.C.M. of the numbers : 2 , 3 , 4 and 6 is 12

$\frac{11}{3}=\frac{44}{12}$, $\frac{11}{2}=\frac{66}{12}$

The integers between $\frac{11}{3}$ and $\frac{11}{2}$ are :

$\frac{48}{12}=4$, $\frac{60}{12}=5$ (1)

$\frac{9}{4}=\frac{27}{12}$, $\frac{25}{6}=\frac{50}{12}$

The integers between $\frac{9}{4}$ and $\frac{25}{6}$ are :

$\frac{36}{12}=3$, $\frac{48}{12}=4$ (2)

The required integer = $\frac{48}{12}=4$

Another solution :

The two numbers $\frac{11}{3}$, $\frac{11}{2}$ are $3\frac{2}{3}$, $5\frac{1}{2}$

The included integers between them are 4 and 5

The two numbers $\frac{9}{4}$, $\frac{25}{6}$ are $2\frac{1}{4}$, $4\frac{1}{6}$

and the included integers between them are 3 and 4

The required integer is 4

13

OA = OB

$\frac{x}{6}=-\frac{5}{3}$

$x=-\frac{5\times 6}{3}=-10$

$\frac{x}{6}=-1\frac{2}{3}$

$3x=-5\times 6$

Answers of Unit 1

Answers of Exercise 3

1

1 zero 2 $-\frac{3}{7}$ 3 $\frac{4}{9}$ 4 $-\frac{6}{11}$
5 -1 6 -1 7 $-\frac{4}{5}$ 8 zero

2

1 $\frac{5}{7}$ 2 zero 3 $\frac{4}{8}=\frac{1}{2}$
4 $-\frac{12}{5}$ 5 $\frac{1}{6}$ 6 $\frac{5}{9}+\frac{4}{9}=\frac{9}{9}=1$

3

1 Since L.C.M. of the denominators is 8
therefore $\frac{1}{4}+\frac{25}{8}=\frac{2}{8}+\frac{25}{8}=\frac{27}{8}$

2 Since L.C.M. of the denominators is 15
therefore $\frac{1}{5}-\frac{2}{3}=\frac{3}{15}-\frac{10}{15}=-\frac{7}{15}$

3 $-\frac{9}{12}=-\frac{3}{4}$
Since L.C.M. of the denominators is 16
therefore $-\frac{3}{4}+\frac{3}{16}=-\frac{12}{16}+\frac{3}{16}=-\frac{9}{16}$

4 Since L.C.M. of the denominators is 10
therefore $-\frac{3}{10}+(-\frac{2}{5})=-\frac{3}{10}+(-\frac{4}{10})=-\frac{7}{10}$

5 $-\frac{15}{18}=-\frac{5}{6}$, $\frac{12}{16}=\frac{3}{4}$
Since L.C.M. of the denominators is 12
therefore $-\frac{5}{6}+\frac{3}{4}=-\frac{10}{12}+\frac{9}{12}=-\frac{1}{12}$

6 $\frac{3}{15}=\frac{1}{5}$
$-\frac{2}{5}-\frac{3}{15}=-\frac{2}{5}-\frac{1}{5}=-\frac{3}{5}$

7 Since L.C.M. of the denominators is 35
therefore $\frac{3}{7}-(-\frac{2}{5})=\frac{15}{35}-(-\frac{14}{35})=\frac{15}{35}+\frac{14}{35}$
"From the definition of subtraction operation" $=\frac{29}{35}$

8 Since L.C.M. of the denominators is 12
therefore $-\frac{5}{6}-(-\frac{3}{4})=-\frac{10}{12}-(-\frac{9}{12})=-\frac{10}{12}+\frac{9}{12}$
"From the defnition of subtracition operation" $=-\frac{1}{12}$

9 Since L.C.M. of the denominators is 100
therefore $\frac{19}{10}+(-\frac{39}{100})=\frac{190}{100}+(-\frac{39}{100})=\frac{151}{100}$

4

1 $5\frac{5}{7}$

2 $9\frac{1}{5}=\frac{46}{5}$, $7\frac{3}{5}=\frac{38}{5}$
$9\frac{1}{5}-7\frac{3}{5}=\frac{46}{5}-\frac{38}{5}=\frac{8}{5}=1\frac{3}{5}$

Another solution :

$9\frac{1}{5}=8\frac{6}{5}$
$9\frac{1}{5}-7\frac{3}{5}=8\frac{6}{5}-7\frac{3}{5}=1\frac{3}{5}$

3 $-10\frac{7}{8}+4\frac{5}{8}=-6\frac{2}{8}=-6\frac{1}{4}$
"From the definition of subtraction operation"

4 Since L.C.M. of the denominators is 8
therefore $\frac{1}{4}+2\frac{3}{8}=\frac{2}{8}+2\frac{3}{8}=2\frac{5}{8}$

5 $6\frac{2}{3}=\frac{20}{3}$, $3\frac{1}{6}=\frac{19}{6}$
Since L.C.M. of the denominators is 6
therefore $\frac{20}{3}-\frac{19}{6}=\frac{40}{6}-\frac{19}{6}=\frac{21}{6}=\frac{7}{2}$

6 $-15\frac{1}{2}=-\frac{31}{2}$, $2\frac{3}{8}=\frac{19}{8}$
Since L.C.M. of the denominators is 8
therefore $-\frac{31}{2}+\frac{19}{8}=-\frac{124}{8}+\frac{19}{8}$
$=-\frac{105}{8}$

7 $-2\frac{1}{2}=-\frac{5}{2}$, $-12\frac{1}{16}=-\frac{193}{16}$
Since L.C.M. of the denominators is 16
therefore $-\frac{5}{2}-\frac{193}{16}=-\frac{40}{16}-\frac{193}{16}=-\frac{233}{16}$

8 $2\frac{3}{8}=\frac{19}{8}$
Since L.C.M. of the denominators is 8
therefore $\frac{19}{8}-\frac{1}{4}=\frac{19}{8}-\frac{2}{8}=\frac{17}{8}$

9 $13\frac{3}{7}=\frac{94}{7}$
therefore $-2+13\frac{3}{7}=-\frac{14}{7}+\frac{94}{7}$
$=\frac{80}{7}$

5

1 $0.2=\frac{2}{10}=\frac{1}{5}$ therefore $\frac{2}{5}+0.2=\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$

2 $|-5\frac{1}{2}|=|-\frac{11}{2}|=\frac{11}{2}$
Since L.C.M. of the denominators is 4
therefore $\frac{11}{2}-\frac{1}{4}=\frac{22}{4}-\frac{1}{4}=\frac{21}{4}$

3 $50\% = \frac{1}{2}$

Since L.C.M. of the denominators is 4

therefore $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

4 $25\% = \frac{1}{4}$

therefore $\frac{1}{4} + (-\frac{1}{4}) = \text{zero}$

5 $0.\dot{3} = \frac{1}{3}$

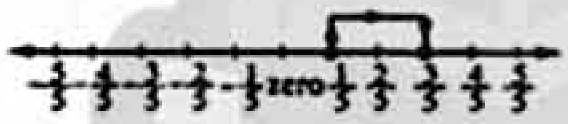
therefore $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

6

1 (c) 2 (a) 3 (c) 4 (c) 5 (b)
6 (c) 7 (b) 8 (d) 9 (c) 10 (b)
11 (c) 12 (b) 13 (c)

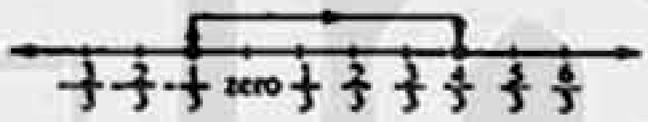
7

1 $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

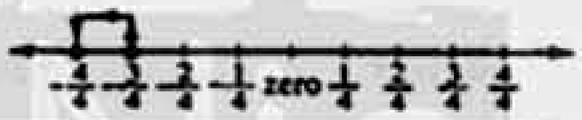


2 $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$

3 $-\frac{1}{3} + \frac{5}{3} = \frac{4}{3}$



4 $-\frac{3}{4} + (-\frac{1}{4}) = -\frac{4}{4} = -1$



8 1 ✗ 2 ✓ 3 ✓ 4 ✗

9

1 Commutative property. 2 Associative property.
3 Additive inverse 4 Additive identity

10

1 $\frac{4}{7} + \text{zero} = \frac{4}{7}$ 2 $\text{zero} + (-\frac{7}{10}) = -\frac{7}{10}$

3 $\text{zero} - (-\frac{17}{4}) = \text{zero} + \frac{17}{4}$

"From the definition of subtraction operation"

$= \frac{17}{4}$

4 $[\frac{1}{4} + (-\frac{1}{4})] + \frac{3}{4} = \text{zero} + \frac{3}{4} = \frac{3}{4}$

5 $\frac{5}{6} + (-\frac{3}{6} + \frac{3}{6}) = \frac{5}{6} + \text{zero} = \frac{5}{6}$

6 $[\frac{2}{9} + (-\frac{4}{9})] + (-\frac{3}{9}) = -\frac{2}{9} + (-\frac{3}{9}) = -\frac{5}{9}$

11

1 $(\frac{1}{4} + \frac{3}{4}) + \frac{1}{2} = 1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

2 $(\frac{2}{7} + \frac{5}{7}) + (\frac{3}{4} + \frac{1}{4}) = \frac{7}{7} + \frac{4}{4} = 2$

3 $(\frac{5}{4} + (-\frac{25}{4})) + (-\frac{13}{5} + \frac{28}{5})$

$= -\frac{20}{4} + \frac{15}{5} = -5 + 3 = -2$

4 $(\frac{5}{8} + \frac{3}{8}) + (-\frac{3}{4} + \frac{3}{4}) = \frac{8}{8} + \text{zero} = 1$

5 $(\frac{2}{13} + \frac{11}{13}) + (\frac{1}{5} + (-\frac{6}{5})) = \frac{13}{13} + (-\frac{5}{5})$

$= 1 + (-1) = \text{zero}$

6 $(-\frac{3}{7} + \frac{1}{2}) + (-\frac{1}{14}) = (-\frac{6}{14} + \frac{7}{14}) + (-\frac{1}{14})$

$= \frac{1}{14} + (-\frac{1}{14}) = \text{zero}$

7 $\frac{12}{18} = \frac{2}{3}$, $-\frac{15}{27} = -\frac{5}{9}$

$\frac{2}{3} + \frac{5}{9} + \frac{1}{3} + (-\frac{5}{9}) = (\frac{2}{3} + \frac{1}{3}) + (\frac{5}{9} + (-\frac{5}{9}))$

$= \frac{3}{3} + \text{zero} = 1$

8 $[\frac{2}{3} + \frac{4}{5}] + \frac{3}{4} = [\frac{10}{15} + \frac{12}{15}] + \frac{3}{4}$

$= \frac{22}{15} + \frac{3}{4} = \frac{88}{60} + \frac{45}{60} = \frac{133}{60}$

9 $\frac{1}{4} + 7 + (-\frac{1}{4}) + (-11)$

$= (\frac{1}{4} + (-\frac{1}{4})) + (7 + (-11)) = \text{zero} + (-4) = -4$

10 $-\frac{1}{8} - 13 + \frac{3}{8} + 7 = (-\frac{1}{8} + \frac{3}{8}) + (-13 + 7)$

$= \frac{2}{8} + (-6) = \frac{1}{4} + (-\frac{24}{4})$

$= -\frac{23}{4}$

12

1 $\frac{5}{6} + \frac{1}{2} = \frac{5}{6} + \frac{3}{6} = \frac{8}{6} = \frac{4}{3}$

2 $\frac{5}{6} + (-\frac{1}{3}) = \frac{5}{6} + (-\frac{2}{6}) = \frac{3}{6} = \frac{1}{2}$

3 $\frac{5}{6} - (-\frac{1}{3}) = \frac{5}{6} + \frac{1}{3}$

"From the definition of subtraction operation"

$= \frac{5}{6} + \frac{2}{6} = \frac{7}{6}$

4 $(-\frac{1}{3} + \frac{1}{2}) - \frac{5}{6} = (-\frac{2}{6} + \frac{3}{6}) - \frac{5}{6}$

$= \frac{1}{6} - \frac{5}{6} = -\frac{4}{6} = -\frac{2}{3}$

13 $[\frac{1}{2} - (-\frac{3}{2})]^3 = (\frac{1}{2} + \frac{3}{2})^3$

"From the definition of subtraction operation"

$= (\frac{4}{2})^3 = (2)^3 = 8$

Answers of Unit 1

14 1) 3 2) $-\frac{7}{16}$

15 1) $\frac{31}{32}$, $\frac{63}{64}$ 2) $3\frac{3}{4}$, 3 , $2\frac{1}{4}$, $1\frac{1}{2}$

16

$$AB = 8\frac{2}{3} - (2\frac{1}{4} + 2\frac{1}{4} + 1\frac{1}{2})$$
$$= 8\frac{2}{3} - (2\frac{1}{4} + 2\frac{1}{4} + 1\frac{2}{4})$$
$$= 8\frac{2}{3} - 6 = 2\frac{2}{3} \text{ cm.}$$

17

1) $x + \frac{1}{5} = \frac{2}{5}$ therfore $x = \frac{1}{5}$
or $x + \frac{1}{5} = -\frac{2}{5}$ therefore $x = -\frac{3}{5}$

2) $\frac{3}{4} - x = \frac{1}{4}$ therefore $x = \frac{1}{2}$
or $\frac{3}{4} - x = -\frac{1}{4}$ therefore $x = 1$

18

Since $(51\frac{1}{2} - 1\frac{1}{2}) = 50$

and $(52\frac{1}{2} - 2\frac{1}{2}) = 50$ .... and so on

Since the expression consists of 50 operation of subtraction , the result of each of them = 50

therefore the expression = $50 \times 50 = 2500$

Answers of Exercise 4

1

1) 1 2) $\frac{7}{3}$ 3) $-\frac{9}{4}$ 4) $-\frac{1}{6}$
5) $\frac{2}{7}$ 6) 2 7) 1 8) -1
9) 1 10) $\frac{5}{3}$ 11) 1 12) zero

2

1) $\frac{2}{3}$ 2) 1 3) 1 4) 1
5) $-\frac{11}{4}$ 6) $\frac{5}{13}$ 7) $\frac{5}{4}$ 8) $-\frac{5}{4}$
9) $\frac{1}{3}$ 10) 4 11) 1 12) 35

3

1) ✗ 2) ✗ 3) ✗ 4) ✗ 5) ✓ 6) ✓

4

1) commutative
2) multiplicative inverse
3) commutative
4) multiplicative identity
5) multiplying by zero

5

1) $\frac{6}{35}$ 2) $-\frac{1}{3}$ 3) $\frac{5}{8}$
4) $-\frac{1}{4}$ 5) $-\frac{5}{12}$ 6) $-\frac{12}{35}$
7) $-\frac{4}{7}$ 8) 6 9) $\frac{1}{3}$

6

1) $\frac{4}{5} \times \frac{7}{3} = \frac{28}{15}$
2) $-\frac{1}{6} \times \frac{2}{5} = -\frac{1}{15}$
3) $-\frac{4}{11} \times (-\frac{11}{4}) = 1$
4) $\frac{5}{27} \times 9 = \frac{5}{3}$
5) $\frac{5}{6} \times (-\frac{2}{15}) = -\frac{1}{9}$
6) $-\frac{5}{16} \times (-\frac{8}{11}) = \frac{5}{22}$
7) $-\frac{5}{8} \times \frac{8}{5} = -1$
8) zero $\times \frac{5}{3}$ = zero
9) $\frac{3}{4} \times (-\frac{1}{9}) = -\frac{1}{12}$

7

1) $\frac{7}{2} \times (-4) = -14$
2) $\frac{3}{2} \times (-\frac{3}{2}) = -\frac{9}{4}$
3) $-\frac{30}{7} \times (-\frac{31}{6}) = \frac{155}{7} = 22\frac{1}{7}$
4) $\frac{25}{8} \times (-\frac{21}{5}) = -\frac{105}{8} = -13\frac{1}{8}$
5) $-\frac{5}{10} \times \frac{2}{5} = -\frac{1}{5}$
6) $\frac{5}{2} \times \frac{8}{10} = 2$
7) $\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}$
8) $|-\frac{2}{3}| \times \frac{4}{3} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

8

1) $-\frac{11}{5} \times \frac{5}{11} = -1$
2) $-\frac{47}{6} \times \frac{100}{47} = -\frac{50}{3}$
3) $-\frac{30}{7} \div \frac{15}{14} = -\frac{30}{7} \times \frac{14}{15} = -4$
4) $-1 \div \frac{9}{4} = -1 \times \frac{4}{9} = -\frac{4}{9}$
5) $-\frac{13}{3} \div (-\frac{13}{4}) = -\frac{13}{3} \times -\frac{4}{13} = \frac{4}{3}$
6) $\frac{5}{10} \div \frac{11}{2} = \frac{5}{10} \times \frac{2}{11} = \frac{1}{11}$
7) $-\frac{11}{4} \div (-\frac{25}{8}) = -\frac{11}{4} \times (-\frac{8}{25}) = \frac{22}{25}$
8) $\frac{25}{4} \times (-\frac{1}{15}) = -\frac{5}{12}$
9) $\frac{13}{5} \div (-\frac{26}{15}) = \frac{13}{5} \times (-\frac{15}{26}) = -\frac{3}{2}$

9

1. $\frac{5}{12}(3+9)=\frac{5}{12}\times 12=5$
2. $\frac{4}{9}(11+16)=\frac{4}{9}\times 27=12$
3. $(4+9+4)\times\frac{8}{17}=17\times\frac{8}{17}=8$
4. $\frac{6}{37}(7+5-11)=\frac{6}{37}\times 1=\frac{6}{37}$
5. $\frac{4}{5}(13-22+9)=\frac{4}{5}\times \text{zero}=\text{zero}$
6. $\frac{7}{12}(5+9-2)=\frac{7}{12}\times 12=7$
7. $\frac{7}{13}(6+8-1)=\frac{7}{13}\times 13=7$
8. $\frac{27}{11}\left(\frac{9}{4}-\frac{1}{4}+9\right)=\frac{27}{11}\times 11=27$
9. $-\frac{3}{7}(8+5+1)=-\frac{3}{7}\times 14=-6$
10. $\frac{5}{2}\left(\frac{13}{11}-\frac{2}{11}+1\right)=\frac{5}{2}\times 2=5$
11. $\frac{22}{25}\left(\frac{7}{11}+\frac{5}{11}-1\right)=\frac{22}{25}\times\frac{1}{11}=\frac{2}{25}$
12. $35\left(\frac{3}{4}+\frac{1}{2}-\frac{1}{4}\right)=35\times 1=35$
13. $\frac{7}{15}\left(\frac{4}{25}+\frac{1}{5}\right)+\frac{16}{25}\left(\frac{2}{3}-\frac{1}{5}\right)$
   $=\frac{7}{15}\times\frac{9}{25}+\frac{16}{25}\times\frac{7}{15}$
   $=\frac{7}{15}\left(\frac{9}{25}+\frac{16}{25}\right)=\frac{7}{15}\times 1=\frac{7}{15}$

10

1. $\frac{8}{8}\times\frac{8}{5}=\frac{8}{5}$
2. $\frac{3}{4}\times\left(\frac{3}{6}-\frac{2}{6}\right)=\frac{3}{4}\times\frac{1}{6}=\frac{1}{8}$
3. $\left(-\frac{18}{5}\times\frac{35}{9}\right)\times\left(-\frac{3}{7}\right)=-14\times\left(-\frac{3}{7}\right)=6$
4. $\frac{12}{35}\times\left(-\frac{14}{9}\right)=-\frac{8}{15}$
5. $\left(-\frac{5}{3}\times\frac{14}{3}\right)\div\frac{55}{9}=-\frac{70}{9}\times\frac{9}{55}=-\frac{14}{11}$
6. $\left(\frac{81}{16}\div\frac{27}{4}\right)\times\left(-\frac{68}{9}\right)$
   $=\left(\frac{81}{16}\times\frac{4}{27}\right)\times\left(-\frac{68}{9}\right)=\frac{3}{4}\times\left(-\frac{68}{9}\right)=-\frac{17}{3}$
7. $-\frac{5}{2}\div\left(\frac{9}{12}+\frac{6}{12}-\frac{4}{12}\right)=-\frac{5}{2}\div\frac{11}{12}$
   $=-\frac{5}{2}\times\frac{12}{11}=-\frac{30}{11}$
8. $\left(\frac{12}{5}\div\frac{3}{4}\right)\left(-\frac{4}{3}\times\frac{1}{2}\right)=\left(\frac{12}{5}\times\frac{4}{3}\right)\times\left(-\frac{2}{3}\right)$
   $=\frac{16}{5}\times\left(-\frac{2}{3}\right)=-\frac{32}{15}$

11

1. 1
2. $\frac{3}{17}$
3. zero
4. 1
5. 5

12

$(a-b)\div c=\left(2-\frac{1}{2}\right)\div\frac{3}{2}=\left(\frac{4}{2}-\frac{1}{2}\right)\div\frac{3}{2}=\frac{3}{2}\times\frac{2}{3}=1$

13

1. $xyz=-\frac{1}{3}\times\frac{3}{4}\times -3=\frac{3}{4}$
2. $xy+yz=-\frac{1}{3}\times\frac{3}{4}+\frac{3}{4}\times -3$
   $=-\frac{1}{4}-\frac{9}{4}=-\frac{10}{4}=-\frac{5}{2}$

14

1. $abc+3=\frac{7}{4}\times\frac{12}{7}\times\frac{2}{3}+3=2+3=5$
2. $ab-c=\frac{7}{4}\times\frac{12}{7}-\frac{2}{3}=3-\frac{2}{3}=\frac{9}{3}-\frac{2}{3}=\frac{7}{3}$

15

$a-b=\frac{3}{4}-\left(-\frac{5}{2}\right)=\frac{3}{4}+\frac{10}{4}=\frac{13}{4}$

$a+b=\frac{3}{4}+\left(-\frac{5}{2}\right)=\frac{3}{4}-\frac{10}{4}=-\frac{7}{4}$

$\frac{a-b}{a+b}=\frac{13}{4}\div\left(-\frac{7}{4}\right)=\frac{13}{4}\times\left(-\frac{4}{7}\right)=-\frac{13}{7}$

16

$(b-a)(b-c)=\left(\frac{1}{2}-\frac{1}{3}\right)\left(\frac{1}{2}-(-2)\right)$

$=\left(\frac{3}{6}-\frac{2}{6}\right)\left(\frac{1}{2}+\frac{4}{2}\right)=\frac{1}{6}\times\frac{5}{2}=\frac{5}{12}$

17

1. $xyz=\frac{3}{2}\times\left(-\frac{1}{4}\right)\times(-2)=\frac{3}{4}$
   $\frac{1}{xyz}=1\div\frac{3}{4}=1\times\frac{4}{3}=\frac{4}{3}$
2. $x-(z\div y)=\frac{3}{2}-\left(-2\div\left(-\frac{1}{4}\right)\right)$
   $=\frac{3}{2}-\left(-2\times\left(-\frac{4}{1}\right)\right)=\frac{3}{2}-8$
   $=\frac{3}{2}-\frac{16}{2}$
   $=-\frac{13}{2}$
3. $\frac{x}{y}=\frac{3}{2}\div\left(-\frac{1}{4}\right)=\frac{3}{2}\times\left(-\frac{4}{1}\right)=-6$
   $\frac{z}{y}=-2\div\left(-\frac{1}{4}\right)=-2\times\left(-\frac{4}{1}\right)=8$
   $\frac{x}{y}-\frac{z}{y}=-6-8=-14$

4 $x + z = \frac{3}{2} + (-2) = \frac{3}{2} - \frac{4}{2} = -\frac{1}{2}$

$y - z = -\frac{1}{4} - (-2) = -\frac{1}{4} + \frac{8}{4} = \frac{7}{4}$

$(x + z) \div (y - z) = -\frac{1}{2} \div \frac{7}{4} = -\frac{1}{2} \times \frac{4}{7} = -\frac{2}{7}$

5 $x + y = \frac{3}{2} + (-\frac{1}{4}) = \frac{6}{4} - \frac{1}{4} = \frac{5}{4}$

$\frac{x+y}{z} = \frac{5}{4} \div (-2) = \frac{5}{4} \times (-\frac{1}{2}) = -\frac{5}{8}$

18

The weight of the man on the moon $= \frac{1}{6} \times 76\frac{4}{5}$

$= \frac{1}{6} \times \frac{384}{5} = \frac{64}{5}$

$= 12\frac{4}{5}$ kg.

19

The capacity of three containers $= 3 \times 20 = 60$ litres
the number of minutes needed to fill the 3 containers

$= 60 \div 2\frac{1}{2} = 60 \div \frac{5}{2} = 60 \times \frac{2}{5} = 24$ minutes

20

The number of pieces

$= 60 \div 3\frac{3}{4} = 60 \div \frac{15}{4} = 60 \times \frac{4}{15} = 16$ pieces

There is not any wire left over.

21

$(\frac{14}{35} - \frac{5}{35}) \div (\frac{4}{35} + \frac{5}{35}) = \frac{9}{35} \div \frac{9}{35} = \frac{9}{35} \times \frac{35}{9} = 1$

the required rational number $= 2 + 1 = 3$

22

$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} \times \ldots\ldots \times \frac{99}{100} = \frac{1}{100}$

and if the last rational number is $\frac{n-1}{n}$

the result will be $\frac{1}{n}$

Answers of Exercise 5

1 Let the required number be L :

1 $L = \frac{3}{8} + \frac{1}{2}|\frac{5}{8} - \frac{3}{8}| = \frac{3}{8} + \frac{1}{2} \times \frac{2}{8}$

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

2 $L = \frac{2}{5} + \frac{1}{2}|\frac{4}{5} - \frac{2}{5}| = \frac{2}{5} + \frac{1}{2} \times \frac{2}{5}$

$= \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

3 $L = \frac{3}{4} - \frac{1}{2}|\frac{3}{4} - (-\frac{3}{4})| = \frac{3}{4} - \frac{1}{2}|\frac{3}{4} + \frac{3}{4}|$

$= \frac{3}{4} - \frac{1}{2} \times \frac{6}{4} = \frac{3}{4} - \frac{3}{4} = \text{zero}$

4 The distance between the two numbers

$= |\frac{1}{2} - \frac{7}{8}| = |\frac{4}{8} - \frac{7}{8}| = |-\frac{3}{8}| = \frac{3}{8}$

Then $L = \frac{4}{8} + \frac{1}{2} \times \frac{3}{8} = \frac{4}{8} + \frac{3}{16} = \frac{8+3}{16} = \frac{11}{16}$

5 The distance between the two numbers

$= |-\frac{1}{2} - (-\frac{3}{4})| = |-\frac{2}{4} + \frac{3}{4}| = |\frac{1}{4}| = \frac{1}{4}$

Then $L = -\frac{2}{4} - \frac{1}{2} \times \frac{1}{4} = -\frac{2}{4} - \frac{1}{8} = \frac{-4-1}{8} = -\frac{5}{8}$

6 The distance between the two numbers

$= |0.1 - (-\frac{2}{5})| = |\frac{1}{10} + \frac{4}{10}| = |\frac{5}{10}| = \frac{1}{2}$

Then $L = \frac{1}{10} - \frac{1}{2} \times \frac{1}{2} = \frac{1}{10} - \frac{1}{4} = \frac{2-5}{20} = -\frac{3}{20}$

7 The distance between the two numbers

$= |-\frac{11}{9} - (-\frac{13}{35})| = |-\frac{385}{315} + \frac{117}{315}| = |-\frac{268}{315}| = \frac{268}{315}$

Then $L = -\frac{385}{315} + \frac{1}{2} \times \frac{268}{315} = -\frac{385}{315} + \frac{134}{315} = -\frac{251}{315}$

8 The distance between the two numbers

$= |-4\frac{3}{7} - 8\frac{1}{3}| = |-\frac{31}{7} - \frac{25}{3}| = |-\frac{93}{21} - \frac{175}{21}|$

$= |-\frac{268}{21}| = \frac{268}{21}$

Then $L = -\frac{93}{21} + \frac{1}{2} \times \frac{268}{21} = -\frac{93}{21} + \frac{134}{21} = \frac{41}{21}$

9 $L = \text{zero} + \frac{1}{2}|\frac{2}{5} - \text{zero}| = \text{zero} + \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

2

1 The distance between the two numbers

$= |\frac{5}{7} - (-\frac{3}{7})| = |\frac{5}{7} + \frac{3}{7}| = \frac{8}{7}$

Then the number $= -\frac{3}{7} + \frac{1}{4} \times \frac{8}{7} = -\frac{3}{7} + \frac{2}{7} = -\frac{1}{7}$

2 The distance between the two numbers

$= |\frac{1}{3} - 1| = |\frac{1}{3} - \frac{3}{3}| = |-\frac{2}{3}| = \frac{2}{3}$

Then the number $= 1 - \frac{1}{4} \times \frac{2}{3} = 1 - \frac{1}{6}$

$= \frac{6-1}{6} = \frac{5}{6}$

3 The distance between the two numbers

$=|-\frac{3}{5}-(-\frac{4}{5})|=|-\frac{3}{5}+\frac{4}{5}|=\frac{1}{5}$

Then the number $=-\frac{3}{5}-\frac{1}{3}\times\frac{1}{5}=-\frac{3}{5}-\frac{1}{15}=\frac{-9-1}{15}$

$=-\frac{10}{15}=-\frac{2}{3}$

4 The distance between the two numbers

$=|\frac{4}{7}-1\frac{3}{4}|=|\frac{4}{7}-\frac{7}{4}|=|\frac{16}{28}-\frac{49}{28}|=\frac{33}{28}$

Then the number $=\frac{16}{28}+\frac{1}{3}\times\frac{33}{28}=\frac{16}{28}+\frac{11}{28}=\frac{27}{28}$

5 The distance between the two numbers

$=|-\frac{1}{2}-(-\frac{2}{5})|=|-\frac{1}{2}+\frac{2}{5}|=|-\frac{5}{10}+\frac{4}{10}|=\frac{1}{10}$

Then the number

$=-\frac{4}{10}-\frac{1}{5}\times\frac{1}{10}=-\frac{4}{10}-\frac{1}{50}=\frac{-20-1}{50}=-\frac{21}{50}$

6 The distance between the two numbers

$=|-\frac{2}{3}-(-\frac{3}{5})|=|-\frac{2}{3}+\frac{3}{5}|=|-\frac{10}{15}+\frac{9}{15}|=\frac{1}{15}$

Then the number $=-\frac{10}{15}+\frac{1}{5}\times\frac{1}{15}=-\frac{10}{15}+\frac{1}{75}$

$=\frac{-50+1}{75}=-\frac{49}{75}$

7 The distance between the two numbers

$=|\frac{5}{6}-\frac{2}{3}|=|\frac{5}{6}-\frac{4}{6}|=\frac{1}{6}$

Then the number $=\frac{4}{6}+\frac{1}{10}\times\frac{1}{6}=\frac{4}{6}+\frac{1}{60}$

$=\frac{40+1}{60}=\frac{41}{60}$

8 The distance between the two numbers

$=|\text{zero}-(-1\frac{1}{2})|=|\text{zero}+\frac{3}{2}|=\frac{3}{2}$

, then the number from the side of the greater

$=\text{zero}-\frac{1}{8}\times\frac{3}{2}=-\frac{3}{16}$

, the number from the side of the smaller

$=-\frac{3}{2}+\frac{1}{8}\times\frac{3}{2}=-\frac{3}{2}+\frac{3}{16}=\frac{-24+3}{16}=-\frac{21}{16}$

3 1 (d) 2 (c) 3 (d) 4 (c) 5 (b)

4

The distance between the tree and the lamp post

$=|7\frac{1}{2}-3.3|=|\frac{15}{2}-\frac{33}{10}|$

$=|\frac{75}{10}-\frac{33}{10}|=\frac{42}{10}=\frac{21}{5}$

Then the distance where the flower bed should be put at $=3.3+\frac{1}{3}\times\frac{21}{5}=\frac{33}{10}+\frac{7}{5}$

$=\frac{33}{10}+\frac{14}{10}=\frac{47}{10}=4.7$ m.

Answers of exams on unit one

Model 1

1

1 $-\frac{3}{5}$ 2 5 3 $-\frac{5}{6}$ 4 $\frac{5}{8}$ 5 1

2

1 (b) 2 (a) 3 (c) 4 (a) 5 (d) 6 (c)

3 [a] zero

[b] -2

4 [a] The numbers are : $\frac{7}{18}$, $\frac{8}{18}$, $\frac{9}{18}$, $\frac{10}{18}$

(There are other solutions)

[b] 1 -22 2 $-\frac{1}{4}$

5 [a] The descending order is :

$\frac{1}{3}$, $\frac{3}{10}$, $\frac{4}{15}$, $\frac{7}{30}$ and $\frac{1}{5}$

[b] $\frac{27}{28}$

Model 2

1

1 (d) 2 (b) 3 (d) 4 (b) 5 (b) 6 (c)

2

1 zero 2 -7 3 $\frac{13}{28}$ 4 zero 5 $\frac{11}{24}$

3 [a] The numbers are : $\frac{9}{12}$, $\frac{10}{12}$, $\frac{11}{12}$ and 1

(There are other solutions)

[b] 6

4 [a] $\frac{13}{20}$

[b] 1 $\frac{7}{20}$ 2 $\frac{5}{33}$ 3 $-\frac{4}{1}$

5 [a] 1 $\frac{7}{2}$ 2 $\frac{37}{42}$

[b] Represent by yourself.

Answers of unit two

Answers of Exercise 6

1

Coefficient	3	7	−8	1
Degree	zero	5	3	3

2

Number of terms	Name	Degree
3	trinomial	3
3	trinomial	4
2	binomial	5
4	4 terms	5

3

1. third , 3
2. $\frac{1}{2}$, sixth
3. 1 , first
4. zero
5. −8 , zero
6. second
7. 3 , second

4

1. (a)
2. (a)
3. (c)
4. (c)
5. (b)
6. (c)
7. (b)
8. (d)

5

1. $5a^5b^3 - 3a^2b^5 + 7ab$
2. $-7 + 5x + x^2 + x^3$

6

Ascendingly according to the indices of a :

$-4 + 3b^3a + 2a^2b^2 - 5ba^3 + 6a^4b^4$

Descendingly according to the indices of b :

$6a^4b^4 + 3b^3a + 2a^2b^2 - 5ba^3 - 4$

7

The expression $= (x \times y) - (\frac{1}{2} \times 1 \times 2) = xy - 1$ of the second degree.

8 The expression $= \frac{1}{2}bh - \pi r^2$ of second degree

9

1. 3
2. 3
3. 2
4. {zero , 1 , 2 , 3}

Answers of Exercise 7

1

1. $5x$
2. $3x$
3. $-7x$
4. $-10x$
5. $-2a^2$
6. x^2y
7. a
8. zero
9. $2x$
10. $\frac{2x}{7}$

2

1. $-3y^2 - y^2 = -4y^2$
2. $9x^2y - (-6x^2y) = 9x^2y + 6x^2y = 15x^2y$
3. $-2x - (-5x) = -2x + 5x = 3x$
4. $3a^2b - a^2b = 2a^2b$
5. $2ab - (-3ab) = 2ab + 3ab = 5ab$
6. $-7x^2y - 6x^2y = -13x^2y$

3

1. $4a$
2. $8x^2$
3. $-2m$
4. $-5x$
5. $2a$
6. $10x$
7. $3x$
8. $-2x$
9. $2x , -2x$

4

1. (c)
2. (b)
3. (b)
4. (d)
5. (d)

5

1. $5a^2$
2. $2x^2$
3. $-2m^2$
4. $-2a^2b$
5. $7a^2b$
6. 11
7. $22x$
8. $14x$

6 The other term $= 12x^2y - 4x^2y = 8x^2y$

7

1. $8a + 6b$
2. $2x - 3y$
3. $-7x - 7y$
4. $30m - 12n$
5. $-4a + 3$
6. $-5b$
7. $-6y - 7x$
8. $6a + 13b$

8

1. $-10x^2 - x + 3$
2. $x^2y - xy^2 + 2x^2y^2$
3. $4a^2 - 2a - 4$
4. $6x^2 - 9x + 5$

9

1. The expression $= 3x + 1 + 3x^2 + x = 3x^2 + 4x + 1$
2. The expression $= 2x^2 + x + 4x + 2 = 2x^2 + 5x + 2$
3. The expression $= 5x^2 + 2x + 15x + 6 = 5x^2 + 17x + 6$

10

(1) The expression $= x+3+5+x+3+x+x+5$
$= 4x+16$

(2) The expression $= y+3+x+2+3+y+2+x$
$= 2y+2x+10$

(3) The expression $= y+x+y+\frac{1}{2}x+x+\frac{1}{2}x+y+y$
$= 4y+3x$

11 The perimeter $= 4+4+4-x+x+x+4-x$
$= 16$ cm.

12 (1) 3 (2) 1 , 2

Answers of Exercise 8

1

(1) $8a+2b+4c$ (2) $4a-3b-c$
(3) $10x-2y+4$ (4) $3a^3-2a^2b-ab^2$

2

(1) $4x+3$ (2) 2ℓ
(3) $2n^2+2n-3$ (4) $2m^2+\ell^2$
(5) $a^2b-3ab^2+2b^3$ (6) $4a^3-2ab^2+4a^2b$

3

(1) $a+2$ (2) $7x+3y+3z$
(3) 1 (4) zero
(5) $-2x^2+4x+9$ (6) x^2+xy-y^2

4

(1) $x-3$ (2) $-11y+9$
(3) $3x^2+2$ (4) $a^3+2a^2b+3b^3-3ab^2$

5

(1) $2a+9b$ (2) $-2x^2-7x+2$
(3) $-8x+2$ (4) $7x^2y-8x$

6

(1) $-5a+2b$ (2) $2x^2-3xy-2y^2$
(3) $a^2+4ab+9b^2$ (4) $2x^2-3x+3$

7 The expression $= 6+x^2-x-(2x-3x^2+5)$
$= 4x^2-3x+1$

8 The expression $= 2x-3y+6z-\ell$
$-(5z-4y+3x-2\ell) = -x+y+z+\ell$

9 The expression $= 0-(3a^2-5ab+2b^2)$
$= -3a^2+5ab-2b^2$

10 The other expression $= 5x-7y+9$
$-(2y+3x-4) = 2x-9y+13$

11 The remainder $= a+5b-2$
The numerical value $= 2+5\times1-2=5$

12 The sum $= 4x-5y-6z$
The remainder $= 5x+5y-z-(4x-5y-6z)$
$= x+10y+5z$

13 The sum $= 5a-7b-7c$
The decrease $= 5a-7b-7c-(2a-8b-c)$
$= 3a+b-6c$

14 The sum $= \ell+2m+2n$
The remainder $= 2\ell-4m+5n-(\ell+2m+2n)$
$= \ell-6m+3n$

15 The sum $= 7x^2-x-23$
The increase $= 3x^2-5+2x-(7x^2-x-23)$
$= -4x^2+3x+18$

16 The sum $= x^2-xy+x-5$
The numerical value $= (-1)^2-(-1)\times2+(-1)-5$
$= 1+2-1-5=-3$

17 $x+y-z = (a-2b+c)+(2a+3b-4c)$
$-(b-4a+c) = 7a-4c$

18 The total surface area of the first solid
= The area of six faces
$= 2\times a\times b+2\times5\times b+2\times5\times a$
$= 2ab+10b+10a$
The total surface area of the second solid
= The area of six faces
$= 2\times a\times b+2\times3\times b+2\times3\times a$
$= 2ab+6b+6a$
The sum of the two areas
$= (2ab+10b+10a)+(2ab+6b+6a)$
$= 4ab+16b+16a$

19 The sum of lengths of the two sides
$= (x^3-3x^2+2x-3)+(x^4-2x^2+4x+1)$
$= (x^4+x^3-5x^2+6x-2)$ cm.

The length of the third side

$= (2x^4 - 3x^2 + 5x - 4)$
$\quad - (x^4 + x^3 - 5x^2 + 6x - 2)$
$= (x^4 - x^3 + 2x^2 - x - 2)$ cm.

20 The remainder distance

$= (3x^3 - 4x^2 + 2x - 5) - (2x^3 + x^2 - 3x + 1)$
$= (x^3 - 5x^2 + 5x - 6)$ km.

21 The width of the picture

$= (8x^2 + 10) - (x^2 - 2x + 3) - (x^2 - 2x + 3)$
$= 8x^2 + 10 - 2x^2 + 4x - 6$
$= (6x^2 + 4x + 4)$ cm.

22 [1] Since , $a + b + b + c = \frac{5}{4} + \frac{3}{4}$

Therefore , $a + 2b + c = \frac{8}{4} = 2$

[2] Since , $a + b + b + c - a - c = \frac{5}{4} + \frac{3}{4} - \frac{1}{2}$

Therefore , $2b = \frac{8}{4} - \frac{2}{4} = \frac{6}{4}$

Therefore , $b = \frac{6}{4} \times \frac{1}{2} = \frac{3}{4}$

Answers of Exercise 9

1

[1] $15xy$ [2] $-21ac$ [3] $-6x^2$
[4] $56y^9$ [5] $-6x^3y$ [6] $10x^4y^6$
[7] $-10a^3b^3$ [8] $2x^3$ [9] $-40a^2$
[10] $6a^2b^2$ [11] $30x^9$ [12] $24x^6y^8$

2

[1] $3a$ [2] -12 [3] 5 [4] $-2x$
[5] $5a^4$ [6] -1 [7] $\frac{3}{2}x^2y^3$ [8] $8b^4$
[9] $-2m^3n$ [10] $3x^2y^3$

3

[1] t^8 [2] $6a^7$ [3] $6a^4b^3$
[4] $\frac{1}{2}x^5$ [5] $6h^4k^8$ [6] $-7m^6$

4

[1] (d) [2] (d) [3] (d) [4] (d)
[5] (b) [6] (a) [7] (a) [8] (b)

5

[1] $2y^2$ [2] $3x^2 - 2x$ [3] $3x$ [4] -5
[5] $3l$ [6] $-35a^5$ [7] $5xy$
[8] $-64x^7y^6$

6

[1] $3a^2b^6$ [2] $3a^4$ [3] $-2c^2d$
[4] $7b^3$ [5] $2a^3b^2$ [6] $7xy^3$

7

[1] $9y$ [2] $-\frac{2}{3}xy^n$

8

The volume of the cuboid $= x \times 2x \times 4x = 8x^3$ cm³

The volume of the small cube $= x \times x \times x = x^3$ cm³

The number of the small cubes $= \frac{8x^3}{x^3} = 8$ cubes.

9

[1] The perimeter $= 2(3ab + 2ab) = 10ab$

The area $= 3ab \times 2ab = 6a^2b^2$

[2] The perimeter $= 2x + x + x + 3x + 3x + 4x = 14x$

The area $= 4x \times 3x - x \times x$
$= 12x^2 - x^2 = 11x^2$

[3] The perimeter $= 4a + a + a + a + a + a + 4a + a$
$+ a + a + a + a = 18a$

The area $= (4a \times a) + (2a \times a) + (4a \times a)$
$= 4a^2 + 2a^2 + 4a^2 = 10a^2$

10 The area of the coloured part $= (3a \times 5a) - (2a \times 2a)$
$= 15a^2 - 4a^2 = 11a^2$

11 The total surface area of the first solid = the sum of areas of its six faces

$= (3y \times 2x \times 2) + (3y \times a \times 2) + (2x \times a \times 2)$
$= 12xy + 6ya + 4xa$

The total surface area of the second solid = the sum of areas of its six faces

$= (x \times y \times 2) + (y \times 3a \times 2) + (x \times 3a \times 2)$
$= 2xy + 6ay + 6xa$

The total surface area of the two solids

$= 12xy + 6ya + 4xa + 2xy + 6ay + 6xa$
$= 14xy + 12ay + 10xa$

12 Let the radius length of the ball be r

Then , the dimensions of the box are :

6 r , 2 r and 2 r

Then , $\frac{\text{Volume of the three balls}}{\text{Volume of the box}} = \frac{3 \times \frac{4}{3} \times \pi r^3}{6r \times 2r \times 2r}$

$= \frac{4\pi r^3}{24 r^3} = \frac{\pi}{6} = \frac{3.14}{6} = \frac{157}{300}$

13 The solid consists of 12 lateral faces equal in area beside the area of two bases.

The lateral area $= (12) \times (3x) \times (x) = 36x^2$

The area of the base = the sum of areas of 5 squares (equal in area) $= 5x^2$

The total area $= 36x^2 + 10x^2 = 46x^2$

The volume = the sum of volumes of 5 cuboids equal in volume where the dimensions of each are x, x and $3x$

$= 5 \times x \times x \times 3x = 15x^3$

Answers of Exercise 10

1

1. $a^2 + a$
2. $a^2 - 2a$
3. $21xy - 12xz$
4. $-3y - 9$
5. $-14c + 6c^2$
6. $6x^3 + 8xy^2$
7. $-10x^2 - 5xy + 15xz$
8. $6x^3y - 15x^3y^2 - 12xy^3$
9. $\ell^3m^2 - 3\ell^2m^3 - 4\ell m^4$
10. $2x^4 - 3x^3y - x^2y^2$

2

1. $4y^3 - 2y^2 - 10y$
2. $-4xy^3 - 3x^2y^2 + 5y^2$
3. $-20x^2y + 16xy^2 - 4x^2y^2$
4. $-2xy$, $-2xy^2$

3

1. 6, $2x^2$
2. $2x$, $15xy$
3. $4x^2$, $10xy$
4. $5x^2y$, $12x^2y^2$
5. $3ab^2$, $4a^3b^2$
6. 5, $6x^2$
7. $2b^2$, $-8a^2b$
8. $2x$, $2xy$
9. $5y$, $2x$
10. a, b, c
11. $2x$, $4y$, $15x^3y$
12. $2mn^2$, $5m$

4 $abc = 5x \times (3xy) \times (x - y)$

$= 15x^2y(x - y) = 15x^3y - 15x^2y^2$

5

1. $3a^2 - 3ab + 8a^2 + 4ab = 11a^2 + ab$
2. $12a^2 - 6a - 12a^2 + 8a = 2a$
3. $12a^2 - 3a + 2a^2 + 6a - 10a^2 + 5a = 4a^2 + 8a$
4. $2x^2 + 2xy - 2xy + y^2 + 2y^2 - 2x^2 = 3y^2$

6 The expression $= 6a^2 - 2a + 3a^2 + 6a = 9a^2 + 4a$

The numerical value $= 9(1)^2 + 4(1) = 9 + 4 = 13$

7 The expression $= 6a^2 + 2ab - 3ab - 3b^2$

$= 6a^2 - ab - 3b^2$

The numerical value $= 6 \times 1^2 - 1 \times 1 - 3 \times (1)^2$

$= 6 - 1 - 3 = 2$

8 The expression $= 2x^2 - xy - 2xy + 2y^2$

$= 2x^2 - 3xy + 2y^2$

The numerical value $= 2 \times 2^2 - 3 \times 2 \times (-1) + 2 \times (-1)^2$

$= 8 + 6 + 2 = 16$

9 The expression $= 2x(3x - 2y) + y(x + y) + (x^2 - y^2)$

$= 6x^2 - 4xy + xy + y^2 + x^2 - y^2$

$= 7x^2 - 3xy$

The numerical value $= 7 \times (-2)^2 - 3 \times (-2) \times (-1)$

$= 28 - 6 = 22$

10 The expression $= 3 - 6x - x^2 + 5x - 3 + 2x^2 + 6x$

$= x^2 + 5x$

The numerical value $= (-2)^2 + 5(-2) = 4 - 10 = -6$

11 The expression $= 3a^2b - 2ab^2 - 2a^2b + 2ab^2$

$+ 4ab^2 - a^2b = 4ab^2$

The numerical value $= 4 \times 1 \times (-3)^2$

$= 4 \times 1 \times 9 = 36$

12 The expression =

$2x[x - 2y + 2x] - 3y[y - 2x + 2y]$

$= 2x(3x - 2y) - 3y(-2x + 3y)$

$= 6x^2 - 4xy + 6xy - 9y^2$

$= 6x^2 + 2xy - 9y^2$

The numerical value $= 6 \times 1^2 + 2 \times 1 \times 1 - 9 \times 1^2$

$= 6 + 2 - 9 = -1$

13 The perimeter of the rectangle

$= 2(2a + b + 4a - 2b) = 2(6a - b)$

$= (12a - 2b)$ cm.

14

1. The expression $= x(3x + 2y) = 3x^2 + 2xy$
2. The expression $= 2y(2y + 2) = 4y^2 + 4y$

3 The expression $= 3x(x+9) - 4x$
$= 3x^2 + 27x - 4x = 3x^2 + 23x$

4 The expression $= 4x(2y+5) - (5 \times 2x)$
$= 8xy + 20x - 10x = 8xy + 10x$

5 The expression $= 6x(5x-1) - 4(2x^2 - 2)$
$= 30x^2 - 6x - 8x^2 + 8 = 22x^2 - 6x + 8$

6 The expression
$= 5x(3x+5) - (x \times x + x \times x + 2 \times x)$
$= 15x^2 + 25x - x^2 - x^2 - 2x$
$= 13x^2 + 23x$

7 The expression $= 3x(2x+8) - \frac{1}{2} \times 3x(2x+8)$
$= 6x^2 + 24x - 3x^2 - 12x$
$= 3x^2 + 12x$

8 The expression $= 4y(3x+4) + xy - xy$
$= 12xy + 16y$

9 The expression $= x(x+5) + (x \times x)$
$= x^2 + 5x + x^2 = 2x^2 + 5x$

10 The expression $= (x \times 3x) + x(x+8)$
$= 3x^2 + x^2 + 8x = 4x^2 + 8x$

15 The width of the rectangle $= x$ cm. , hence
The length of the rectangle $= (2x+3)$ cm.
The area of the rectangle $= x(2x+3)$
$= (2x^2 + 3x)$ cm².

16 The volume = length × width × height
$= 3x \times 3x \times (2x^2+3) = 9x^2(2x^2+3)$
$= (18x^4 + 27x^2)$ cm³.

17 The lateral surface area of the cuboid
= the perimeter of the base × height
$= 2(3x + 4x + y) \times 5y = 10y(7x+y)$
$= (70xy + 10y^2)$ cm².
Its volume = length × width × height
$= 3x \times (4x+y) \times 5y = 15xy(4x+y)$
$= (60x^2y + 15xy^2)$ cm³.

18 The area of the coloured part
$= 8x(x^2 + 4x) - (x \times 3x + 3x \times 5x)$
$= 8x^3 + 32x^2 - 3x^2 - 15x^2$
$= (8x^3 + 14x^2)$ cm².

Answers of exams on the first part of unit two

Model 1

1

1 (c) 2 (c) 3 (d) 4 (d) 5 (a) 6 (c)

2

1 $2y^2$ 2 $5y, 4x$ 3 zero
4 $30a^4b^5$ 5 $2x$

3 [a] $4x - 8$
[b] $-3a^2 - 4a + 3$

4 [a] $-9 + 4x + 5x^2 + x^3$
[b] The expression $= x^2 + x + 2$
The numerical value = 4

5 [a] $2x + 6y + 2z$
[b] 1 $16x^7$ 2 $-2a^3b$

Model 2

1

1 (b) 2 (a) 3 (b) 4 (a) 5 (d) 6 (c)

2

1 $4y^4$ 2 $2ab, 15a^2b^2$ 3 third
4 $-6a^2$ 5 $100x^2$

3 [a] The expression $= 3n^2 + 26n$
The numerical value $= -23$
[b] $-4x^2 - 4x + 5$

4 [a] $7x + 14y$
[b] 1 $\frac{3}{2}x^2y^2$ 2 $5x$

5 [a] $3x^4 - 4x^3y + 2x^2y^2$
[b] $(6x - 9y + 15z)$ cm.

Answers of Exercise 11

1

1. x^2 2. $-3x$ 3. y^2 , 20
4. 10 a , 21 5. $2x^2$, $9x$ 6. $14xy$

2

1. x^2+6x+8 2. $y^2-3y-10$
3. $30m^2-7m-2$ 4. $8x^2+14x+3$
5. $6a^2-11ab-10b^2$
6. $6x^2+5xy-4y^2$ 7. b^4-2b^2-8
8. $6m^4+7m^2-24$ 9. $-x^2+8xy-7y^2$
10. $\frac{9}{4}a^2-3ab-24b^2$

3

1. a^2+6a+9 2. $4y^2+12y+9$
3. $16m^2-56m+49$ 4. $9x^2+6xy+y^2$
5. $x^2-6xy+9y^2$ 6. $l^2+2lm+m^2$
7. $16a^2+56a+49$ 8. $4x^2+12xy+9y^2$
9. $16x^4-4x^2y^2+\frac{1}{4}y^4$

4

1. a^2-9 2. $16m^2-49$
3. $36x^2-4y^2$ 4. a^4-81
5. $9x^4-25y^4$ 6. $l^2m^2-36n^2$
7. $\frac{1}{4}x^2-\frac{1}{9}y^2$ 8. $4x^2-9y^2$
9. y^4-81 10. x^4-16y^4

5

1. x^3+4x^2+4x+3 2. x^3+1
3. $2y^3+3y^2+11y+5$ 4. $8x^3-4x+21$
5. $4x^3-8x^2y+5xy^2-y^3$
6. $3a^6-11a^4b^2+11a^2b^4-15b^6$
7. $2a^4+4a^3-11a^2-2a+5$
8. $-3a^3+4a^2+8$
9. $3x^3+26x^2+64x+32$
10. $27x^3+54x^2y+36xy^2+8y^3$

6

1. (b) 2. (a) 3. (a) 4. (c)
5. (d) 6. (a) 7. (d) 8. (c)
9. (b) 10. (c) 11. (b)

7

1. $4x^2$ 2. $x+5$
3. y , $3x$ 4. 3 , $8x$
5. 4 , a^2 , 8 a 6. 2 , $6x^2$, $19x$
7. 1 , $2x$, $18x$ 8. 3 b , 4 a , 2 a b
9. x , 3 , 12 10. 4 a , $16a^2$, $9b^2$

8

1. x^2-6x 2. $50a^3-32ab^2$
3. $3m^2-9m-30$ 4. $-4-x$ 5. -11
6. $5x^2+15x-1$ 7. 1 8. $-4x+8$
9. $3x^2+4xy-5y^2$ 10. $7a^2+b^2$
11. $-40xy$ 12. $67x+11$

9

1. The expression $= x^2-25y^2$
The numerical value $= 1^2-25\times(-2)^2$
$= 1-100 = -99$
2. The expression $= 3x^2+10xy+3y^2$
The numerical value
$= 3\times1^2+10\times1\times(-2)+3\times(-2)^2$
$= 3-20+12 = -5$
3. The expression $= 3x^2+14x+8$
The numerical value $= 3\times(1)^2+14\times(1)+8$
$= 3+14+8 = 25$
4. The expression $= 6y^2+29y+28$
The numerical value $= 6\times(-2)^2+29\times(-2)+28$
$= 24-58+28 = -6$
5. The expression $= |x^2-4y^2|$
The numerical value $= |1^2-4\times(-2)^2|$
$= |1-16| = |-15| = 15$

10

The expression $= x^2-2xy+y^2+2xy = x^2+y^2$
The numerical value $= (-1)^2+(2)^2 = 5$

11

The expression $= 4x^2-8x+4+x^2-4 = 5x^2-8x$
The numerical value $= 5\times(-1)^2-8\times(-1) = 5+8 = 13$

12

The remainder $= (2x^2 + 19x + 9) - (x^2 - 6x + 9)$
$= 2x^2 + 19x + 9 - x^2 + 6x - 9 = x^2 + 25x$

13 The expression $= (3x - 4)(x + 2) - (2x - 3)^2$
$= 3x^2 + 2x - 8 - (4x^2 - 12x + 9) = -x^2 + 14x - 17$
The numerical value $= -17$

14 Since : $2a^2 = 2(16x^2 - 24x + 9)$
$= 32x^2 - 48x + 18$,
$-3b^2 = -3(4x^2 + 4x + 1)$
$= -12x^2 - 12x - 3$,
$bc = (2x + 1)(3x - 2) = 6x^2 - x - 2$
Therefore : $2a^2 - 3b^2 + bc = 32x^2 - 48x$
$+ 18 - 12x^2 - 12x - 3 + 6x^2 - x - 2$
$= 26x^2 - 61x + 13$

15

1 The area of the coloured part
$= 2(x - y)(2x + 3y) - (x - y)(x + y)$
$= 4x^2 + 2xy - 6y^2 - x^2 + y^2$
$= (3x^2 + 2xy - 5y^2)$ cm².

2 The area of the coloured part
$= 2x(x + 5) - (x - 1)(x + 2)$
$= 2x^2 + 10x - x^2 - x + 2$
$= (x^2 + 9x + 2)$ cm².

16

1 The perimeter of the coloured region
$= 2x + 5 + 3x - 1 + x + 2x + 5$
$+ 3x + 5 + 5x + 4$
$= 16x + 18$
The area of the coloured region
$= (5x + 4)(3x + 5) - x(3x - 1)$
$= 15x^2 + 37x + 20 - 3x^2 + x$
$= 12x^2 + 38x + 20$

2 The perimeter of the coloured region
$= x + y + x - 2y + 2x + y + 3x + y + 3x$
$+ 2y + 4x - y$
$= 14x + 2y$
The area of the coloured region
$= (3x + 2y)(4x - y) - (2x + y)(x - 2y)$
$= 12x^2 + 5xy - 2y^2 - 2x^2 + 3xy + 2y^2$
$= 10x^2 + 8xy$

3 The perimeter of the coloured region
$= x - y + x + 2y + 2x - y + x + y + 3x$
$- 2y + 2x + 3y$
$= 10x + 2y$
The area of the coloured region
$= (2x + 3y)(3x - 2y) - (2x - y)(x + 2y)$
$= 6x^2 + 5xy - 6y^2 - 2x^2 - 3xy + 2y^2$
$= 4x^2 + 2xy - 4y^2$

17

1 $(101)^2 = (100 + 1)^2 = 10000 + 200 + 1 = 10201$
2 $(10\frac{1}{2})^2 = (10 + \frac{1}{2})^2 = 100 + 10 + \frac{1}{4} = 110\frac{1}{4}$
3 $(99)^2 = (100 - 1)^2 = 10000 - 200 + 1 = 9801$
4 $64 \times 56 = (60 + 4)(60 - 4) = 3600 - 16 = 3584$
5 $98 \times 102 = (100 - 2)(100 + 2) = 10000 - 4 = 9996$
6 $19 \times 21 = (20 - 1)(20 + 1) = 400 - 1 = 399$
7 $201 \times 199 = (200 + 1)(200 - 1)$
$= 40000 - 1 = 39999$
8 $(49)^2 = (50 - 1)^2 = 2500 - 100 + 1 = 2401$
9 $(41)^2 = (40 + 1)^2 = 1600 + 80 + 1 = 1681$

18 Since : $3x - 2z = 3(2a - 5b) - 2(a - 3b)$
$= 6a - 15b - 2a + 6b = 4a - 9b$
Therefore : $y(3x - 2z) = (3a + 4b)(4a - 9b)$
$= 12a^2 - 11ab - 36b^2$

19

$(2 - y)^3 = 8 - 12y + 6y^2 - y^3$
$(2 - y)^4 = (2 - y)^3(2 - y) = (8 - 12y + 6y^2 - y^3)(2 - y)$
$= 16 - 24y + 12y^2 - 2y^3 - 8y + 12y^2 - 6y^3 + y^4$
$= 16 - 32y + 24y^2 - 8y^3 + y^4$

20 Since : $b + c = 2x + y + x - 3y = 3x - 2y$
Therefore : $a(b + c) = (5x + 2y)(3x - 2y)$
$= 15x^2 - 4xy - 4y^2$ (1)
$ab = (5x + 2y)(2x + y) = 10x^2 + 9xy + 2y^2$

$a\,c = (5x + 2y)(x - 3y) = 5x^2 - 13xy - 6y^2$

Therefore : $a\,b + a\,c = 15x^2 - 4xy - 4y^2$ (2)

From (1) and (2) :

Therefore : $a(b + c) = a\,b + a\,c$

21 The area of the square

$= (2x + 5)^2 = (4x^2 + 20x + 25)$ cm².

The length of the rectangle

$= 2x + 5 + x - 1 = (3x + 4)$ cm.

The width of the rectangle

$= 2x + 5 - (x - 1) = (x + 6)$ cm.

The area of the rectangle $= (3x + 4)(x + 6)$

$= (3x^2 + 22x + 24)$ cm².

Answers of Exercise 12

1

1. $a - 2$
2. $-4x - 5y$
3. $2a + 3$
4. $-4x + 3$
5. $3a + 5b$
6. $4ab - 6b$
7. $-5x^3 + 4x^7 + 1$
8. $-4x^2 + 6 - 9x^4$
9. $-a + 2b - 4$
10. $a - 2b + 3$

2

1. $13x + 7x^3$
2. $-9m^2 - 16$
3. $6x - 10$
4. $3\ell^2 m^2 - 6$
5. $-4a^2 + 3a - 2$
6. $-\frac{5}{3}x^2 y - \frac{2}{3}y + \frac{1}{3}$
7. $-\ell^2 m^3 + 4m^2 n^2 + 3m n^4$
8. $-3x^2 + 7x^3 y^2 - 5x^4 y^3$

3

1. (d)
2. (c)
3. (b)
4. (c)
5. (b)
6. (d)

4

1. $-5n$, $3m^4$
2. $2a + 1$
3. $-2x + y$, zero
4. $12x^3y^3$, $24x^2y^4$, $2x^2y$, $\frac{3}{2}xy^2$, $3y^3$
5. $3a^2b^2$, a^3b^2
6. $-3a^2b^2$, $-3ab$
7. 3
8. 6

5 The product $= 12x^5y^3 - 24x^4y^3$

The quotient $= x\,y - 2y$

6 The quotient $= -x^2 - 2xy + 7$,

The sum $= -6x^2 + 3y^2 + 7$

7 The quotient $= 3y^2 - 2y$

The absolute value $= \left|3\left(\frac{1}{2}\right)^2 - 2 \times \frac{1}{2}\right|$

$= \left|\frac{3}{4} - 1\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$

8 The quotient $= 3x - y$

The numerical value $= 3 \times 1 - (-1) = 4$

9 The quotient $= 4x^2 - 3x + 2$

The sum $= 3x^2 + 9$

The numerical value $= 3(1)^2 + 9 = 12$

10 The length of the rectangle

$= \frac{\text{The area of the rectangle}}{\text{The width}} = \frac{24x^3 + 18x^2 + 42x}{6x}$

$= (4x^2 + 3x + 7)$ cm.

11 The width of the rectangle

$= \frac{\text{The area of the rectangle}}{\text{The length}} = \frac{8a^4b^3 + 12a^3b^4 - 8a^2b^2}{4a^2b^2}$

$= 2a^2b + 3ab^2 - 2$

When : $a = 1$, $b = 2$

Therefore , the width $= 2 \times 1^2 \times 2 + 3 \times 1 \times 2^2 - 2$

$= 14$ cm.

12 The height of the triangle

$= \frac{2 \times (\text{The area of the triangle})}{\text{The length of the base}}$

$= \frac{2(12x^2 + 9x)}{3x} = \frac{24x^2 + 18x}{3x} = (8x + 6)$ cm.

13 The area of the base $= (2x) \times (2x) = 4x^2$ cm².

The height $= \frac{\text{Volume}}{\text{Base's area}} = \frac{12x^3 + 8x^2y}{4x^2}$

$= (3x + 2y)$ cm.

When : $x = 1$ and $y = 2$

Therefore , the height $= 3 \times 1 + 2 \times 2 = 7$ cm.

14

$AD = (xy + 10)$ cm.

The area of the rectangle ABCD

$= 4xy(xy + 10) = 4x^2y^2 + 40xy$ and

the area of the rectangle MNEF

= The area of the rectangle ABCD – the area of the coloured part $= 4x^2y^2 + 40xy - (3x^2y^2 + 35xy)$

$= 4x^2y^2 + 40xy - 3x^2y^2 - 35xy$

$= x^2y^2 + 5xy$

$$FE = \frac{\text{The area of the rectangle MNEF}}{EN} = \frac{x^2y^2+5xy}{xy}$$

$= (xy + 5)$ cm.

Answers of Exercise 13

1

1
$$\begin{array}{l|l} x+2 & x^2+5x+6 \\ \hline x+3 & \overset{\ominus}{x^2}\overset{\ominus}{+2x} \\ \hline & 3x+6 \\ & \overset{\ominus}{3x}\overset{\ominus}{+6} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= x + 3$

2
$$\begin{array}{l|l} y-4 & y^2-9y+20 \\ \hline y-5 & \overset{\ominus}{y^2}\overset{\oplus}{-4y} \\ \hline & -5y+20 \\ & \overset{\oplus}{-5y}\overset{\ominus}{+20} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= y - 5$

3
$$\begin{array}{l|l} x-7 & x^2-5x-14 \\ \hline x+2 & \overset{\ominus}{x^2}\overset{\oplus}{-7x} \\ \hline & 2x-14 \\ & \overset{\ominus}{2x}\overset{\oplus}{-14} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= x + 2$

4
$$\begin{array}{l|l} x+5 & 2x^2+13x+15 \\ \hline 2x+3 & \overset{\ominus}{2x^2}\overset{\ominus}{+10x} \\ \hline & 3x+15 \\ & \overset{\ominus}{3x}\overset{\ominus}{+15} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= 2x + 3$

5
$$\begin{array}{l|l} 3x-4 & 3x^2+2x-8 \\ \hline x+2 & \overset{\ominus}{3x^2}\overset{\oplus}{-4x} \\ \hline & 6x-8 \\ & \overset{\ominus}{6x}\overset{\oplus}{-8} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= x + 2$

6
$$\begin{array}{l|l} x+2 & x^2-x-6 \\ \hline x-3 & \overset{\ominus}{x^2}\overset{\ominus}{+2x} \\ \hline & -3x-6 \\ & \overset{\oplus}{-3x}\overset{\oplus}{-6} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= x - 3$

7
$$\begin{array}{l|l} 7+2x & 14-17x-6x^2 \\ \hline 2-3x & \overset{\ominus}{14}\overset{\ominus}{+4x} \\ \hline & -21x-6x^2 \\ & \overset{\oplus}{-21x}\overset{\oplus}{-6x^2} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= 2 - 3x$

8
$$\begin{array}{l|l} 4x-3y & 8x^2+6xy-9y^2 \\ \hline 2x+3y & \overset{\ominus}{8x^2}\overset{\oplus}{-6xy} \\ \hline & 12xy-9y^2 \\ & \overset{\ominus}{12xy}\overset{\oplus}{-9y^2} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= 2x + 3y$

9
$$\begin{array}{l|l} 2x-4y & 4x^2-16xy+16y^2 \\ \hline 2x-4y & \overset{\ominus}{4x^2}\overset{\oplus}{-8xy} \\ \hline & -8xy+16y^2 \\ & \overset{\oplus}{-8xy}\overset{\ominus}{+16y^2} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= 2x - 4y$

10
$$\begin{array}{l|l} x+1 & x^2 \quad -1 \\ \hline x-1 & \overset{\ominus}{x^2}\overset{\ominus}{+x} \\ \hline & -x-1 \\ & \overset{\oplus}{-x}\overset{\oplus}{-1} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= x - 1$

11
$$\begin{array}{l|l} 4y-2x & 16y^2 \quad -4x^2 \\ \hline 4y+2x & \overset{\ominus}{16y^2}\overset{\oplus}{-8xy} \\ \hline & 8xy-4x^2 \\ & \overset{\ominus}{8xy}\overset{\oplus}{-4x^2} \\ \hline & 00 \quad 00 \end{array}$$

Then, the quotient $= 4y + 2x$

2

1 $x^2+3x+1 \,\big)\, x^3+5x^2+7x+2$; $x+2$

x^3+3x^2+x

$2x^2+6x+2$

$2x^2+6x+2$

$00 \quad 00 \quad 00$

Then , the quotient $= x+2$

2 $3x^2-4x+1 \,\big)\, 6x^3+7x^2-18x+5$; $2x+5$

$6x^3-8x^2+2x$

$15x^2-20x+5$

$15x^2-20x+5$

$00 \quad 00 \quad 00$

Then , the quotient $= 2x+5$

3 $x^2-7x-4 \,\big)\, 2x^3-9x^2-43x-20$; $2x+5$

$2x^3-14x^2-8x$

$5x^2-35x-20$

$5x^2-35x-20$

$00 \quad 00 \quad 00$

Then , the quotient $= 2x+5$

4 $x^2-1 \,\big)\, x^3+3x^2-x-3$; $x+3$

$x^3 \qquad -x$

$3x^2 \qquad -3$

$3x^2 \qquad -3$

$00 \quad 00$

Then , the quotient $= x+3$

5 $4x^2+2 \,\big)\, 8x^3-20x^2+4x-10$; $2x-5$

$8x^3 \qquad +4x$

$-20x^2 \qquad -10$

$-20x^2 \qquad -10$

$00 \quad 00$

Then , the quotient $= 2x-5$

6 $x^2+1 \,\big)\, x^4+3x^2+2$; x^2+2

x^4+x^2

$2x^2+2$

$2x^2+2$

$00 \quad 00$

Then , the quotient $= x^2+2$

7 $x-1 \,\big)\, x^3-x$; x^2+x

x^3-x^2

x^2-x

x^2-x

$00 \quad 00$

Then , the quotient $= x^2+x$

8 $4x^2+2x+1 \,\big)\, 8x^3-1$; $2x-1$

$8x^3+4x^2+2x$

$-4x^2-2x-1$

$-4x^2-2x-1$

$00 \quad 00 \quad 00$

Then , the quotient $= 2x-1$

3

1 $x+2 \,\big)\, x^3+5x^2+7x+2$; x^2+3x+1

x^3+2x^2

$3x^2+7x+2$

$3x^2+6x$

$x+2$

$x+2$

$00 \quad 00$

Then , the quotient $= x^2+3x+1$

2 $x-4 \,\big)\, x^3-x^2-9x-12$; $3x^2+3x+3$

x^3-4x^2

$3x^2-9x-12$

$3x^2-12x$

$3x-12$

$3x-12$

$00 \quad 00$

Then , the quotient $= x^2+3x+3$

3 $2x-3 \,\big)\, 6x^3-5x^2-14x+12$; $3x^2+2x-4$

$6x^3-9x^2$

$4x^2-14x+12$

$4x^2-6x$

$-8x+12$

$-8x+12$

$00 \quad 00$

Then , the quoient $= 3x^2+2x-4$

4

$3x+2 \,\big|\, 6x^3-5x^2+9x+10$

$2x^2-3x+5$

$6x^3+4x^2$

$-9x^2+9x+10$

$-9x^2-6x$

$15x+10$

$15x+10$

$00 \quad 00$

Then, the quotient $=2x^2-3x+5$

5

$5-4x \,\big|\, 15+3x-7x^2-4x^3$

$3+3x+x^2$

$15-12x$

$15x-7x^2-4x^3$

$15x-12x^2$

$5x^2-4x^3$

$5x^2-4x^3$

$00 \quad 00$

Then, the quotient $=3+3x+x^2$

6

$x-1 \,\big|\, 3x^3 \quad -4x+1$

$3x^2+3x-1$

$3x^3-3x^2$

$3x^2-4x+1$

$3x^2-3x$

$-x+1$

$-x+1$

$00 \quad 00$

Then, the quotient $=3x^2+3x-1$

7

$x-3 \,\big|\, x^3 \quad -27$

x^2+3x+9

x^3-3x^2

$3x^2 \quad -27$

$3x^2-9x$

$9x-27$

$9x-27$

$00 \quad 00$

Then, the quotient $=x^2+3x+9$

8

$3a-2 \,\big|\, 27a^3 \quad -8$

$9a^2+6a+4$

$27a^3-18a^2$

$18a^2 \quad -8$

$18a^2-12a$

$12a-8$

$12a-8$

$00 \quad 00$

Then, the quotient $=9a^2+6a+4$

9

$x^2+2x-7 \,\big|\, x^4 \quad -18x^2 \quad +49$

x^2-2x-7

$x^4+2x^3-7x^2$

$-2x^3-11x^2 \quad +49$

$-2x^3-4x^2+14x$

$-7x^2-14x+49$

$-7x^2-14x+49$

$00 \quad 00 \quad 00$

Then, the quotient $=x^2-2x-7$

10

$3x^2+5x-2 \,\big|\, -9x^4 \quad +37x^2 \quad -4$

$-3x^2+5x+2$

$-9x^4-15x^3+6x^2$

$15x^3+31x^2 \quad -4$

$15x^3+25x^2-10x$

$6x^2+10x-4$

$6x^2+10x-4$

$00 \quad 00 \quad 00$

Then, the quotient $=-3x^2+5x+2$

4

1 Since, $13xy+6(x^2+y^2)$

$=13xy+6x^2+6y^2$

$=6x^2+13xy+6y^2$

Then, $2x+3y \,\big|\, 6x^2+13xy+6y^2$

$3x+2y$

$6x^2+9xy$

$4xy+6y^2$

$4xy+6y^2$

$00 \quad 00$

Then, the quotient $=3x+2y$

2

$a^2-2ab-5b^2 \,\big|\, a^4 \quad -6a^2b^2 \quad -16ab^3-15b^4$

$a^2+2ab+3b^2$

$a^4-2a^3b-5a^2b^2$

$2a^3b-a^2b^2 \quad -16ab^3-15b^4$

$2a^3b-4a^2b^2-10ab^3$

$3a^2b^2-6ab^3-15b^4$

$3a^2b^2-6ab^3-15b^4$

$00 \quad 00 \quad 00$

Then, the quotient $=a^2+2ab+3b^2$

5

$$\begin{array}{l|l}
x+3 & 2x^2+3x-9 \\
\hline
2x-3 & 2x^2+6x \\
 & \overline{-3x-9} \\
 & -3x-9 \\
 & \overline{00 \quad 00}
\end{array}$$

Then , the other factor $= 2x-3$

6

$$\begin{array}{l|l}
x^2+3x+3 & x^3-x^2-9x-12 \\
\hline
x-4 & x^3+3x^2+3x \\
 & \overline{-4x^2-12x-12} \\
 & -4x^2-12x-12 \\
 & \overline{00 \quad 00 \quad 00}
\end{array}$$

Then , the other factor $= x-4$

7

$$\begin{array}{r}
3x^3-5x^2+7x+1 \\
3x^3 \qquad -x+7 \\
\hline
6x^3-5x^2+6x+8
\end{array}$$

$$\begin{array}{l|l}
3x+2 & 6x^3-5x^2+6x+8 \\
\hline
2x^2-3x+4 & 6x^3+4x^2 \\
 & \overline{-9x^2+6x+8} \\
 & -9x^2-6x \\
 & \overline{12x+8} \\
 & 12x+8 \\
 & \overline{00 \quad 00}
\end{array}$$

Then , the quotient $= 2x^2-3x+4$

8

$$\begin{array}{l|l}
2x+3 & 2x^3-x^2-2x+6 \\
\hline
x^2-2x+2 & 2x^3+3x^2 \\
 & \overline{-4x^2-2x+6} \\
 & -4x^2-6x \\
 & \overline{4x+6} \\
 & 4x+6 \\
 & \overline{00 \quad 00}
\end{array}$$

Then , the quotient $= x^2-2x+2$

, the numerical value $= (1)^2-2(1)+2 = 1$

9

$$\begin{array}{l|l}
x-2 & 2x^2-7x+m \\
\hline
2x-3 & 2x^2-4x \\
 & \overline{-3x+m} \\
 & -3x+6 \\
 & \overline{m-6}
\end{array}$$

Then , $m-6=0$ Then , $m=6$

10

$$\begin{array}{l|l}
x^2+4x+3 & x^3-3x^2-25x+k \\
\hline
x-7 & x^3+4x^2+3x \\
 & \overline{-7x^2-28x+k} \\
 & -7x^2-28x-21 \\
 & \overline{k+21}
\end{array}$$

Then , $k+21=0$

So , $k=-21$

11

$$\begin{array}{l|l}
3x-5 & 6x^3-13x^2-13x+k \\
\hline
2x^2-x-6 & 6x^3-10x^2 \\
 & \overline{-3x^2-13x+k} \\
 & -3x^2+5x \\
 & \overline{-18x+k} \\
 & -18x+30 \\
 & \overline{k-30}
\end{array}$$

Then , $k-30=0$ So , $k=30$

12

$$\begin{array}{l|l}
x^2+x+2 & x^3+2x^2+3x+2 \\
\hline
x+1 & x^3+x^2+2x \\
 & \overline{x^2+x+2} \\
 & x^2+x+2 \\
 & \overline{00 \quad 00 \quad 00}
\end{array}$$

Then , the required expression $= x+1$

13

$$\begin{array}{l|l}
3x-2 & 15x^2+11x-14 \\
\hline
5x+7 & 15x^2-10x \\
 & \overline{21x-14} \\
 & 21x-14 \\
 & \overline{00 \quad 00}
\end{array}$$

Then , the length $= (5x+7)$ cm.

14

$$\begin{array}{l|l}
x+5 & 2x^2+7x-15 \\
\hline
2x-3 & 2x^2+10x \\
 & \overline{-3x-15} \\
 & -3x-15 \\
 & \overline{00 \quad 00}
\end{array}$$

Answers of Unit 2

Then , the width = $(2x - 3)$ length units

When , $x = 3$

Then , the length = $3 + 5 = 8$ length units

, The width = $2 \times 3 - 3 = 3$ length units

Then , the perimeter = $(8 + 3) \times 2 = 22$ length units

15

$$\begin{array}{r|l} x - 4 & x^2 - kx + 12 \\ x + (4 - k) & \ominus x^2 \oplus 4x \\ \hline & (4 - k)x + 12 \\ & \ominus (4 - k)x \oplus - 16 \ominus + 4k \\ \hline & 12 + 16 - 4k \end{array}$$

Then , $28 - 4k = 0$

Then , $28 = 4k$ then , $k = 7$

16 Let the number that will be added be m

$$\begin{array}{r|l} 2x - 5 & 6x^2 - 11x - 17 + m \\ 3x + 2 & \ominus 6x^2 \oplus - 15x \\ \hline & 4x - 17 + m \\ & \ominus 4x \oplus - 10 \\ \hline & -7 + m \end{array}$$

Then , $-7 + m = 0$ Then , $m = 7$

17 Since , the area = $\frac{1}{2} \times$ the length of $\overline{BC}$

$\times$ the corresponding height of $\overline{BC}$

Then , the corresponding height of

$\overline{BC} = 2 \times \frac{\text{The area}}{\text{The length of } \overline{BC}}$

, Since

$$\begin{array}{r|l} 2x + 1 & 6x^2 + 7x + 2 \\ 3x + 2 & \ominus 6x^2 \ominus + 3x \\ \hline & 4x + 2 \\ & \ominus 4x \ominus + 2 \\ \hline & 00 \quad 00 \end{array}$$

Then , the corresponding height of $\overline{BC}$

$= 2(3x + 2) = (6x + 4)$ cm.

Answers of Exercise 14

1

1. $5(a + b)$
2. $3(x - y)$
3. $5(y - 2)$
4. $4(2y^3 - x^2)$
5. $7y(x + z)$
6. $5b(a - 3c)$
7. $3x(x + 2)$
8. $5a(7 + 2a)$
9. $2a^2(3a - 2b^2)$
10. $7b^2(7 - b)$
11. $5xy(7x^2 + y)$
12. $5a^2b(3a - b)$

2

1. $5(a - b + c)$
2. $2(3a + 4b + 5c)$
3. $x(x^2 + 2x + 5)$
4. $2a(4a^2 - 2a + 3)$
5. $2y(x^2 + 3xy - 1)$
6. $3m^2n^2(3m^2 - 2mn + 4n^2)$
7. $2x(-x^4 + 2x - 3 + x^2)$
8. $8xy(4x^2y^2 + 2xy + 1)$
9. $6abc(3a - 1 + 5c - 4bc)$
10. $3a^2b^2(5ab^2 + 2a^3b - 1)$

3

1. $(a + b)(3x + 7)$
2. $(a + 3)(a + b)$
3. $(x + 4)(x^2 + y^2)$
4. [illegible] $(x + y)(2a - 3b)$
5. $2a(x - 1)(3a - 4)$
6. $4x(x + 1)(3x - 2y)$
7. $24a^2b^3(a - 2) + 36a^3b^2(a - 2)$
   $= 12a^2b^2(a - 2)(2b + 3a)$
8. $(x - 7)(3x^2 + 2x + 5)$
9. $(2x + y)(4m^2 - 3m - 7)$
10. $8a^2b(a + b + 2)(2b - 1)$

4

1. $48(45 + 55) = 48 \times 100 = 4800$
2. $52(43 - 33) = 52 \times 10 = 520$
3. $7(123 + 35 - 18) = 7 \times 140 = 980$
4. $15(17 + 13 - 30) = 15 \times$ zero = zero
5. $12(5 + 4 + 1) = 12 \times 10 = 120$
6. $(1 + 14 - 5) \times 35 = 10 \times 35 = 350$
7. $\frac{5}{18} \times 11 + \frac{5}{18} \times 7 = \frac{5}{18} \times (11 + 7) = \frac{5}{18} \times 18 = 5$
8. $58(58 + 42) = 58 \times 100 = 5800$
9. $256(256 - 156) = 256 \times 100 = 25600$
10. $2 \times 15(3 \times 15 + 9 - 4) = 30(45 + 5)$
    $= 30 \times 50 = 1500$
11. $(5 \times 48 + 7 + 53) \times 48 = (240 + 60) \times 48$
    $= 300 \times 48 = 14400$

12 $31(31+23-54)=31\times \text{zero}=\text{zero}$

13 $51(17+33)+49(21+29)=51\times 50+49\times 50$
$=(51+49)\times 50$
$=100\times 50=5000$

14 $49(49+1)+50(50+1)=49\times 50+50\times 51$
$=50(49+51)=50\times 100=5000$

5

1 $2a, 4b$ 2 ab 3 $4xy, 4y$
4 x, y 5 7 6 x, y
7 15 8 3 9 $4x, 1$ 10 25

6 1 (d) 2 (b) 3 (b) 4 (d)
5 (c) 6 (b) 7 (b) 8 (b)

7 The expression $=(2a+b)(2a+b)$
The numerical value $=3\times 3=9$

8 The expression $=2(a+c)(a+c)$
The absolute value $=|2\times -3\times -3|=18$

9 The expression $=(a-b)(x+y)=-(b-a)(x+y)$
The numerical value $=-4\times 3=-12$

10

1 $\frac{19(19-2+1)}{9}=\frac{19\times 18}{9}=38$

2 $\frac{(5\times 9+11-1)\times 9}{45}=\frac{45+10}{5}=\frac{55}{5}=11$

3 $\left|\frac{(36)^2(5-3)}{-2\times(36)^2}\right|=\left|\frac{5-3}{-2}\right|=|-1|=1$

11 The other factor $=4c-2bc^3+3$

12 The area $=(8a^2b+12a^2bc+16a^3b)\text{ cm}^2$
or $4a^2b(2+3c+4a)\text{ cm}^2$

13 The expression $=b(x-1+c-2x-c+x)=-b$
The numerical value $=-8$

14 $2m(2x+3y)+2n(2x+3y)=16$
$2(m+n)(2x+3y)=16$
Therefore : $4(m+n)=16$
Therefore : $m+n=4$

15 The expression $=abc(a+b+c-1)$
The numerical value $=12\times(8-1)=84$

Answers of exams on the second part of unit two

Model 1

1

1 (b) 2 (a) 3 (c) 4 (d) 5 (b) 6 (d)

2

1 $y^2, 15$ 2 $4x+3$ 3 x^2-3y
4 25 5 x

3 [a] $5xy(3x-5y+2)$
[b] $6x^2y-3xy^2+4$

4 [a] $-3x+9$
[b] $k=6$

5 [a] x^2
[b] 1 39996 2 961

Model 2

1

1 (c) 2 (d) 3 (a) 4 (d) 5 (d) 6 (d)

2

1 $9a, 14$ 2 $25b^2$ 3 $x-y$
4 $9x^2$ 5 21

3 [a] $2x^2+3x-1$
[b] $9x^3(3x-2)$

4 [a] Width $=(x+5)$ length units
, perimeter $=38$ length units
[b] The expression $=a^2+b^2$
The numerical value $=5$

5 [a] 15700
[b] The expression $=5x^2-4x$
The numerical value $=9$

Answers of unit three

Answers of Exercise 15

1

1. The mean $= \frac{4+6}{2} = 5$
2. The mean $= \frac{3+5}{2} = 4$
3. The mean $= \frac{3+4}{2} = 3\frac{1}{2}$
4. The mean $= \frac{2+4+6}{3} = 4$
5. The mean $= \frac{1+3+5}{3} = 3$
6. The mean $= \frac{1+2+3+4+5}{5} = 3$
7. The mean $= \frac{6+10}{2} = 8$
8. The mean $= \frac{\frac{1}{2}+1}{2} = \frac{3}{4}$
9. The mean $= \frac{35+50+60+55}{4} = 50$

2

The mean $= \frac{124+130+122+126+128}{5}$

$= \frac{630}{5} = 126$ cm.

3

The mean $= \frac{89+91+96}{3} = 92$ marks.

4

The mean $= \frac{25^\circ+27^\circ+31^\circ+23^\circ+22^\circ+22^\circ+18^\circ}{7}$

$= 24^\circ$

5

The mean $= \frac{3+2+0+6+1+6}{6} = 3$ goals.

6

The mean $= \frac{3\frac{1}{2}+3+2\frac{1}{2}+3+4}{5} = 3\frac{1}{5}$ hours.

7

1. 20.75
2. 3
3. 3
4. 4
5. 6

8

1. (d)
2. (a)
3. (a)
4. (d)
5. (d)
6. (a)
7. (c)

9

Let the required number be L

1. $L = \left(\frac{1}{3}+\frac{2}{3}\right) \div 2 = \frac{3}{3} \times \frac{1}{2} = \frac{1}{2}$
2. $L = \left(-\frac{3}{5}+\left(-\frac{1}{5}\right)\right) \div 2$

   $= -\frac{4}{5} \times \frac{1}{2} = -\frac{2}{5}$
3. $L = \left(1\frac{1}{2}+2\right) \div 2 = 3\frac{1}{2} \times \frac{1}{2}$

   $= \frac{7}{2} \times \frac{1}{2} = \frac{7}{4} = 1\frac{3}{4}$

10

The sum of Youssif's marks in the first 3 tests

$= 3 \times 16 = 48$ marks

The sum of Youssif's marks in the next 2 tests

$= 2 \times 18 = 36$ marks

The sum of Youssif's marks in the 5 tests

$= 48 + 36 = 84$ marks

The mean = sum of marks ÷ number of tests

$= \frac{84}{5} = 16.8$ marks

11

The sum of Magdi's marks in 4 tests $= 4 \times 16$

$= 64$ marks

Let the mark of Magdi in the 5th test $= X$

$\frac{64+X}{5} = 18$ $\quad$ $64 + X = 90$

$X = 90 - 64 = 26$ marks

The mark of Magdi in the 5th test should be 26 marks

12

The mean

$= \frac{(6\times4)+(9\times7)+(12\times8)+(15\times5)+(17\times6)}{30}$

$= \frac{24+63+96+75+102}{30} = 12$ marks

Answers of Exercise 16

1

1 (b) 2 (c) 3 (c) 4 (b) 5 (a)
6 (c) 7 (c) 8 (c) 9 (d) 10 (c)

2

1 The order is : $-2, -1, 0, 1, 5$
The median $= 0$

2 The order is : $-12, -2, -2, 8, 10, 18$
The median $= \frac{-2+8}{2} = 3$

3 The order is : $1, \frac{1}{2}, \frac{1}{4}$
The median $= \frac{1}{2}$

4 The order is : $\frac{5}{6}, \frac{7}{15}, \frac{2}{5}, \frac{3}{10}$
The median $= \frac{\frac{7}{15}+\frac{2}{5}}{2} = \frac{13}{30}$

5 The order is : $0.2, 2.3, 2.8, 2.9, 3.2$
The median $= 2.8$

6 The order is : $0.8, \frac{3}{5}, \frac{1}{2}, 0.4, 0.25, \frac{5}{25}$
The median $= 0.45$

3

The order is : $6, 6, 7, 8, 10$
The median of the number of absent pupils $= 7$ pupils

4

• The ascending order of studying hours of Sally is :
$2, 3, 3.5, 4.5, 5, 7$
The median of Sally $= \frac{3.5+4.5}{2} = 4$ hours.
• The ascending order of studying hours of Basma is :
$2, 3, 3, 4, 4.5, 6$
The median of Basma $= \frac{3+4}{2} = 3.5$ hours.

5

The ascending order of heights is : 116 , 117 , 118 , 120 , 120 , 121 , 122 , 123 , 124 , 125 , 126 , 127 , 128 , 128 , 131 , 133 , 133 , 134 , 135 , 135

The median $= \frac{125+126}{2} = 125.5$ cm.

6

1 The descending order of the marks is :
48 , 47 , 44 , 41 , 37 , 35
The median $= \frac{44+41}{2} = 42.5$ marks.

2 The mean $= \frac{41+35+47+37+44+48}{6}$
$= 42$ marks.

7

1 2 2 7

Answers of Exercise 17

1

1 the most common data 2 2 3 11
4 4 5 red 6 pen 7 3 8 7
9 8 10 10

2 The mode mark $= 18$ marks.

3 The mode number of studying hours $= 27$ hours.

4 The mode temperature degree $= 21$ degrees.

5

1 • The mean $= \frac{2+5+8+12+13+5+4}{7} = 7$
• The ascending order of the values is :
2 , 4 , 5 , 5 , 8 , 12 , 13
The median $= 5$
• The mode $= 5$

2 • The mean $= \frac{5+4+10+3+3+4+7+4+6+5}{10}$
$= 5.1$
• The descending order of the values is :
10 , 7 , 6 , 5 , 5 , 4 , 4 , 4 , 3 , 3
The median $= \frac{5+4}{2} = 4.5$
• The mode $= 4$

6

The mode mark $= 7$ marks

1 The number of pupils who obtained a mark more than the mode mark = the number of pupils who obtained 8 , 9 and 10 marks $= 6+3+2 = 11$ pupils

2 The number of pupils who obtained a mark less than the mode mark = the number of pupils who obtained 6 and 5 marks $= 4+8 = 12$ pupils

Answers of Unit 3

Answers of activities on unit three

1 30

2

The sum of marks of Karem in 5 tests $= 5 \times 84 = 420$

The sum of marks of Karem in the first three tests $= 3 \times 80 = 240$

The sum of marks of Karem in the last two tests $= 420 - 240 = 180$

, Then the mean of his marks in the last two tests $= \frac{180}{2} = 90$ marks.

3

1. The mean = 5.5 The median = 5.5
2. The mean = 6 The median = 6
3. The mean = 50.5 The median = 50.5
4. The mean = 51 The median = 51
5. The mean = 5 The median = 5
6. The mean = 50 The median = 50

* There is not a mode for any one from the previous.

Answers of exams on unit three

Model 1

1

1. 3
2. the third
3. good
4. 72
5. 3

2

1. (c)
2. (b)
3. (b)
4. (a)
5. (c)

3

1. 15 minutes
2. 18 minutes
3. 17 minutes

4

1. 7 hours.
2. The median number of hours for Mahmoud = 7 hours
   The median number of hours for Mohamed = 8 hours
3. 9 hours.

Model 2

1

1. (b)
2. (d)
3. (a)
4. (b)
5. (d)

2

1. 48
2. $7\frac{1}{2}$
3. The most common data
4. 10
5. 5

3

1. $41\frac{1}{2}$ marks
2. 41 marks.

4

The mode = 22 years.

Answers of accumulative basic skills

1

1. $\frac{1}{2}$
2. $\{0\}$
3. 70
4. $3x$
5. $x + 1$
6. 4
7. 1 : 3
8. 200
9. 21 kg. , 33 kg.
10. 2
11. $\frac{6}{7}$, $\frac{50}{51}$
12. 13

2

1. (c)
2. (c)
3. (a)
4. (d)
5. (d)
6. (d)
7. (a)
8. (b)
9. (c)
10. (d)
11. (c)
12. (b)

Answers of unit four

Answers of Exercise 1

1

1 ∈ 2 ∉ 3 ∈ 4 ∉

5 ⊂ 6 ⊄ 7 ⊂ 8 ⊂

2

1 acute 2 obtuse 3 right

4 reflex 5 straight 6 acute

7 right 8 reflex

3

1 60° 2 30° 3 42° 4 $67\frac{1}{2}$°

5 $36\frac{3}{4}$° 6 zero° 7 64° 8 90°

4

1 160° 2 90° 3 28° 4 170°

5 $87\frac{1}{2}$° 6 180° 7 zero° 8 38°

5

m (∠ ABC)	50°	30°	105°	123°	179°	10°
m (reflex ∠ ABC)	310°	330°	255°	237°	181°	350°

6

1 The union of two rays with the same starting point

2 180° , 0° 3 90° 4 90° , 0°

5 90° 6 180° 7 supplementary

8 complementary 9 supplementary

10 on the same straight line

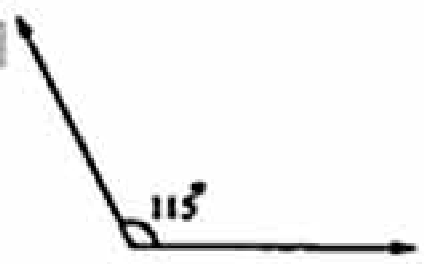
11 180° , a straight 12 40° , 130°

13 60° , 120° 14 30° , 60°

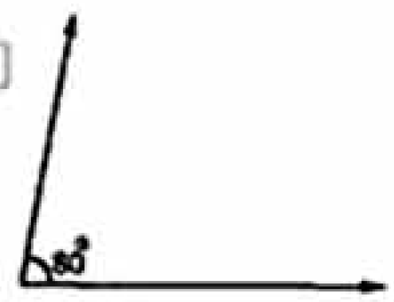
15 an acute , an obtuse 16 right , straight

17 zero , right 18 an acute

7

1


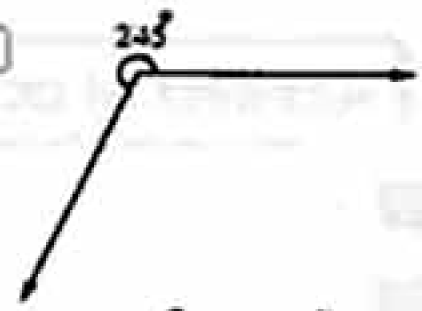
«obtuse angle»

2


«acute angle»

3


«reflex angle»

4


«reflex angle»

5


«straight angle»

8

1 240° 2 80° 3 120°

9

[redacted] ∠ AFC 2 ∠ CFB

[redacted] CFB 4 CFA , EFD

[redacted] a straight , a right 6 AFC , CFD

10

1 25° 2 60° 3 25°

11

1 114° 2 55° 3 80°

4 40° 5 120° 6 140°

12

1 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.

The reason : m (∠ ACD) + m (∠ DCB) = 114° + 66° = 180°

2 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line.

The reason : m (∠ ACD) + m (∠ DCB) = 62° + 116° = 178°

3 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line.

The reason : m (∠ ACD) + m (∠ DCE) + m (∠ ECB)
= 58° + 85° + 39° = 182°

4 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.

The reason : m (∠ ACF) = m (∠ FCE) = m (∠ ECB) = 60°
, m (∠ ACF) + m (∠ FCE) + m (∠ ECB)
= 60° + 60° + 60° = 180°

5 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.

The reason : m (∠ ACD) + m (∠ DCE)
+ m (∠ ECF) + m (∠ FCB)
= 28° + 32° + 64° + 56° = 180°

6 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line.

The reason : m (∠ ACD) = m (∠ DCE) = 40° ,
m (∠ ECF) = m (∠ FCB) = 51°
, m (∠ ACD) + m (∠ DCE) + m (∠ ECF)
+ m (∠ FCB)
= 40° + 40° + 51° + 51° = 182°

Answers of Unit 4

13

1 (b) 2 (c) 3 (b) 4 (a)
5 (a) 6 (b) 7 (c) 8 (d)

14

1 equal in measure 2 45° 3 90°
4 complementary 5 140° 6 90°
7 122° 8 35°
8 123° , 142°

15 $\overline{CA}$ and $\overline{CH}$ are on the same straight line.

The reason : Since the value of the part $= \frac{60°}{3} = 20°$

Therefore : m (∠ ACB) = 2 × 20° = 40°

, m (∠ DCH) = 4 × 20° = 80°

, m (∠ ACB) + m (∠ BCD) + m (∠ DCH)

= 40° + 60° + 80° = 180°

Answers of Exercise 2

1

1 70° 2 60° 3 110° 4 115°
5 60° 6 120° 7 140° 8 100°
9 100° 10 120° 11 50° 12 80°
13 38° 14 130° 15 90°

2

1 equal in measure. 2 360° 3 50°
4 135° 5 70° 6 50°

3

1 (c) 2 (a) 3 (a) 4 (c)
5 (b) 6 (b) 7 (d)

4 m (∠ ABD) = 45° , m (∠ DBE) = 90°
, m (∠ CBE) = 135°

5 m (∠ BME) = 45° , m (∠ DME) = 90°
, m (∠ AMC) = 45° , m (∠ AME) = 135°

6 1 m (∠ CMD) = 50° 2 m (∠ AMC) = 110°

7 m (∠ DMX) = 110°

8 1 m (∠ AMD) = 125° 2 m (∠ DMY) = 45°
3 m (∠ BMY) = 100°

9 m (∠ AMD) = 101°

10

$\overrightarrow{OA}$ and $\overrightarrow{OD}$ are on the same straight line.

The reason :

m (∠ AOH) + m (∠ HOD) = 140° + 40° = 180°

, m (∠ BOC) = 60°

11 m (∠ FBC) = 125°

12 m (∠ DCY) = 124°

13 m (∠ BAC) = 50° , m (∠ ABC) = 70° , m (∠ C) = 60°

14

m (∠ AOB) + m (∠ AOH) = 80°

, m (∠ AOB) = $\frac{2}{5}$ × 80° = 32°

, m (∠ AOH) = $\frac{3}{5}$ × 80° = 48°

15

Since : $\overleftrightarrow{AB} \cap \overleftrightarrow{HO} = \{M\}$

, then m (∠ AMO)

= m (∠ BMH) (V.O.A.)

, then m (∠ AMO) = 140° ÷ 2 = 70°

, since M ∈ $\overleftrightarrow{CD}$

, then m (∠ AMC) + m (∠ AMO) +

m (∠ DMO) = 180°

, then m (∠ AMC) + 70° + m (∠ DMO) = 180°

, then m (∠ AMC) + m (∠ DMO) = 180° − 70° = 110°

, since m (∠ AMC) : m (∠ DMO) = 2 : 3

, then the sum of the parts = 2 + 3 = 5

, then the value of one part = 110° ÷ 5 = 22°

, then m (∠ DMO) = 3 × 22° = 66°

, since $\overleftrightarrow{CD} \cap \overleftrightarrow{HO} = \{M\}$

, then m (∠ CMH) = m (∠ DMO) (V.O.A.)

, then m (∠ CMH) = 66°

Answers of Exercise 3

1

1. they are equal in length.
2. they are equal in measure.
3. side , angle , the other polygon
4. congruent 5. CD 6. zero 7. 50°
8. 90° 9. 45° 10. =
11. $\overline{LX}$, YZL
12. their sides , their dimensions.

2

1. R 2. ROHES 3. 4
4. C 5. 5 6. H

3

1. ED 2. EF 3. A 4. D
5. FCB 6. 110° 7. 10 8. 90°
9. 140° 10. $\overline{CF}$

4

1. XC 2. AD 3. BY 4. a common
5. 120° 6. 85° 7. 65° 8. 90°

5

1. 90° 2. 80° 3. 6 4. AFDE , BFDC

6 32 cm. **7** 40 cm.

8

1. M 2. MDE 3. F
4. 125° 5. 55° 6. 9

9

1. M 2. EFM 3. 90° 4. 3 5. 180°

Answers of exams on the first part of unit four

Model 1

1

1. (a) 2. (c) 3. (c)
4. (a) 5. (b) 6. (c)

2

1. equal in measure 2. 40° , 130°
3. supplementary 4. zero
5. equal in measure

3

[a] $X = 50°$ [b] m (∠ ABD) = 50°

4

1. AF 2. DCE 3. 120°
4. $\overline{DC}$ 5. 90° 6. 120°
7. $\overrightarrow{FE}$

5

[a] m (∠ EMD) = 60°

[b] No , because

m (∠ ACD) + m (∠ DCE) + m (∠ ECF) + m (∠ FCB)
$= 27° + 65° + 49° + 38° = 179° \neq 180°$

Model 2

1

1. (c) 2. (c) 3. (a)
4. (b) 5. (c) 6. (c)

2

1. complementary 2. a zero
3. equal in length
4. 360° 5. 45°

3

[a] m (∠ CMD) = 120°

[b] m (∠ ACE) = 30°

4

[a] m (∠ AMC) = 110°

[b] 1. The perimeter of the polygon ABCD = 18 cm.
2. m (∠ F) = 90° , m (∠ E) = 90°

5

[a] $X = 50°$ [b] m (∠ FBC) = 70°

Answers of Exercise 4

1

1. the included angle in one of them are congruent to their corresponding elements in the other.

Answers of Unit 4

2 the side drawn between their vertices.

3 side in one of the two triangles , side.

4 the hypotenuse and one side in one of them are congruent to their corresponding elements in the other.

5 congruent 6 XY , C 7 ABC , LMN

2

1 The two triangles are congruent (Two sides and included angle).

2 The two triangles are congruent (Three sides).

3 The two triangles are congruent (Two sides and included angle).

4 The two triangles are congruent (Three sides).

5 The two triangles are congruent (Two sides and included angle).

6 The two triangles are not congruent because the two congruent sides are not corresponding.

7 The two triangles are congruent (Hypotenuse and one side).

8 The two triangles are congruent (Hypotenuse and one side).

9 The two triangles are not congruent because the two congruent sides are not corresponding.

10 The two triangles are congruent (Two angles and one side).

11 The two triangles are congruent (Hypotenuse and one side).

12 The two triangles are congruent (Two angles and one side).

13 The data is not enough to prove the congruence of the two triangles.

14 The data is not enough to prove the congruence of the two triangles.

15 The two triangles are congruent (Two sides and included angle).

16 The two triangles are congruent (Three sides).

17 The two triangles are not congruent because the given angle is not included between the two sides.

18 The two triangles are congruent (Hypotenuse and one side).

19 The two triangles are congruent (Three sides).

20 The two triangles are not congruent because the two congruent sides are not corresponding.

21 The two triangles are congruent (Two sides and included angle).

22 The two triangles are congruent (Two angles and one side).

23 The two triangles are congruent (Two sides and included angle).

24 The two triangles are congruent (Hypotenuse and one side).

25 The two triangles are congruent (Two angles and one side).

26 The data in not enough to prove the congruence of the two triangles.

3 $X = 52°$

4 1 ACD 2 30° 3 7 4 125°

5 45°

6

1 50° 2 35° 3 Δ EFD 4 $\overline{ED}$ 5 7

7 1 ABC 2 100° 3 6

8 1 DCB 2 30° 3 ACB

9 1 55° 2 55° 3 110°

10 1 (d) 2 (d) 3 (b) 4 (d) 5 (b)

11

$\Delta ABC \equiv \Delta ADC$ (Two sides and included angle).

i.e. $AD = AB = 5$ cm. , $m(\angle D) = m(\angle B) = 57°$

$m(\angle DAC) = 180° - (90° + 57°) = 33°$

12

From ΔCDB : $m(\angle CDB) = 180° - (110° + 30°) = 40°$

i.e. $\Delta ADB \equiv \Delta CDB$ (Two sides and included angle).

The length of $\overline{BC} = BA = 7$ cm.

$m(\angle BAD) = m(\angle BCD) = 110°$

13

From ΔADB : $m(\angle ABD) = 180° - (50° + 110°) = 20°$

$\Delta ABD \equiv \Delta CBD$ (Three sides).

i.e. $m(\angle ABD) = m(\angle CBD) = 20°$

, $m(\angle ABC) = 20° + 20° = 40°$

14

$\Delta ABD \equiv \Delta CBD$ (Two angles and a side).

i.e. The length of $\overline{CB} = AB = 8$ cm.

The length of $\overline{AD} = CD = 6$ cm.

15

Yes : Δ AMC $\equiv$ Δ BMD

Because : $\begin{cases} AM = BM \\ CM = DM \\ m(\angle AMC) = m(\angle BMD) \end{cases}$

"vertically opposite angles"

16

Yes : Δ ACE $\equiv$ Δ DBE

Because : $\begin{cases} AE = ED \\ m(\angle A) = m(\angle D) \\ m(\angle AEC) = m(\angle DEB) \end{cases}$

"vertically opposite angles"

and we deduce that : CE = EB

17

From Δ ABC : m ($\angle$ ACB) = 180° – (90° + 57°) = 33°

Δ ABC $\equiv$ Δ EDA (Hypotenuse and one side).

m ($\angle$ E) = m ($\angle$ BAC) = 57° ,

m ($\angle$ EAD) = m ($\angle$ ACB) = 33°

18

Δ AEC $\equiv$ Δ ADC (Three sides).

m ($\angle$ AEC) = m ($\angle$ ADC) = 90°

i.e. $\overline{AE} \perp \overline{BC}$

From Δ ABE : m ($\angle$ BAE) = 180° – (30° + 90°) = 60°

19

Since $\angle$ ABD supplements $\angle$ ABC ,

$\angle$ ACE supplements $\angle$ ACB ,

Where m ($\angle$ ABC) = m ($\angle$ ACB) ,

then m ($\angle$ ABD) = m ($\angle$ ACE)

In $\Delta\Delta$ ABD , ACE :

Since m ($\angle$ DAB) = m ($\angle$ EAC)

, m ($\angle$ ABD) = m ($\angle$ ACE)

, then m ($\angle$ D) = m ($\angle$ E)

In $\Delta\Delta$ ABD , ACE :

$\begin{cases} BD = CE \\ m(\angle ABD) = m(\angle ACE) \\ m(\angle D) = m(\angle E) \end{cases}$

, then Δ ABD $\equiv$ Δ ACE

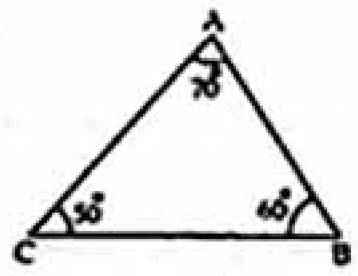
, then we deduce that : AD = AE .

20

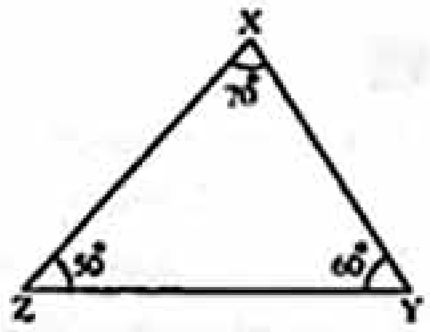
1. 70°
2. 50°
3. 60°
4. 90°
5. 3 cm.

21

[a]



[b]



Yes since Δ ABC is not congruent to Δ XYZ

22

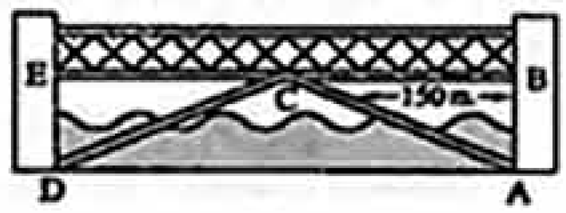
1. Δ RPQ $\equiv$ Δ BCA (Two sides and included angle)
   X = QR = AB = 4.8 cm. , Y° = m ($\angle$ R) = m ($\angle$ B) = 42°
2. Δ DEF $\equiv$ Δ CAB (Two sides and included angle)
   X = DE = CA = 16 cm. , Y° = m ($\angle$ D)
   but : DF = DE = 16 cm. i.e. m ($\angle$ E) = m ($\angle$ F) = 75°
   Therefore : Y° = 180° – (75° + 75°) = 30°
3. Δ ABC $\equiv$ Δ RST (Two angles and a side).
   X = BC = ST = 69 cm. , Y° = m ($\angle$A) = m ($\angle$ R) = 83°
4. Δ ABC $\equiv$ Δ EDB (Three sides).
   Y° = m ($\angle$ A) = m ($\angle$ BED) = 64°
   X° = m ($\angle$ C) = 180° – (64° + 30°) = 86°
5. Δ ABC $\equiv$ Δ PNA (Two angles and a side).
   X = AN = CB = 22 cm. , Y = 39 – 22 = 17 cm.

23

1. The two triangles are congruent (Two sides and included angle).
2. The data is not enough.
3. The two triangles are congruent (Three sides).
4. The data is not enough because the given angle is not included between the two sides.
5. The two triangles are congruent (Two angles and a side).
6. The data is not enough because the two sides $\overline{AC}$ and $\overline{PG}$ are not corresponding sides.

24

In $\Delta\Delta$ ABC and DEC :



$\begin{cases} AB = DE \\ AC = CD \\ m(\angle ABC) = m(\angle DEC) \end{cases}$

, then $\Delta ABC \equiv \Delta DEC$, then we deduce that :

$BC = CE = 150$ m.

Then , the length of the bridge = 300 m.

25

Since $\overline{AE} \cap \overline{DB} = \{C\}$

Then , $m(\angle DCE) = m(\angle ACB)$ « V.O.A.»

In $\Delta\Delta$ ABC and EDC :

$$\begin{cases} DC = BC \\ m(\angle DCE) = m(\angle BCA) \\ m(\angle D) = m(\angle B) = 90^\circ \end{cases}$$

Then $\Delta ABC \equiv \Delta EDC$, then we deduce that :

$AB = DE = 400$ m.

Then , the width of the river = 400 m.

26

From the square ABCD :

$m(\angle XAB) = 90^\circ - 70^\circ = 20^\circ$

, $\Delta ABX \equiv \Delta BCY$ "Two sides and included angle"

Then : $m(\angle YBC) = m(\angle XAB) = 20^\circ$

Answers of Exercise 5

1

1 perpendicular. 2 parallel.
3 equal in measure. 4 equal in measure.
5 supplementary. 6 parallel.
7 parallel. 8 parallel.
9 equal in length.

2

Fig. 1 : $m(\angle CFE) = 110^\circ$

Fig. 2 : $m(\angle DFY) = 63^\circ$

Fig. 3 : $m(\angle XEB) = 116^\circ$, $m(\angle EFD) = 116^\circ$

3

Fig. 1 : $m(\angle ABD) = 60^\circ$, $m(\angle D) = 60^\circ$

Fig. 2 : $m(\angle D) = 51^\circ$

4

Fig. 1 : 2 cm. Fig. 2 : 8 cm. Fig. 3 : 15 cm.

5

Fig. 1 : $m(\angle AEM) = 180^\circ - m(\angle MEB)$

$= 180^\circ - 122^\circ = 58^\circ$

i.e. $m(\angle AEM) = m(\angle CFE) = 58^\circ$

and they are two corresponding angles

, then $\overline{AB} // \overline{CD}$

Fig. 2 : $m(\angle BEF) = 180^\circ - m(\angle BEM)$

$= 180^\circ - 100^\circ = 80^\circ$

i.e. $m(\angle BEF) = m(\angle DFN) = 80^\circ$

and they are two corresponding angles

, then $\overline{AB} // \overline{CD}$

Fig. 3 : $m(\angle BEF) = m(\angle AEM) = 132^\circ$ (V.O.A)

i.e. $m(\angle BEF) = m(\angle DFN) = 132^\circ$

and they are two corresponding angles

, then $\overline{AB} // \overline{CD}$

6

Fig. 1 : $m(\angle B) = m(\angle BCY) = 60^\circ$

i.e. $m(\angle B) = m(\angle XAD) = 60^\circ$

and they are two corresponding angles

, then $\overline{AD} // \overline{BC}$

Fig. 2 : $m(\angle C) = m(\angle EBC) = 110^\circ$

i.e. $m(\angle C) + m(\angle D) = 180^\circ$

and they are two interior angles in the same side of the transversal.

, then $\overline{AD} // \overline{BC}$

Fig. 3 : $m(\angle C) = 180^\circ - m(\angle B) = 180^\circ - 124^\circ = 56^\circ$

i.e. $m(\angle C) = m(\angle CDE) = 56^\circ$

and they are two alternate angles

, then $\overline{AD} // \overline{BC}$

7

1 (c)	2 (b)	3 (d)	4 (c)
5 (c)	6 (a)	7 (b)	8 (b)
9 (c)	10 (a)	11 (a)	12 (b)
13 (b)			

8

Since : $\overline{AO} // \overline{HD} // \overline{YX} // \overline{CB}$

, $\overline{AB}$ and $\overline{AC}$ are two transversals to them.

, $AD = DX = XB$

, then : $AH = HY = YC = \frac{18}{3} = 6$ cm.

, then : $AY = 12$ cm.

9

Since : $\overline{AB} // \overline{EF} // \overline{CD}$, $\overline{AD}$ and $\overline{BC}$ are transversals to them

, AE = ED

, then : BE = EC

, then : BE = 4 cm.

10

m (∠ AEF) = m (∠ A) = 42° (alternate angles)

, m (∠ CEF) = 180° − m (∠ ECD)

= 180° − 117° = 63°

, then : m (∠ AEC) = 42° + 63° = 105°

11

m (∠ ACD) = m (∠ A) = 40° (alternate angles)

m (∠ DCE) = m (∠ E) = 55° (alternate angles)

, then : m (∠ ACE) = 40° + 55° = 95°

12

m (∠ B) = m (∠ DAB) = 50° (alternate angles)

m (∠ C) = m (∠ EAD) = 70° (corresponding angles)

m (∠ BAC) = 180° − (50° + 70°) = 60°

13

1. m (∠ ACD) = m (∠ A) = 35°
   Since $\overline{CD}$ bisects ∠ ACE
   , then : m (∠ DCE) = m (∠ ACD) = 35°
2. m (∠ CEF) + m (∠ DCE) = 180°
   (Two interior angles in the same side of the transversal)
   , then : m (∠ CEF) = 180° − 35° = 145°

14

m (∠ ABC) = m (∠ BAE) = 2 × 56° = 112°

(alternate angles)

, then m (∠ C) + m (∠ ABC) = 180°

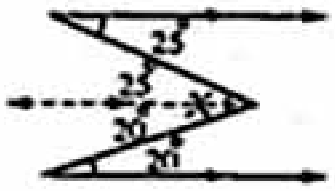
(Two interior angles in the same side of the transversal)

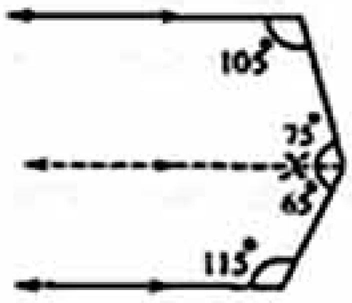
, then m (∠ C) = 180° − 112° = 68°

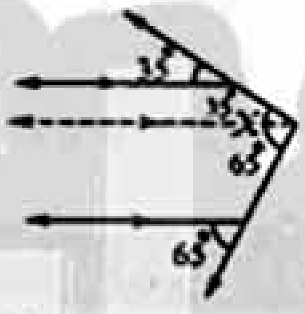
15

1. m (∠ X) = m (∠ XYM) = 100° (alternate angles)
2. m (∠ Z) = m (∠ XYM) = 100° (corresponding angles)
3. m (∠ L) + m (∠ X) = 180°
   (Two interior angles in the same side of the transversal)
   m (∠ L) = 180° − 100° = 80°

16

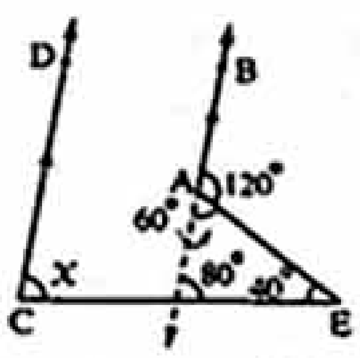
1. X = 60°
2. X = 20°
3. X = 80°
4. 

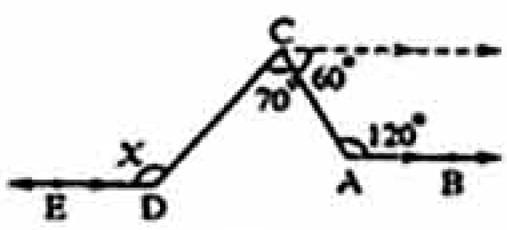
   , then : X = 25° + 20° = 45°
5. 

   , then : X = 75° + 65° = 140°
6. 

   , then : X = 35° + 65° = 100°

17

1. X = 70°
2. X = 40°
3. X = 240°
4. X = 105°
5. 

   , then X = 80°
6. 

   , then X = 60° + 70° = 130°

18

1. $\overleftrightarrow{OX} \parallel \overleftrightarrow{EZ}$ 2. $\overleftrightarrow{KC} \parallel \overleftrightarrow{DE}$ 3. $\overleftrightarrow{AS} \parallel \overleftrightarrow{LT}$

19 Yes ,

m (∠ DCE) = m (∠ D) = 125° (alternate angles)

Then : m (∠ FCE) = 125° – 50° = 75°

i.e. m (∠ FCE) = m (∠ B) = 75°

and they are two corresponding angles.

Then : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CF}$

20

Since : m (∠ XAB) = m (∠ B) = 60°

and they are two alternate angles

, then : $\overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$

m (∠ EDB) + m (∠ B) = 120° + 60° = 180°

and they are interior angles in the same side of the transversal

, then : $\overleftrightarrow{ZL} \parallel \overleftrightarrow{BC}$

i.e. : $\overleftrightarrow{XY} \parallel \overleftrightarrow{ZL} \parallel \overleftrightarrow{BC}$

, AD = DB

, then : AE = $\frac{18}{2}$ = 9 cm.

21 Yes ,

since A ∈ $\overline{BE}$

i.e. m (∠ BAC) = 180° – 100° = 80°

, since $\overrightarrow{AD}$ bisects ∠ BAC

Then : m (∠ DAC) = 80° ÷ 2 = 40°

Then : m (∠ DAC) = m (∠ ACE) = 40°

and they are two alternate angles.

Then : $\overleftrightarrow{AD} \parallel \overleftrightarrow{CE}$

22 Yes ,

i.e. m (∠ B) = m (∠ FAB) = 60° (alternate angles)

, since C ∈ $\overline{BD}$

, then : m (∠ ECD) = 180° ÷ 3 = 60°

i.e. m (∠ B) = m (∠ ECD) = 60°

and they are two corresponding angles.

, then : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CE}$

23 Yes ,

m (∠ A) = m (∠ C) = 45°

and they are two alternate angles.

Then : $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

m (∠ D) + m (∠ E) = 72° + 108° = 180°

and they are two interior angles in the same side of the transversal.

, then : $\overleftrightarrow{DC} \parallel \overleftrightarrow{EF}$ Then : $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC} \parallel \overleftrightarrow{EF}$

24

1. Yes ,

Δ AMB ≡ Δ CMD

Because $\begin{cases} MB = MD \\ MA = MC \\ m(\angle AMB) = m(\angle CMD) \quad (V.O.A) \end{cases}$

2. Yes ,

Δ AMB ≡ Δ CMD

We deduce from the congruence that :

m (∠ B) = m (∠ D)

and they are two alternate angles

, then : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

25 Yes ,

Δ ADB ≡ Δ CBD

Because $\begin{cases} AD = CB \\ \overline{DB} \text{ is a common side} \\ m(\angle ADB) = m(\angle CBD) = 90° \end{cases}$

We deduce from the congruence that :

m (∠ ABD) = m (∠ CDB)

and they are two alternate angles

, then : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

26 Yes ,

AB = CD

AB + BC = CD + BC

AC = BD

Δ ACL ≡ Δ BDM

Because $\begin{cases} AL = BM \\ LC = MD \\ AC = BD \end{cases}$

We deduce from the congruence that :

m (∠ A) = m (∠ MBD) and they are two corresponding angles

, then : $\overleftrightarrow{AL}$ // $\overleftrightarrow{BM}$

, m (∠ D) = m (∠ LCA) and they are two corresponding angles

, then : $\overleftrightarrow{CL}$ // $\overleftrightarrow{DM}$

27 Yes ,

BF = CE

BF + FC = CE + FC

BC = EF

Δ ABC ≡ Δ DEF

Because $\begin{cases} m(\angle B) = m(\angle E) \text{ (alternate angles)} \\ m(\angle ACB) = m(\angle DFE) \text{ (alternate angles)} \\ BC = EF \end{cases}$

We deduce from the congruence that :

$\overline{AB} \equiv \overline{DE}$

28

m (∠ AFG) = m (∠ B)

and they are two corresponding angles

, then : $\overleftrightarrow{FG}$ // $\overleftrightarrow{BK}$

, m (∠ B) = m (∠ K)

and they are two alternate angles.

, then : $\overleftrightarrow{BF}$ // $\overleftrightarrow{KM}$

, m (∠ K) = m (∠ M)

and they are two alternate angles

, then : $\overleftrightarrow{BK}$ // $\overleftrightarrow{ME}$

Since $\overleftrightarrow{FG}$ // $\overleftrightarrow{BK}$, $\overleftrightarrow{BK}$ // $\overleftrightarrow{ME}$

, then : $\overleftrightarrow{FG}$ // $\overleftrightarrow{ME}$

29 Yes ,

i.e. $\overleftrightarrow{BC}$ // $\overleftrightarrow{ED}$

, Then : m (∠ 2) = m (∠ 3) (alternate angles)

, m (∠ 1) = m (∠ 4)

, then : m (∠ 1) + m (∠ 2) = m (∠ 3) + m (∠ 4)

, then : m (∠ ABD) = m (∠ FDB)

and they are alternate angles.

, then : $\overleftrightarrow{BA}$ // $\overleftrightarrow{DF}$

30 Yes ,

i.e. $\overleftrightarrow{AM}$ // $\overleftrightarrow{DE}$ // $\overleftrightarrow{BC}$, AD = DB

, then : FE = EC , ME = EL

, then : Δ FEM ≡ Δ CEL

Because $\begin{cases} FE = CE \\ EM = EL \\ m(\angle FEM) = m(\angle CEL) \end{cases}$ «V.O.A.»

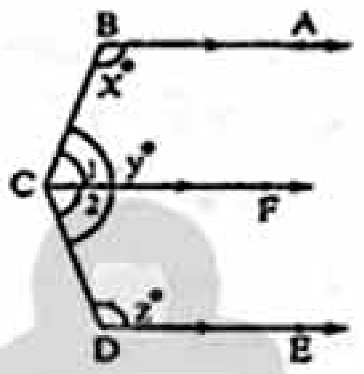
, then : FM = LC

31

1 $\overleftrightarrow{YQ}$ // $\overleftrightarrow{XD}$, $\overleftrightarrow{YR}$ // $\overleftrightarrow{XB}$

2 $\overleftrightarrow{EF}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{GH}$ // $\overleftrightarrow{AB}$ and J // P

32



Construction :

Draw $\overrightarrow{CF}$ // $\overrightarrow{BA}$ // $\overrightarrow{DE}$

From the figure

m (∠ B) + m (∠ 1) = 180°

(Two interior angles in the same side of the transversal)

m (∠ 2) + m (∠ D) = 180°

(Two interior angles in the same side of the transversal)

i.e. m (∠ B) + m (∠ 1) + m (∠ 2) + m (∠ D)

= 180° + 180° = 360°

i.e. m (∠ B) + m (∠ C) + m (∠ D) = 360°

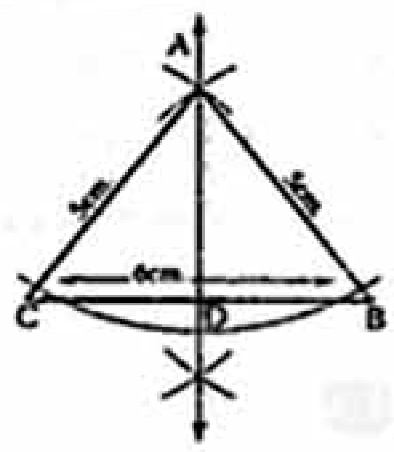
, then : $x + y + z = 360°$

Answers of Exercise 6

1

From drawing and by measuring

AD = 4 cm.



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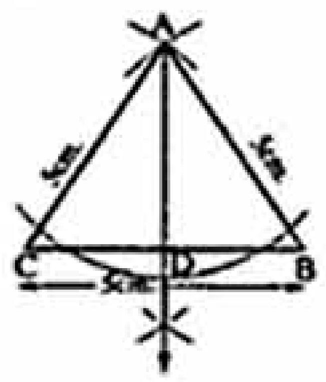


Answers of Unit 4

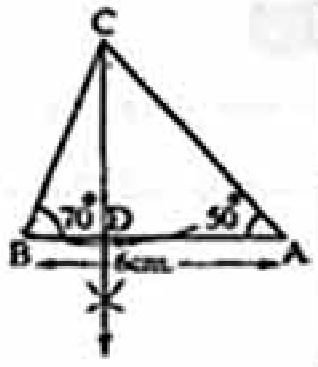
3

From the drawing and by measuring the length of $\overline{CD} = 5$ cm.

the area of

$\Delta ABC = \frac{1}{2} \times 6 \times 5$

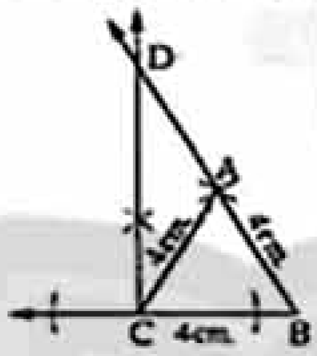
$= 15 \text{ cm}^2.$



4

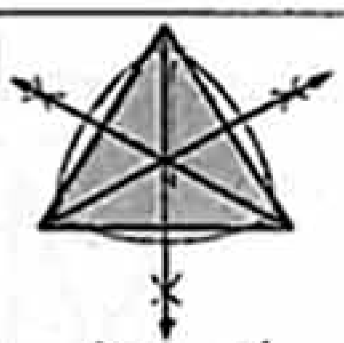
From the figure and by measuring :

DA = 4 cm.



5

1



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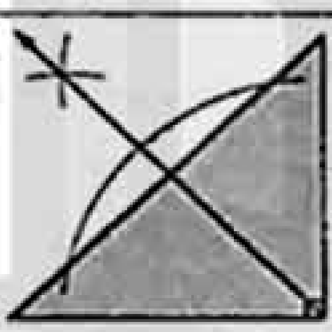

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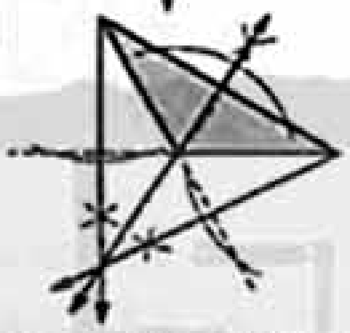
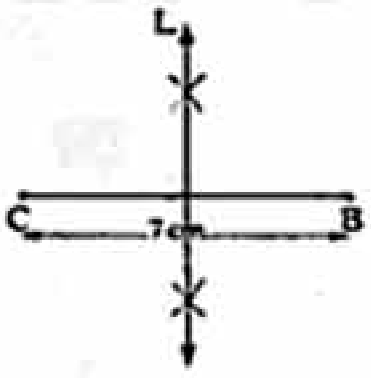


We notice that the straight lines carrying the altitudes of the triangle are concurrent at a point. This point lies :

- Inside the acute-angled triangle.
- At the vertex of the right angle in the right-angled triangle.
- Outside the obtuse-angled triangle.

6



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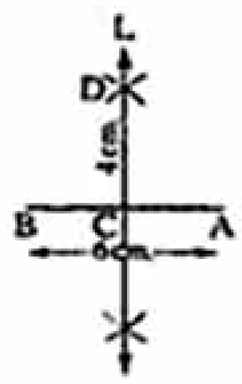
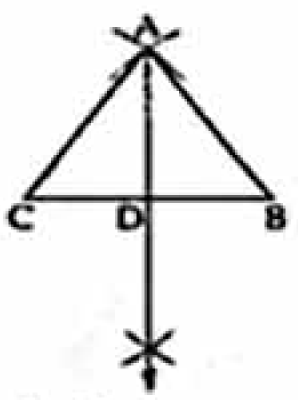


From the figure

DA = DB = 5 cm.

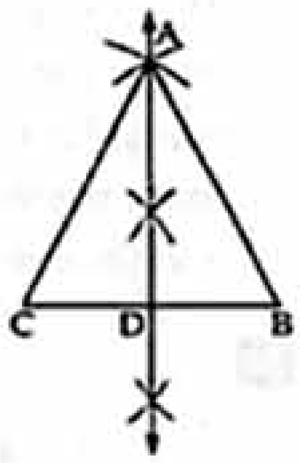
8



From the figure we notice that AB = AC

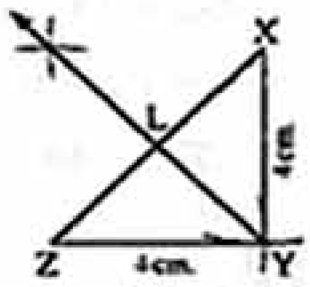
9

Yes , $\overline{AD} \perp \overline{BC}$



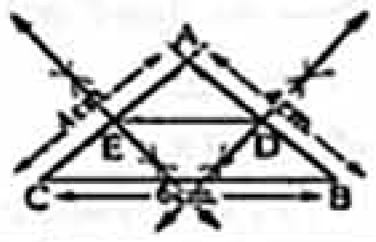
10 By measuring we find that

$m(\angle XLY) = 90°$



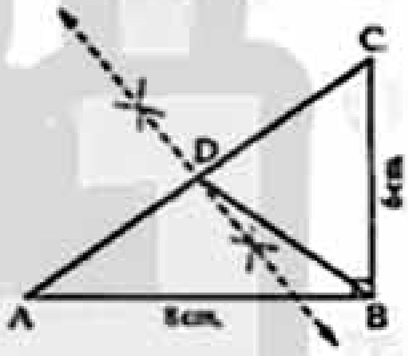
11

From the figure DE = 3 cm.



12

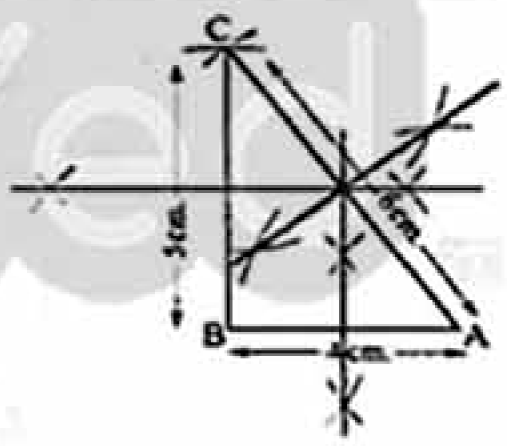
Yes , $BD = \frac{1}{2} AC$



13

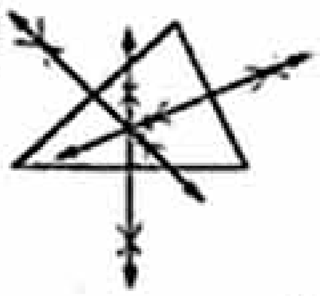
From the figure :

We notice that the bisectors of the sides of the triangle meet at one point.

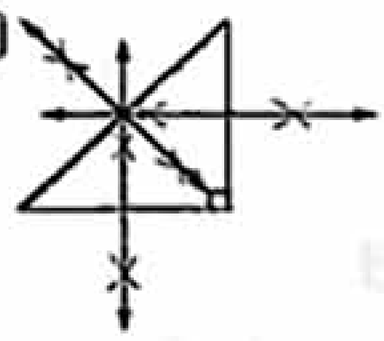


14

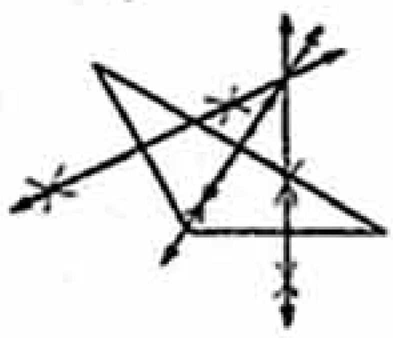
1



2



3

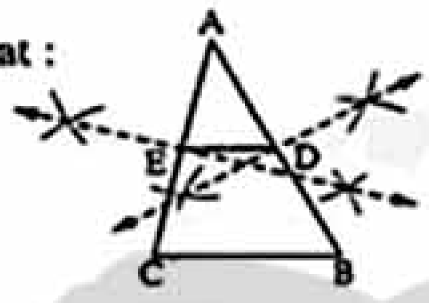


The symmetry axes of the sides of the triangle are meeting at one point and this point lies inside the triangle if it is an acute-angled. At the mid-point of the hypotenuse if it is a right-angled and outside the triangle if it is an obtuse-angled triangle.

15

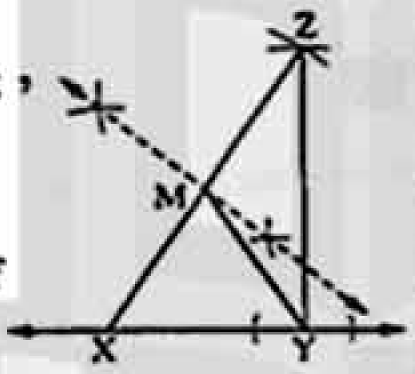
1 From the figure , we find that :
BC = 2 DE

2 Yes , ∠ ABC ≡ ∠ ADE
, $\overline{DE} \parallel \overline{BC}$

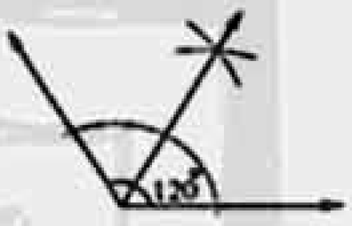


16

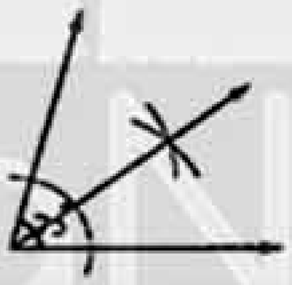
From the figure and by measuring , we find that :
MX = MY = MZ
Draw other triangles by yourself and notice that in each time :
MX = MY = MZ



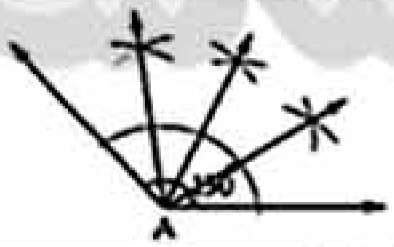
17



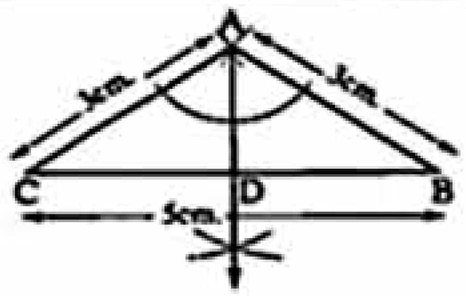
18



19

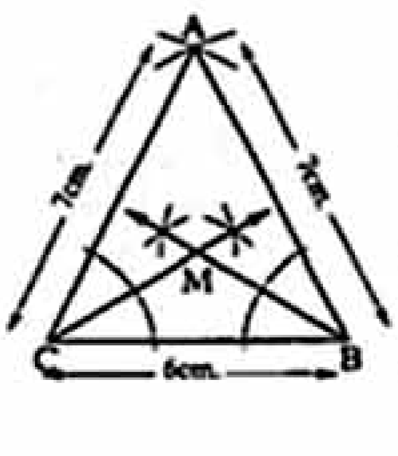


20



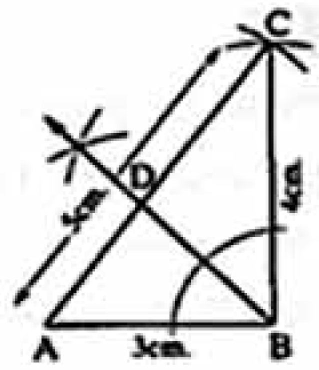
21

From drawing and by measuring :
MB = MC

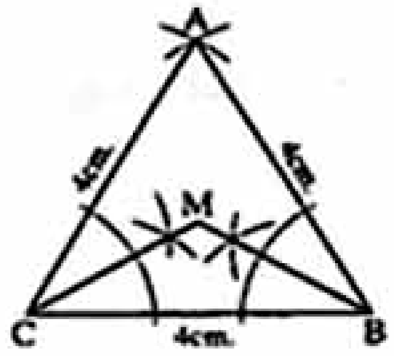


22

From drawing and by measuring :
BD ≃ 2.4 cm.



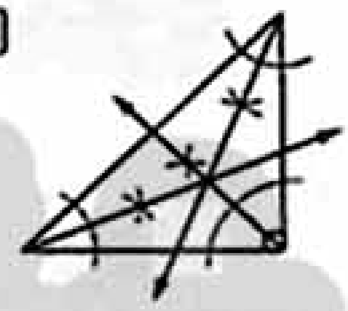
23



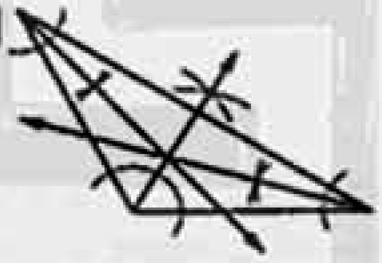
From the drawing and by measuring m (∠ BMC) = 120°

24 1

2

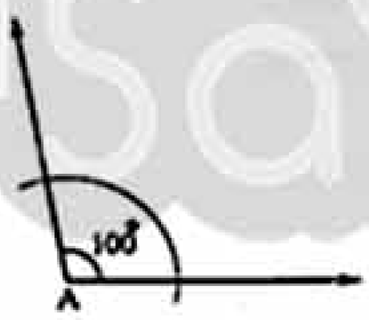


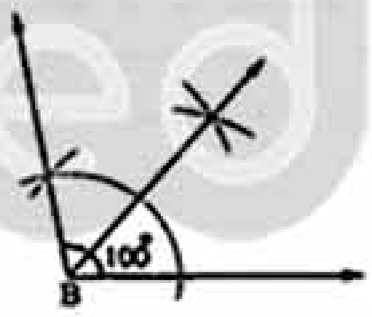
3



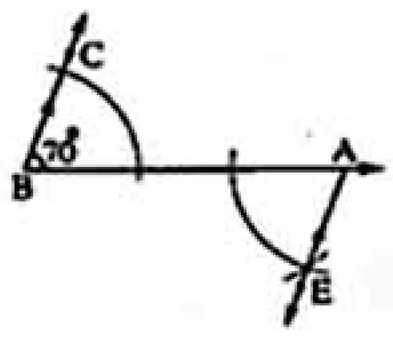
We notice that the three bisectors of the angles of the triangle are concurrent.

25

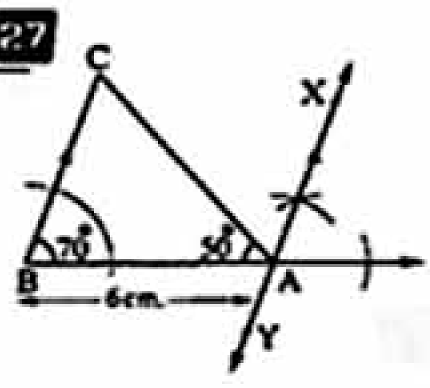




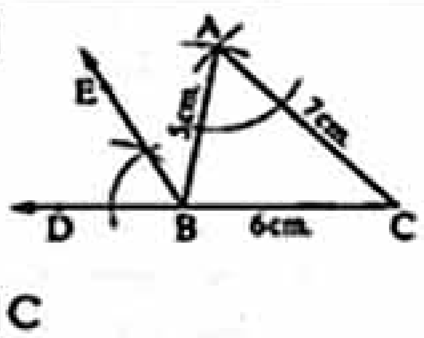
26



27



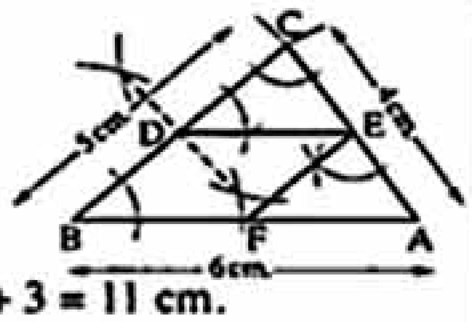
28 1



2 C

Answers of Unit 4

29

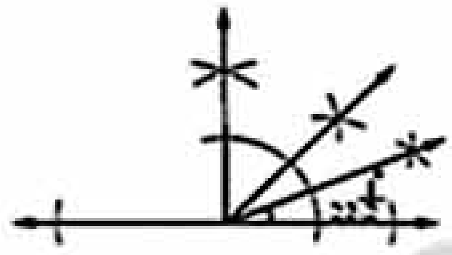


From the figure and by measure ED = 3 cm. , EF = 2.5 cm.

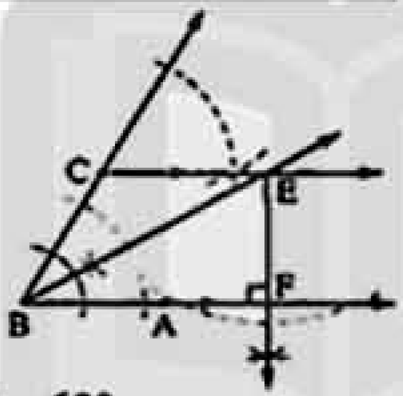
The figure DEFB is a parallelogram

Its perimeter = 2.5 + 3 + 2.5 + 3 = 11 cm.

30



31



Since , m (∠ ABC) = 60°

Then , m (∠ ABE) = 30°

, since m (∠ EFB) = 90°

Then , in Δ EFB :

m (∠ FEB) = 180° − (90° + 30°) = 60°

Then , m (∠ ABC) = m (∠ FEB)

Answers of exams on the second part of unit four

Model 1

1

1 (c)	2 (d)	3 (c)
4 (c)	5 (a)	6 (b)

2

1 side of one triangle 2 supplementary

3 perpendicular to it from its midpoint

4 parallel

5 hypotenuse and a side in one of them are congruent to their corresponding elements of the other triangle.

3

[a] m (∠ B) = 140°

[b] 1 Yes , Δ BAC ≡ Δ DAC

because AB = AD , $\overline{AC}$ is a common side

, m (∠ BAC) = m (∠ DAC)

2 m (∠ D) = 40°

3 CD = 5 cm.

4

[a] Draw by yourself.

[b] 1 m (∠ Y) = 65°

2 Yes , $\overline{YX}$ // $\overline{ZL}$ because m (∠ Y) + m (∠ Z)

= 65° + 115° = 180°

(interior angles on the same side of the transversal).

5

[a] 1 AB = BC

AD = CD

$\overline{BD}$ is a common side.

2 m (∠ ADC) = 60°

[b] Draw by yourself , AD = 4 cm.

Model 2

1

1 (b)	2 (b)	3 (a)
4 (a)	5 (c)	6 (d)

2

1 The side drawn between their vertices of one triangle are congruent to the corresponding elements of the other triangle.

2 parallel

3 axis of symmetry of it

4 perpendicular 5 15

3

[a] AD = 12 cm.

[b] Draw by yourself.

4

[a] Δ ACE ≡ Δ DBE , because m (∠ A) = m (∠ D)

, m (∠ AEC) = m (∠ DEB) (V.O.A)

, AE = DE

[b] m (∠ C) = 64°

5

[a] Draw by yourself.

[b] Δ XFY $\equiv$ Δ XFZ , $\overline{XF}$ bisects $\angle$ X because from congruence.

m ($\angle$ YXF) = m ($\angle$ ZXF)

Answers of accumulative basic skills

1

1 22	2 170
3 5	4 5 length units
5 an infinite number	6 30°
7 28	8 the centre of the circle
9 14	10 16
11 42	12 17.85

2

1 (a)	2 (c)	3 (a)	4 (c)
5 (c)	6 (b)	7 (c)	8 (d)
9 (c)	10 (b)	11 (d)	12 (a)

Guide Answers

of The Notebook

- Worksheets.
- Final Examinations.

Answers of the worksheets on Algebra and Statistics

Worksheet (1)

1

1 a   2 b   3 b
4 a   5 c   6 c

2

1 $\frac{9}{4}$ 2 $\frac{7}{20}$

3 $\frac{-11}{11}$, $\frac{0}{7}$

4

[a] 1 $\frac{3}{4}$ 2 $\frac{2}{5}$

[b] 1 $\frac{5}{7} = \frac{10}{14} = \frac{15}{21} = \frac{20}{28}$ "There are other solutions".

2 $\frac{2}{9} = \frac{4}{18} = \frac{6}{27} = \frac{8}{36}$ "There are other solutions".

Worksheet (2)

1

1 b   2 b   3 c
4 a   5 b   6 d

2

[a] $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

[b] Represent by yourself

The order is : $-\frac{1}{4}$, $\frac{1}{2}$, 1 and $2\frac{1}{2}$

3

1 $-\frac{62}{9}$ 2 $\frac{0}{1}$

4

1 $\frac{3}{10}$ "Rational number".

2 $\frac{5}{0}$ "Not rational number".

3 $\frac{0}{3}$ "Rational number".

Worksheet (3)

1

1 zero   2 $\frac{4}{9}$ 3 4   4 40
5 commutative
6 the additive neutral element

2

1 a   2 a   3 d   4 c

3

1 2   2 1   3 $-\frac{16}{5}$ 4 1

4

1 $-\frac{1}{3}$ 2 1

Worksheet (4)

1

1 1   2 $\frac{5}{7}$ 3 $-\frac{1}{2}$ 4 -5
5 $-\frac{4}{3}$ 6 1   7 1   8 $\frac{2}{11}$

2

1 $-\frac{3}{8}$ 2 $-\frac{5}{12}$ 3 $1\frac{5}{9}$

3 1 zero   2 $\frac{2}{25}$

4

[a] $\frac{5}{12}$, $\frac{6}{12}$, $\frac{7}{12}$ (There are other solutions)

[b] $-\frac{29}{11}$

Worksheet (5)

1

1 c   2 c   3 b
4 a   5 a   6 d

2 [a] 1   [b] 10

3

[a] $-\frac{3}{4}$ [b] 1

4

$\frac{7}{8}$, $\frac{8}{8}$, $\frac{9}{8}$ (There are other solutions)

Worksheet 6

1

1 d 2 b 3 b
4 c 5 d 6 b

2

[a] 1 $5a^5b^3 - 3a^2b^5 + 7ab$
2 $3y^2 + 5xy^3 + 6x^2y - 4x^3y^4$
[b] 3

3

1 $\frac{4}{81}$ 2 $-\frac{7}{3}$

4 $\frac{7}{12}$

Worksheet 7

1

1 b 2 d 3 c 4 d
5 b 6 d 7 b 8 c

2

[a] $8ab^2$ [b] $7x + 14y$

3

[a] 5 [b] $y^3 + 4xy^2 + 4x^2y - z + 3$

Worksheet 8

1

1 b 2 c 3 b
4 a 5 b 6 c

2

[a] The remainder = $x^2 + 2$
The numerical value = 38
[b] -2

3

[a] $4x^2 + 12x - 9$
[b] $\frac{7}{9}$, $\frac{29}{36}$ (There are other solutions)

4

[a] $a + b - 2c$ [b] 1 -1 2 $\frac{1}{8}$

Worksheet 9

1

1 a 2 a 3 b 4 b
5 c 6 c 7 d 8 a

2

[a] $3a - 2b + 6c$ [b] $8x^5y^5$

3 The perimeter of the figure = $10x$
it is an algebraic term of first degree
The area of the figure = $5x^2$ it is an algebraic term of second degree

4 $-y + 6z$

Worksheet 10

1

1 a 2 b 3 c
4 b 5 d 6 b

2 The expression = $3n^2 + 26n$
The numerical value = -23

3

[a] $\frac{57}{140}$, $\frac{58}{140}$, $\frac{59}{140}$
(There are other solutions)
[b] $3x$

4

[a] $\frac{3}{7}$ [b] The remainder = $2x^2$

Worksheet 11

1

1 c 2 d 3 a
4 b 5 d 6 d

2

[a] The expression = $4a^2 - 2$
The numerical value = 2
[b] $5a - 7b - 2c$

3 [a] 5 [b] $\frac{7}{12}$

4

1 $\frac{3}{4}$ 2 $-\frac{5}{2}$

Worksheet 12

1

1 d 2 c 3 a
4 d 5 d 6 d

2

[a] $2x + 3y - 4xy^2$ [b] -25

3

[a] $4a^2 + 3ab - 6b^2$ [b] -4

4

[a] $\frac{7}{30}$, $\frac{4}{15}$ (There are other solutions)

[b] $(6a^2 + 4ab^2 - 8)$ cm.

Worksheet 13

1

1 b 2 d 3 c
4 a 5 b 6 d

2

1 $6x^4y$ 2 15
3 $6xy$ 4 $\frac{7}{24}$

3

[a] $x^2 + 2$

[b] $10x + 34$, the numerical value = 4

4

[a] zero [b] $8x^2 - 6x - 5$

Worksheet 14

1

1 d 2 a 3 d
4 b 5 b 6 a

2

1 $5a^3x^2(2a + 3a^2x^2 - 6x)$

2 The expression = $3(a - 2b)^2$

The numerical value = $\frac{1}{3}$

3

[a] 170 [b] -6

4

[a] $5a^2 - 12a + 5$ [b] $x + 5$

Worksheet 15

1

1 d 2 d 3 b
4 a 5 b 6 d

2 16 marks.

3

[a] $4x(4x^3 - x + 2)$ [b] zero

4

[a] $4a^2 + 3ab - 6b^2$ [b] $\frac{23}{40}$

Worksheet 16

1

1 a 2 c 3 c
4 a 5 b 6 c

2

[a] $-\frac{1}{2}$

[b] $\frac{4}{15}$, $\frac{5}{15}$ and $\frac{6}{15}$ (There are other solutions)

3

[a] 1 $3x + y - 2$ 2 $6x^2 + 10x - 9$

[b] The median number of hours of Sarah = 5 hours.
The median number of hours of Gamal = 6 hours.

Worksheet 17

1

1 c 2 d 3 b
4 c 5 b 6 a

2

[a] $3x^2 + 14x$ [b] 15

3

[a] $2x - 5$

[b] The mean = 12 , the median = 11 , the mode = 15

Answers of the school book models on Algebra and Statistics

Model 1

1

1 $\frac{5}{11}$ 2 27 3 -0.12

4 $3y^3$ 5 $7x$

2

1 c 2 a 3 a 4 b 5 b 6 d

3

[a] $x^2 + xy + 2y^2 - 1$ [b] $\frac{27}{7}$

4

[a] $4x^2 - 2$, the numerical value $= 2$

[b] $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$ (there are other solutions)

5

[a] $x^2 - 2$ [b] $39\frac{2}{3}$

Model 2

1

1 $4x^2y^3$ 2 $5x$ 3 13

4 4 5 $x + 3y$

2

1 c 2 c 3 c 4 a 5 b 6 a

3

[a] 3

[b] $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$ (there are other solutions)

4

[a] $5x - y$ [b] $2x - 5y + 1$

5

[a] The expression $= x^2$
, the numerical value $= 25$

[b] $k = 2$

Model examination for the merge students

1

1 second 2 $x + 3$ 3 zero

4 4 5 zero

2

1 a 2 c 3 b 4 a 5 d

3

[a] $\frac{5}{7}(8 + 5 + 1) = \frac{5}{7}(14) = 10$

[b] $b \div a = (-2) \div \left(\frac{1}{2}\right) = (-2) \times (2) = -4$

4

1 ✓ 2 ✗ 3 ✓ 4 ✓ 5 ✗

5

1 7 2 3 3 $7x$ 4 50 5 1

Answers of schools examinations on Algebra and Statistics

1 Cairo

1
1 b   2 c   3 b
4 b   5 c   6 c

2
1 $\frac{7}{23}$, $\frac{-7}{23}$ 2 6   3 $3x$
4 $12x^2+3x$ 5 6 , 7

3
[a] $2x^2+2xy-5y^2-7$

[b] $\frac{22}{25}\left(\frac{7}{11}+\frac{5}{11}-1\right)=\frac{22}{25}\times\frac{1}{11}=\frac{2}{25}$

[c] $3b^2-a+2a^2b^2$

4
[a] L.C.M. of the denominators = 12

$\frac{1}{3}=\frac{4}{12}$, $\frac{1}{4}=\frac{3}{12}$

$\frac{4}{12}=\frac{16}{48}$, $\frac{3}{12}=\frac{12}{48}$

The numbers are : $\frac{13}{48}$, $\frac{14}{48}$, $\frac{15}{48}$

(there are other solutions)

[b] $2x^2y(3y-2xy^3+1)$

[c] The expression $=4y^2+x^2-9y^2$

$=x^2-5y^2$

5
[a]

$$\begin{array}{r|l} x+2 & x^2+5x+6 \\ x+3 & \ominus x^2 \ominus 2x \\ \hline & 3x+6 \\ & \ominus 3x \ominus 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $=x+3$

[b] The mean $=\dfrac{3\frac{1}{2}+3+2\frac{1}{2}+2+4}{5}=3$ hours.

2 Cairo

1
1 b   2 d   3 a
4 c   5 a

2
1 0   2 third   3 $5x+3y$
4 $\frac{4}{7}$ 5 17

3
[a] $\frac{4}{19}(12+9-2)=\frac{4}{19}\times 19=4$

[b] $4m^2n+2mn^2-3$

[c] $2x^2z+5$

4
[a] 1 The expression $=5x^2-2x-15x+6$

$=5x^2-17x+6$

2 The expression $=x^2-2x+1+x^2-9$

$=2x^2-2x-8$

[b] L.C.M. of the denominators = 6

$\frac{1}{2}=\frac{3}{6}$, $\frac{1}{3}=\frac{2}{6}$

$\frac{3}{6}=\frac{9}{18}$, $\frac{2}{6}=\frac{6}{18}$

The numbers are : $\frac{7}{18}$, $\frac{8}{18}$

(there are other solutions)

5
[a] $5l^2m(lm-6)$

[b] The mode = 9 marks

[c] $4c-ab=4\times\frac{1}{2}-(-2)\times 3=2+6=8$

3 Cairo

1
1 c   2 c   3 c
4 a   5 c   6 d

2
1 3 , 9   2 third   3 27
4 $2xy$ 5 -0.12

3
[a] The expression $=4x^2-25+16$

$=4x^2-9$

The numerical value $=4\times(-3)^2-9=36-9=27$

[b] The sum $=-16x^3y^2+8x^2y^3+24x^2y^2$

The quotient $=-2x+y+3$

4

[a] $\frac{22}{7}\left(\frac{5}{7}+\frac{1}{5}+\frac{3}{35}\right)=\frac{22}{7}\times\frac{35}{35}=\frac{22}{7}$

[b] $x+\frac{1}{5}=\frac{2}{5}$ Therefore $x=\frac{1}{5}$

or $x+\frac{1}{5}=\frac{-2}{5}$ Therefore $x=\frac{-3}{5}$

[c] The fraction $=\frac{48}{56}=\frac{6}{7}$

5

[a] $\frac{35\,(35-4+1)}{32\,(32+3)}=\frac{35\times 32}{32\times 35}=1$

[b] The remainder $=(2x^2+5x+2)-(x^2-6x+9)$

$=2x^2+5x+2-x^2+6x-9$

$=x^2+11x-7$

The numerical value $=1^2+11\times 1-7=5$

[c] The mode $=60$

The mean $=$

$\frac{(20\times 18)+(30\times 20)+(40\times 22)+(50\times 25)+(60\times 27)+(70\times 8)}{120}$

$=\frac{360+600+880+1250+1620+560}{120}$

$=43.91\dot{6}$

4 Giza

1 (1) fourth (2) $+x$ (3) $\neq 3$
(4) 60 (5) $2xy$

2 (1) c (2) d (3) c
(4) a (5) b (6) c

3

[a] $4a-9b+9c$

[b] 0

[c] $3m^3-9mn+6m$

4

[a] $\left(\frac{16}{7}\div\frac{4}{7}\right)\left(\frac{12}{4}-\frac{1}{4}\right)=\left(\frac{16}{7}\times\frac{7}{4}\right)\left(\frac{11}{4}\right)$

$=4\times\frac{11}{4}=11$

[b] The experssion $=4x^2-9+7=4x^2-2$

The numerical value $=4\times(-2)^2-2=14$

5

[a]
$$\begin{array}{r|l}
x+3 & x^3+x^2-10x-12 \\
x^2-2x-4 & \ominus\ \ \ominus \\
 & x^3+3x^2 \\ \hline
 & -2x^2-10x-12 \\
 & \oplus\ \ \oplus \\
 & -2x^2-6x \\ \hline
 & -4x-12 \\
 & \oplus\ \ \oplus \\
 & -4x-12 \\ \hline
 & 0\ \ \ 0
\end{array}$$

The quotient $=x^2-2x-4$

[b] The mean $=\frac{30+35+40+35+40}{5}=36$

5 Giza

1 (1) a (2) b (3) c
(4) b (5) c (6) a

2 (1) 1 (2) 75 (3) 4
(4) 0 , 1 (5) 11

3

[a] $\frac{23}{45}\left(\frac{7}{12}+\frac{17}{12}-2\right)=\frac{23}{45}\times 0=0$

[b] L.C.M. of the denominators = 12

$\frac{1}{2}=\frac{3}{6}$, $\frac{1}{3}=\frac{2}{6}$

$\frac{3}{6}=\frac{12}{24}$, $\frac{2}{6}=\frac{8}{24}$

The numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

(there are other solutions)

[c] x^2+9x-6

4

[a] $4x^2+3xy-2y^2$

[b] $2x+3y-4xy^2$

[c] The experssion $=x^2+4x+4-x^2+4=4x+8$

5

[a] $5xy\,(2x-y)$

[b] The mean $=\frac{2+5+3+6+4}{5}=4$

6 Alexandria

1 1 zero 2 1 3 fourth 4 100 5 $(a+b)^2$

2 1 c 2 c 3 d 4 d 5 d 6 b

3

[a] L.C.M. of the denominators = 30

$\frac{1}{5}=\frac{6}{30}$, $\frac{7}{6}=\frac{35}{30}$

The numbers are : $\frac{7}{30}$, $\frac{8}{30}$, $\frac{9}{30}$, $\frac{10}{30}$

(there are other solutions)

[b] $3xy(5x-1+3y)$

[c] $-4x^2+xy+2y^2$

4

[a]

$$\begin{array}{r|l} 3x-4 & 3x^2+2x-8 \\ \hline x+2 & 3x^2-4x \\ & \quad 6x-8 \\ & \quad 6x-8 \\ & \quad 0 \quad 0 \end{array}$$

The quotient = $x+2$

[b] $3a^3-4a^2-8$

[c] The mean = $\frac{2+5+6+3}{4}=4$

5

[a] $\frac{3}{4}+\frac{1}{4}+\left(\frac{-1}{3}\right)+\left(\frac{-2}{9}\right)=\frac{3}{4}+\frac{1}{4}+\left(\frac{-3}{9}\right)+\left(\frac{-2}{9}\right)$

$=\left(\frac{3}{4}+\frac{1}{4}\right)+\left[\frac{-3}{9}+\left(\frac{-2}{9}\right)\right]=1+\left(\frac{-5}{9}\right)=\frac{4}{9}$

[b] The area = $(x+5)(x+5)=(x^2+10x+25)$

The perimeter = $4(x+5)=4x+20$

[c] The ascending order is :

6 , 7 , 8 , 9 , 10 , 11 , 12 , 13 , 14

The median = 10

7 Alexandria

1 1 c 2 a 3 c 4 b 5 d 6 c

2 1 6 2 −2 3 $-12x$ 4 -1 , $\frac{1}{2}$ 5 5

3

[a] The distance between the numbers = $\left|\frac{7}{4}-\frac{4}{7}\right|=\frac{33}{28}$

, the number = $\frac{4}{7}+\frac{1}{3}\times\frac{33}{28}=\frac{27}{28}$

[b] The sum = x^2+7x+3

The increase = $3x^2-7x-5-(x^2+7x+3)$

$=2x^2-14x-8$

4

[a] 1 $6a^2b(2+3ab)$

2 $\frac{23}{45}\left(\frac{7}{12}+\frac{17}{12}-2\right)=\frac{23}{45}\times 0=0$

[b] The experssion = $3-6x-x^2+5x-3=-x^2-x$

The numerical value = $-(-1)^2-(-1)$

$=-1+1=0$

5

[a] $a+2b+c=a+b+b+c=\frac{5}{4}+\frac{3}{4}=\frac{8}{4}=2$

[b] The mean = $\frac{30+35+42+38+44+51}{6}=40$

The ascending order is :

30 , 35 , 38 , 42 , 44 , 51

The median = $\frac{38+42}{2}=40$

8 El-Kalyoubia

1 1 c 2 d 3 d 4 d 5 c

2 1 9 2 zero 3 4 4 3 5 fourth

3

[a] $5a-2b+2c$ [b] $2x+3y$

[c] The number = $\left(\frac{1}{2}+\frac{5}{8}\right)\div 2=\frac{9}{8}\times\frac{1}{2}=\frac{9}{16}$

4

[a] $\frac{2}{7}\left(\frac{1}{3}+\frac{3}{4}-1\right)=\frac{2}{7}\times\frac{1}{12}=\frac{1}{42}$

[b] $3x(x+5y)$

5

[a] The expression $= x^2 - 4$

The numerical value $= 3^2 - 4 = 9 - 4 = 5$

[b] $\frac{27 + 8 + 16 + 24 + 6 + k}{6} = 14$

$81 + k = 84 \qquad k = 3$

9 El-Sharkia

1 1 a 2 a 3 a 4 c 5 a 6 a

2 1 5 2 fifth 3 10 4 $\frac{4}{7}$ 5 $-\frac{3}{5}$

3

[a] L.C.M. of the denominators = 6

$\frac{1}{2} = \frac{3}{6} , \frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} = \frac{12}{24} , \frac{2}{6} = \frac{8}{24}$

The numbers are : $\frac{9}{24} , \frac{10}{24} , \frac{11}{24}$

(There are other solutions)

[b] $4ab(3a^2b + 4ab^2 - 1)$

4

[a] $5a - 2b$

[b]
$$\begin{array}{r|l} x+5 & x^2 + 6x + 5 \\ x+1 & \ominus x^2 \ominus 5x \\ \hline & x + 5 \\ & \ominus x \ominus 5 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x + 1$

5

[a] $\frac{3}{11}(5 + 7 - 1) = \frac{3}{11} \times 11 = 3$

[b] 1 The mean $= \frac{30 + 34 + 32 + 36 + 35 + 25}{6} = 32$

2 The asceneling order is :

25 , 30 , 32 , 34 , 35 , 36

The median $= \frac{32 + 34}{2} = 33$

10 El-Monofia

1 1 d 2 d 3 b 4 d 5 d 6 c

2 1 13 2 6 3 $\frac{-3}{7}$ 4 $\frac{2}{7}$ 5 7

3

[a] $-x + 3y + 4z$

[b] $\frac{7}{9}\left(\frac{3}{7} + \frac{4}{7} - 1\right) = \frac{7}{9} \times 0 = 0$

4

[a] $3x(2x^2 + 1)$

[b] The length $= (2x^2 + x + 1)$ cm.

5

[a] The expression $= x^2 + 4x + 4 + x^2 - 1$

$= 2x^2 + 4x + 3$

The numerical value $= 2 \times (1)^2 + 4 \times 1 + 3$

$= 2 + 4 + 3 = 9$

[b] 1 The ascending order is :

45 , 59 , 59 , 61 , 65 , 70

The median $= \frac{59 + 61}{2} = 60$

2 The mode = 59

11 El-Dakahlia

1 1 d 2 c 3 a 4 b 5 a 6 d

2 1 1 , sixth 2 $\frac{-3}{7}$ 3 4 4 $\frac{9}{16}$ 5 -1

3

[a] The area $= (3x + 2y)(x + 3y)$

$= (3x^2 + 11xy + 6y^2)$ cm^2

$= 3 \times (2)^2 + 11 \times 2 \times 1 + 6 \times (1)^2$

$= 3 \times 4 + 22 + 6 = 40$ cm^2

[b] $\frac{5}{17}(10 + 23 + 1) = \frac{5}{17} \times 34 = 10$

4

[a] Dividing $2x^3 - 5x^2 - 22x - 15$ by $2x + 3$:

$$\begin{array}{r|l} 2x+3 & 2x^3 - 5x^2 - 22x - 15 \\ x^2 - 4x - 5 & \ominus\ 2x^3 + 3x^2 \\ \hline & -8x^2 - 22x - 15 \\ & \oplus\ -8x^2 - 12x \\ \hline & -10x - 15 \\ & \oplus\ -10x - 15 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x^2 - 4x - 5$

[b] The distance between the two numbers

$= \left|\frac{7}{4} - \frac{4}{7}\right| = \frac{33}{28}$

, the number $= \frac{4}{7} + \frac{1}{3} \times \frac{33}{28} = \frac{27}{28}$

5

[a] $(2x + y)(4m^2 - 3m - 7)$

[b] $\frac{8+7+5+9+4+3+k+2}{7} = 6$

$k + 38 = 42 \qquad k = 4$

12 Ismailia

1

1 b	2 d	3 b
4 d	5 b	6 d

2

1 -1	2 6	3 80
4 3	5 5 a	

3

[a] Dividing $x^2 + 5x + 6$ by $x + 3$:

$$\begin{array}{r|l} x+3 & x^2 + 5x + 6 \\ x+2 & \ominus\ x^2 + 3x \\ \hline & 2x + 6 \\ & \ominus\ 2x + 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x + 4$

[b] L.C.M. of the denominators = 8

$\frac{1}{2} = \frac{4}{8} = \frac{12}{24}$, $\frac{5}{8} = \frac{15}{24}$

The numbers are : $\frac{13}{24}$, $\frac{14}{24}$

(there are other solutions)

4

[a] $\frac{3}{7}(10 + 5 - 1) = \frac{3}{7} \times 14 = 6$

[b] $4a + 2b - 2c$

5

[a] The expression $= 4x^2 + 12x + 9 - 12x = 4x^2 + 9$

The numerical value $= 4(-2)^2 + 9$

$= 4 \times 4 + 9 = 25$

[b] The mode = 8 marks.

13 Damietta

1

1 c	2 c	3 b
4 b	5 b	6 c

2

1 -5	2 $\frac{-2}{3}$	3 the mode
4 0	5 4	

3

[a] $7x - 3y + 2$

[b] L.C.M. of the denominators = 15

$\frac{4}{5} = \frac{12}{15}$, $\frac{2}{3} = \frac{10}{15}$

$\frac{12}{15} = \frac{48}{60}$, $\frac{10}{15} = \frac{40}{60}$

The numbers are : $\frac{41}{60}$, $\frac{42}{60}$, $\frac{43}{60}$, $\frac{44}{60}$

(there are other solutions)

[c] $6x^2 + 5xy - 21y^2$

4

[a] $\frac{-3}{7}(8 + 5 + 1) = \frac{-3}{7} \times 14 = -6$

[b] $5x(2x^3 + 4x^2 - 3)$

5

[a] The ascending order is :

3 , 3 , 4 , 4 , 4 , 5 , 5 , 6 , 7 , 9

, the median $= \frac{4+5}{2} = 4.5$

, the mode = 4

[b] Dividing $x^2 + 5x + 6$ by $x + 3$:

$$\begin{array}{r|l} x+3 & x^2 + 5x + 6 \\ x+2 & \ominus\ x^2 + 3x \\ \hline & 2x + 6 \\ & \ominus\ 2x + 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x + 2$

14 El-Fayoum

1
1 5 2 15 3 4
4 third , – 2 5 5

2
1 a 2 a 3 b
4 a 5 c 6 b

3
[a] $\frac{6}{37}(7+5-11)=\frac{6}{37}\times 1=\frac{6}{37}$
[b] $9x^3(3x-2)$

4
[a] 1 $2x^2+3y^2$ 2 x^3+1
[b] $2x^2-9$

5
[a] $(x+y)-(z+y)=\left(\frac{2}{3}\div\frac{1}{6}\right)-(-3\div 6)$
$=\left(\frac{2}{3}\times 6\right)-(-3\times 6)$
$=4+18=22$
[b] $\frac{8+7+5+9+4+3+k+4}{7}=6$
$k+40=42$ $k=2$

15 Luxor

1
1 c 2 b 3 d
4 a 5 b 6 c

2
1 315.7 2 $\frac{5}{11}$ 3 4.
4 $3y^3$ 5 0

3
[a] $\frac{5}{7}(8+5+1)=\frac{5}{7}\times 14=10$
[b] $2x^2-5y+1$

4
[a] L.C.M. of the denominators = 6
$\frac{1}{3}=\frac{2}{6}$, $\frac{1}{2}=\frac{3}{6}$
$\frac{2}{6}=\frac{8}{24}$, $\frac{3}{6}=\frac{12}{24}$
The numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$
(there are other solutions)
[b] $3m^2n^2(m^2-2mn+3n^2)$

5
[a] $5x+3y+4$
[b] 1 The mode = 4
2 The ascending order is : 3 , 4 , 5 , 7 , 8
The median = 5

Answers of the worksheets on Geometry

Worksheet (1)

1

1 b 2 b 3 b 4 c

2

1 an obtuse 2 300° 3 180° 4 complementary

3

1 Yes , give the reason by yourself
2 No , give the reason by yourself

4 30°

Worksheet (2)

1

1 equal in measure 2 supplementary
3 25° 4 90°

2 1 b 2 d 3 c 4 c

3 1 45° 2 45° 3 135°

4 115°

Worksheet (3)

1

1 b 2 d 3 b 4 d

2

1 $\overline{FE}$ 2 3 3 A
4 110° 5 90° 6 24

3

1 are equal in measure 2 LMN
3 45° 4 100°

4 70°

Worksheet (4)

1

1 c 2 b 3 a 4 d

2

1 zero° 2 180° , 360° 3 X 4 360°

3 Yes , $\Delta ABD \equiv \Delta ACD$, give the reason by yourself

4 150°

Worksheet (5)

1

1 equal in measure 2 80°
3 parallel 4 supplementary

2

1 b 2 a 3 b 4 b

3 110°

4 Yes , $\overline{CD} \parallel \overline{EF}$, give the reason by yourself

Worksheet (6)

1

1 d 2 a 3 d 4 b

2

1 on the same straight line.
2 the perpendicular to it from its midpoint
3 supplementary
4 20°

3 Draw by yourself.

4 Draw by yourself.

Answers of the school book models on Geometry

Model 1

1

1 axis of symmetry of it
2 40°
3 255°
4 40°
5 hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

2

1 b 2 d 3 d 4 b 5 a 6 c

3

[a] The conditions of congruency of the two right-angled traingles Δ ABD and Δ CBD are

$\begin{cases} AB = CB \\ \overline{BD} \text{ is a common hypotenuse} \end{cases}$

, CD = AD = 3 cm.

[b] AY = 6 cm.

4

[a] m (∠ ACE) = 95° [b] m (∠ BMC) = 120°

5

[a] The conditions that make Δ AMB ≡ Δ DMC are :

$\begin{cases} AM = DM \\ BM = CM \\ m(\angle AMB) = m(\angle DMC) \text{ (V.O.A.)} \end{cases}$

[b]

Model 2

1

1 360°
2 equal in measure
3 250°
4 hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.
5 supplementary.

2

1 a 2 d 3 a 4 b 5 b 6 a

3

[a] Mention by yourself.

[b] Prove by yourself , m (∠ ABD) = 60°

4

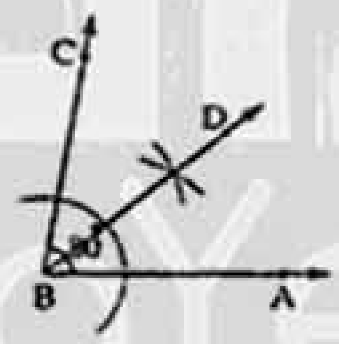
[a] m (∠ C) = m (∠ D) = 70° "Alternate angles"

Yes : $\overleftrightarrow{AB} \,/\!/\, \overline{CD}$

because : m (∠ A) + m (∠ C) = 110° + 70° = 180°

(Two interior angles in the same side of the transversal).

[b]



5

[a] X = 65° [b] m (∠ A) = 85°

Model examination for the merge students

1

1) 260° 2) 40° 3) parallel

4) the included angle between them of one triangle are congruent to the corresponding parts of the other triangle.

5) C

2

1) d 2) a 3) c 4) a 5) a

3

1) ✗ 2) ✓ 3) (a) ✗ (b) ✓ (c) ✗

4

[a] $\overrightarrow{BA} // \overrightarrow{CD}$

, then m (∠ ABC) = m (∠ BCD) "alternate angles"

, m (∠ BCD) = 50°

[b] 1) DCM 2) 5 3) C

5

1) 60° 2) 80° 3) 40° 4) 50° 5) 1

Answers of schools examinations on Geometry

1 Cairo

1

1 d 2 d 3 c 4 d 5 c 6 b

2

1 70° 2 150° 3 25°

4 the hypotenuse and a side in one triangle are congruent to the corresponding parts in the other triangle.

5 45°

3

[a] $\Delta ABC \equiv \Delta ADC$

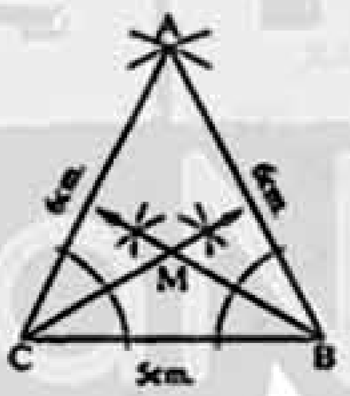
because $\begin{cases} AB = AD \\ m(\angle CAB) = m(\angle CAD) \\ \overline{AC} \text{ is a common side} \end{cases}$

and we deduce that

$m(\angle ACD) = m(\angle ACB) = 180° - (100° + 30°) = 50°$

, CD = CB = 7 cm.

[b]



4

[a]

Fig. 1 since : $\overleftrightarrow{CB} \parallel \overleftrightarrow{DH}$

, then $m(\angle D) = 180° - 80° = 100°$ (interior angles)

, since : $\overleftrightarrow{CD} \parallel \overleftrightarrow{HK}$

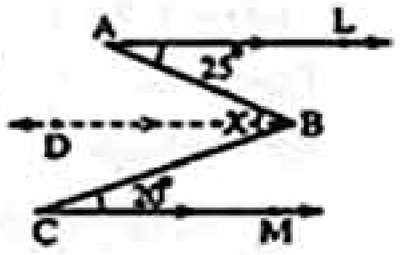
, then : $m(\angle H) = m(\angle D) = 100°$ (alternate angles)

, since : $\overleftrightarrow{DH} \parallel \overleftrightarrow{KL}$

, then : $m(\angle K) = 180° - 100° = 80°$ (interior angles)

then $X = 80°$

Fig. 2



Constr. : Draw $\overrightarrow{BD} \parallel \overrightarrow{AL} \parallel \overrightarrow{CM}$

Solution : Since : $\overrightarrow{AL} \parallel \overrightarrow{BD}$

, then $m(\angle ABD) = m(\angle A)$

$= 25°$ (alternate angles)

, since : $\overrightarrow{BD} \parallel \overrightarrow{CM}$

, then $m(\angle DBC) = m(\angle C) = 20°$ (alternate angles)

, then $m(\angle ABC) = 25° + 20° = 45°$

[b] $\Delta ADC \equiv \Delta ADB$

because $\begin{cases} AC = AB \\ CD = BD \\ \overline{AD} \text{ is a common side} \end{cases}$

and we deduce that :

$m(\angle CAD) = m(\angle BAD) = 50°$

, $m(\angle BAC) = 50° + 50° = 100°$

5

[a] $m(\angle C) = m(\angle D) = 70°$ "Alternate angles"

Yes : $\overrightarrow{AB} \parallel \overline{CD}$

because : $m(\angle A) + m(\angle C) = 110° + 70° = 180°$

(Two interior angles in the same side of the transversal)

[b] $m(\angle BMD) = 180° - 50° = 130°$

Since : $\overrightarrow{MC}$ bisects $\angle BMD$

Then : $m(\angle BMC) = 130° \div 2 = 65°$

$m(\angle AMB) = m(\angle DMH) = 50°$ (V. O. A)

$m(\angle AMC) = 50° + 65° = 115°$

2 Cairo

1

1 b 2 d 3 d 4 b 5 c 6 d

2

1 200° 2 equal 3 congruent

4 parallel 5 8

3

[a] $m(\angle CHD) = 360° - (90° + 110° + 30°) = 130°$

[b] $m(\angle A) = 180° - 127° = 53°$ (interior angles)

Since : $m(\angle A) = m(\angle CBH) = 53°$

and they are corresponding angles

then : $\overline{BC} \parallel \overline{AD}$

4

[a] Since : $\overrightarrow{AE}$ bisects $\angle BAH$

Then : $m(\angle BAH) = 2 \times 58° = 116°$

$m(\angle B) = m(\angle BAH) = 116°$ (alternate angles)

$m(\angle C) = 180° - m(\angle B) = 180° - 116° = 64°$

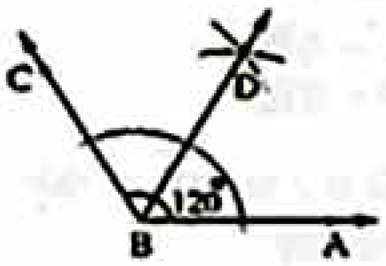
(interior angles)

[b] $\Delta AMC \equiv \Delta BMD$

because $\begin{cases} AM = BM \\ m(\angle AMC) = m(\angle BMD) \text{ (V.O.A)} \\ CM = DM \end{cases}$

5

[a]



[b] $\Delta ABD \equiv \Delta CBD$ because $\begin{cases} AD = CD \\ \overline{BD} \text{ is a common hypotenuse} \\ m(\angle A) = m(\angle C) = 90° \end{cases}$

and we deduce that

$BC = BA = 5$ cm.

, $m(\angle CBD) = m(\angle ABD) = 31°$

3 Cairo

1

1 c 2 d 3 b 4 c 5 a 6 a

2

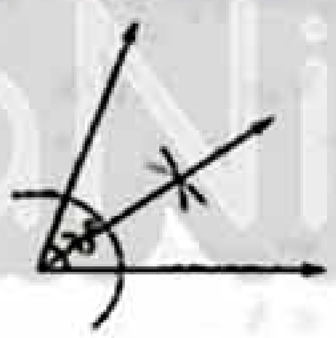
1 on the same straight line 2 equal in measure

3 the perpendicular from its midpoint

4 parallel 5 90°

3

[a]



[b] $m(\angle A) = 22.5°$

4

[a] $m(\angle ABC) = 360° - (150° + 90°) = 120°$

[b] $\Delta ABD \equiv \Delta ACD$

because $\begin{cases} AB = AC \\ BD = CD \\ \overline{AD} \text{ is a common side} \end{cases}$

and we deduce that

$m(\angle BAD) = m(\angle CAD) = 30°$

$m(\angle CAB) = 30° + 30° = 60°$

5

[a] Answer by yourself.

[b] $m(\angle C) = m(\angle B) = 55°$ (alternate angles)

Since : $m(\angle C) + m(\angle D) = 55° + 125° = 180°$

and they are interior angles

then : $\overline{BC} \parallel \overline{DE}$

4 Giza

1

1 a 2 b 3 d 4 d 5 a 6 d

2

1 equal in measure 2 an obtuse

3 the hypotenuse , one side

4 100° 5 70°

3

[a] $m(\angle ACD) = m(\angle A) = 50°$ (alternate angles)

Since : $\overrightarrow{CD}$ bisects $\angle ACE$

Then : $m(\angle DCE) = m(\angle ACD) = 50°$

, $m(\angle CEF) = 180° - m(\angle DCE)$

$= 180° - 50° = 130°$ (interior angles)

[b] $m(\angle CMD) = 360° - (70° + 90° + 150°) = 50°$

4

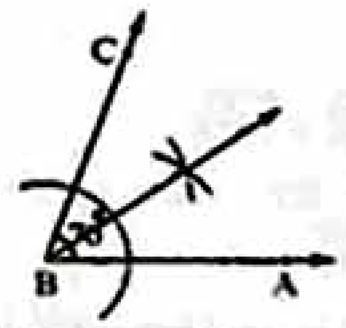
[a] $\Delta ABC \equiv \Delta ADC$

because $\begin{cases} AB = AD \\ BC = DC \\ \overline{AD} \text{ is a common side} \end{cases}$

and we deduce that

$m(\angle D) = m(\angle B) = 35°$

[b]



5

[a] equal in measure

[b] Since : $\overline{AB} \parallel \overline{CD} \parallel \overline{EF} \parallel \overline{GH}$

, $BD = DF = FH$

Then : $AC = CE = EG = \frac{21}{3} = 7$

$AE = 14$ cm.

$m(\angle ABD) = 180° - m(\angle BDC)$

$= 180° - 70° = 110°$ (interior angles)

, m (∠ DFE) = m (∠ BDC)

= 70° (corresponding angles)

m (∠ HFL) = m (∠ DFE) = 70° (V. O.A)

5 Giza

1

1 equal in measure 2 270° 3 135°

4 the included angle 5 supplementary

2

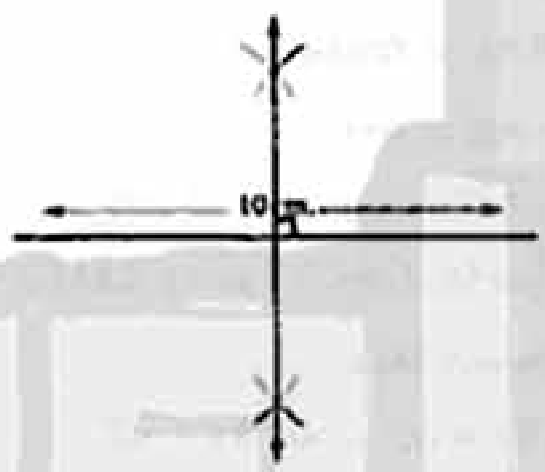
1 d 2 b 3 b

4 b 5 c

3

[a] m (∠ XMY) = 360° – (120° + 90°) = 150°

[b]



4

[a] m (∠ L) = m (∠ Z) = 60° (alternate angles)

, m (∠ M) = 180° – m (∠ L)

= 180° – 60° = 120° (interior angles)

[b] Since : $\overline{AF} // \overline{DX} // \overline{EY} // \overline{BC}$

, AX = XY = YC

Then : AD = DE = EB = 5 cm.

, AB = 3 EB = 15 cm.

5

[a] m (∠ ABD) = m (∠ DBE) = m (∠ EBF)

$= \frac{180° - 30°}{3} = 50°$

, m (∠ ABE) = 50° + 50° = 100°

[b] Δ LME ≡ Δ NME

because $\begin{cases} LM = NM \\ \overline{ME} \text{ is a common side} \\ (m (\angle L) = m (\angle N)) \end{cases}$

and we deduce that

NE = LE = 3 cm.

6 Alexandria

1

1 245° 2 360° 3 perpendicular

4 axis of symmetry

5 the hypotenuse and one side in one of them are congruent to their corresponding elements in the other triangle.

6 45°

2

1 c 2 d 3 b 4 b 5 d

3

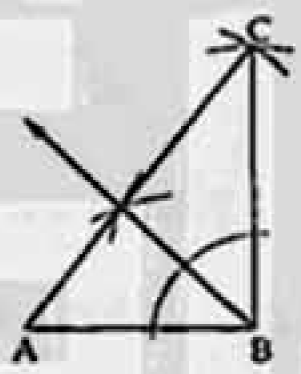
[a] In Δ BCD : m (∠ C) = 180° – (120° + 35°) = 25°

Since : $\overline{BD}$ bisects ∠ B

Then : m (∠ B) = 2 × 35° = 70°

In Δ ABC : m (∠ A) = 180° – (70° + 25°) = 85°

[b]



4

[a] The two right-angled triangles ABD , CBD are congruent

because $\begin{cases} AB = CB \\ \overline{BD} \text{ is a common side} \end{cases}$

, CD = AD = 4 cm.

[b] m (∠ DEC) = 360° – (110° + 90° + 30°) = 130°

5

[a] 1 75° 2 100°

[b] Since : $\overline{AD} // \overline{XY} // \overline{CB}$

, AY = YC

Then : AX = XB = 3 cm.

7 Alexandria

1

1 c 2 a 3 a

4 d 5 c 6 a

2

1 140° 2 230° 3 equal in measure.
4 12 5 60°

3

[a] 1 m (∠ DBC) = 180° − 50° = 130°

2 m (∠ ABE) = m (∠ DBC) = 130° (V.O.A)

3 Since : $\overrightarrow{BF}$ bisects ∠ ABE

Then : m (∠ ABF) = m (∠ FBE) = 130° ÷ 2 = 65°

m (∠ EBC) = m (∠ ABD) = 50° (V.O.A)

m (∠ FBC) = 65° + 50° = 115°

4 m (∠ DBF) = 65° + 50° = 115°

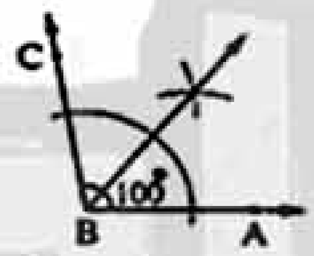
[b] Since : $\overline{AF}$ // $\overline{ED}$ // $\overline{YX}$ // $\overline{CB}$

, AD = DX = XB Then : AE = EY = YC = 3 cm.

, EY = 6 cm.

4

[a]



[b] m (∠ CBD) = 180° − (100° + 30°) = 50°

Yes , Δ CBD ≡ Δ ABD

because $\begin{cases} CB = AB \\ CD = AD \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that :

m (∠ ABD) = m (∠ CBD) = 50°

5

[a] m (∠ ACD) = m (∠ CAB) = 70° (alternate angles)

, m (∠ DCH) = 180° − m (∠ H) = 180° − 100° = 80° (interior angles)

, m (reflex ∠ ACH) = 360° − (70° + 80°) = 210°

[b] Δ ABD ≡ Δ ACD (two sides and included angle)

m (∠ ABD) = m (∠ ACD) = 40°

8 El-Kalyoubia

1

1 210° 2 180° 3 XY
4 130° 5 equal

2

1 c 2 a 3 b 4 a 5 c

3

[a] Each two alternate angles are equal in measure

, each two corresponding angles are equal in measure

, each two interior angles in the same side of the transversal are supplementry.

[b] 1 m (∠ CEF) = m (∠ ABC) = 83° (alternate angles)

2 m (∠ DCE) = m (∠ ABC) = 83° (corresponding angles)

4

[a] Mention by yourself.

[b] m (∠ ACB) = 180° − (100° + 30°) = 50°

Yes , Δ BAC ≡ Δ DAC

because $\begin{cases} AB = AD \\ \overline{AC} \text{ is a common side} \\ m (\angle BAC) = m (\angle DAC) = 30° \end{cases}$

and we deduce that :

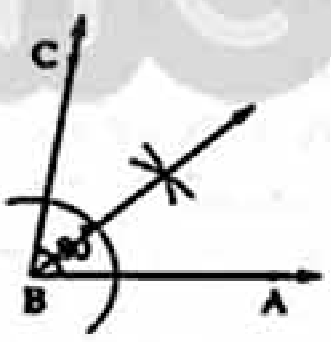
m (∠ ACD) = m (∠ ACB) = 50°

CD = CB = 7 cm.

5

[a] m (∠ CED) = 360° − (100° + 90° + 50°) = 120°

[b]



9 El-Gharbia

1

1 b 2 c 3 d
4 b 5 d 6 c

2

1 m (∠ BAC) = m (∠ DAE) 2 74°
3 perpendicular 4 8 5 0

3

[a] Yes , $\Delta ALC \equiv \Delta BMD$

because $\begin{cases} AL = BM \\ LC = MD \\ AC = BD \end{cases}$

and we deduce that : $m(\angle A) = m(\angle MBD)$

and they are corresponding angles , then $\overline{AL} \parallel \overline{BM}$

[b] 1 $m(\angle C) = m(\angle A) = 60°$ (alternate angles)

2 Yes $\overline{AC} \parallel \overline{DE}$, because

$m(\angle C) + m(\angle D) = 60° + 120° = 180°$

and they are interior angles.

4

[a] Since : $\overrightarrow{BD}$ bisects $\angle ABC$

Then : $m(\angle ABD) = m(\angle CBD)$

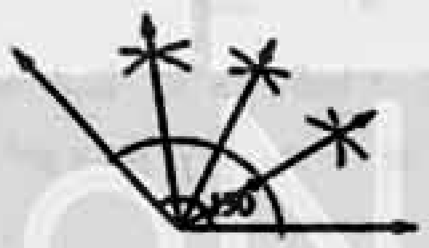
1 Yes , $\Delta ABD \equiv \Delta CBD$

because $\begin{cases} m(\angle ABD) = m(\angle CBD) \\ \overline{BD} \text{ is a common side} \\ m(\angle ADB) = m(\angle CDB) \end{cases}$

2 $CB = AB = 8$ cm.

, $AD = CD = 6$ cm,

[b]



5

[a] $m(\angle CAD) = 360° - (90° + 120°) = 150°$

[b] $m(\angle CBE) = m(\angle C) = 110°$ (alternate angles)

Since : $\overrightarrow{BF}$ bisects $\angle CBE$

Then : $m(\angle FBE) = 110° \div 2 = 55°$

$m(\angle A) = m(\angle FBE) = 55°$ (corresponding angles)

10 El-Dakahlia

1

1 d   2 c   3 b   4 b   5 b   6 a

2

1 the included angle.

2 perpendicular.   3 $70°$

4 the perpendicular bisector of it.   5 $280°$

3

[a] $m(\angle ACB) = 180° - (30° + 40°) = 110°$

$\Delta ABC \equiv \Delta ADC$

because $\begin{cases} AB = AD \\ \overline{AC} \text{ is a common side} \\ m(\angle BAC) = m(\angle DAC) \end{cases}$

and we deduce that :

$m(\angle ACD) = m(\angle ACB) = 110°$

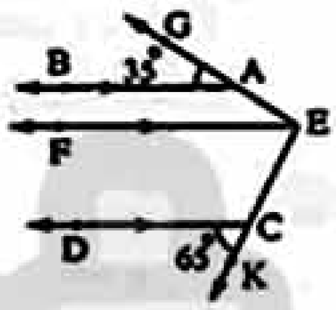
, $m(\angle BCD) = 360° - (110° + 110°) = 140°$

[b] Since : $\overleftrightarrow{AF} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, $AY = YE = EC$

Then : $AX = XD = DB$

$BC = 23 - (9 + 6) = 8$ cm.

4



[a] Draw $\overleftrightarrow{EF} \parallel \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$m(\angle AEF) = m(\angle GAB) = 35°$

(corresponding angles)

$m(\angle CEF) = m(\angle KCD) = 65°$

(corresponding angles)

$m(\angle AEC) = 35° + 65° = 100°$

[b] $\Delta AMC \equiv \Delta BMD$

because $\begin{cases} AM = BM \\ CM = DM \\ m(\angle AMC) = m(\angle BMD) \ (\text{V.O.A}) \end{cases}$

and we deduce that :

$m(\angle A) = m(\angle B)$ and they are alternate angles

, then $\overline{AC} \parallel \overline{BD}$

5

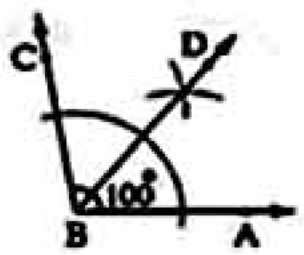
[a] $m(\angle BMD) = m(\angle AMC) = 40°$ (V.O.A)

Since : $\overrightarrow{MD}$ bisects $\angle BME$

Then : $m(\angle BME) = 2 \times 40° = 80°$

$m(\angle AME) = 180° - 80° = 100°$

[b]



11 Suez

1

1 d   2 c   3 a   4 b   5 a   6 c

2

1 equal in measure.   2 AC

3 the perpendicular bisector of it.

4 obtuse   5 45°

3

[a] Mention by yourself.

[b] The conditions of the two right-angled triangles ABD , CBD to be congruent are

$\begin{cases} AB = BC \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that : CD = AD = 3 cm.

4

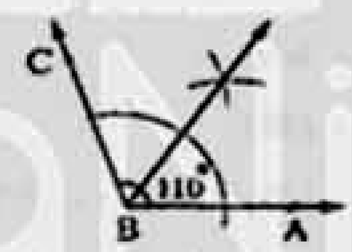
[a] m (∠ BMC) = 360° – (100° + 50° + 90°) = 120°

[b] m (∠ DCA) = m (∠ A) = 120°   (alternate angles)

m (∠ ECF) = 180° – (120° + 40°) = 20°

5

[a]



[b] m (∠ C) = 180° – (120° + 35°) = 25°

Since : $\overline{BD}$ bisects ∠ B

, then : m (∠ B) = 2 × 35° = 70°

m (∠ A) = 180° – (70° + 25°) = 85°

12 Port Said

1

1 b   2 d   3 c   4 a   5 b   6 c

2

1 25°   2 60°   3 M   4 equal   5 12

3

[a] Mention by yourself.

[b] Yes , Δ ADC ≡ Δ ADB

because $\begin{cases} AC = AB \\ DC = DB \\ \overline{AD} \text{ is a common side} \end{cases}$

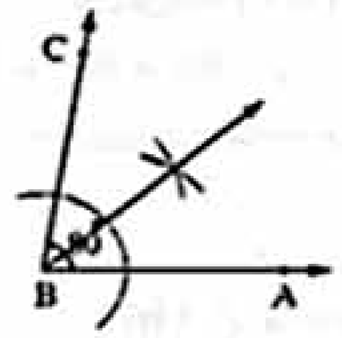
and we deduce that

m (∠ CAD) = m (∠ BAD) = 25°

m (∠ CAB) = 25° + 25° = 50°

4

[a]



[b] 1 m (∠ MYZ) = 180° – 130° = 50°

2 Yes , $\overrightarrow{YM}$ // $\overrightarrow{ZL}$

because : m (∠ Z) = m (∠ MYZ)

and they are alternate angles

5

[a] Yes , $\overrightarrow{CA}$, $\overrightarrow{CB}$ are on the same straight line

because , m (∠ ACD) + m (∠ DCE) + m (∠ ECB)

= 60° + 60° + 60° = 180°

[b] m (∠ C) = m (∠ EBA) = 75° (corresponding angles)

m (∠ D) = 180° – m (∠ C) = 180° – 75° = 105°

(interior angles)

3 x = 105° x = 35°

13 Kafr El-Sheikh

1

1 d   2 b   3 a   4 c   5 d   6 c

2

1 equal in measure   2 XC

3 110°   4 perpendicular

5 equal in measure

3

[a] m (∠ ABE) = 180° – m (∠ A) = 180° – 120°

= 60° (interior angles)

m (∠ EBF) = m (∠ F) = 40° (alternate angles)

m (∠ ABF) = 60° + 40° = 100°

[b] Since : $\overline{AF} \parallel \overline{DE} \parallel \overline{BC}$

, AD = BD

, then : AE = EC = 4 cm.

, then : AC = 8 cm.

4

[a] m (∠ BME) = 180° − 110° = 70°

Since : $\overline{MD}$ bisects ∠ EMB

, then : m (∠ BMD) = 70° ÷ 2 = 35°

, m (∠ CMB) = m (∠ AME) = 110° (V.O.A)

[b] Δ ABD ≡ Δ ACD

because $\begin{cases} AB = AC \\ BD = CD \\ \overline{AD} \text{ is a common side} \end{cases}$

and we deduce that

m (∠ BAD) = m (∠ CAD)

$\overline{AD}$ bisects ∠ BAC

5

[a]

[b] m (∠ C) = m (∠ B) = 30° (alternate angles)

Yes , $\overline{DE} \parallel \overline{CB}$

because

m (∠ C) + m (∠ CDE) = 30° + 150° = 180°

and they are interior angles.

14 El-Menia

1

1 c 2 b 3 b 4 a 5 b 6 d

2

1 280° 2 40° 3 360°

4 equal 5 equal in measure

3

[a] State by yourself

[b] m (∠ DBC) = 180° − (40° + 80°) = 60°

Yes , Δ CBD ≡ Δ ABD

because $\begin{cases} CB = AB \\ CD = AD \\ \overline{BD} \text{ is a common side} \end{cases}$

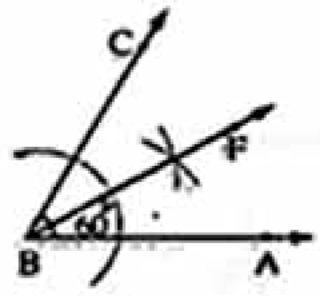
and we deduce that

m (∠ ABD) = m (∠ CBD) = 60°

4

[a] m (∠ BMC) = 360° − (110° + 90° + 40°) = 120°

[b]



5

[a] m (∠ C) = 180° − m (∠ A) (interior angles)

= 180° − 115° = 65°

Yes , $\overline{AC} \parallel \overline{DE}$

because m (∠ C) = m (∠ D) = 65°

and they are alternate angles

[b] m (∠ ABD) = 180° − 135° = 45°

Since $\overline{BA}$ bisects ∠ DBE

, then : m (∠ EBA) = m (∠ ABD) = 45°

, m (∠ DBE) = 45° + 45° = 90°

, m (∠ CBE) = 180° − 45° = 135°

15

1

1 260° 2 40° 3 parallel 4 C 5 360°

2

1 b 2 b 3 a 4 c 5 a

3

[a] m (∠ DBC) = 180° − (40° + 80°) = 60°

Δ CBD ≡ Δ ABD

because $\begin{cases} CB = AB \\ CD = AD \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that

m (∠ ABD) = m (∠ CBD) = 60°

[b] 150° , 90° , 240° , 120°

4

[a] m (∠ CBD) = 180° – 50° = 130°

$2x = 130°$ $\quad x = \frac{130}{2} = 65°$

[b] m (∠ C) = 180° – (120° + 35°) = 25°

Since : $\overline{BD}$ bisects ∠ B

Then : m (∠ B) = 2 × 35° = 70°

m (∠ A) = 180° – (70° + 25) = 85°

5

[a] Mention by yourself.

[b]